Abbreviations:

NRA = Normal Retirement Age

OFP = Optional Form of Payment

ERA = Early Retirement Age

NRB = Normal Retirement Benefit

NFP = Normal Form of Payment

ERB = Early Retirement Benefit

ERF = Early Retirement Factor

1) Age | Salary | (ee) Employee Contribution \( \leq 6\% \)

\[
\begin{align*}
25 & \quad 40000 \\
26 & \quad 40000 (1.03) \\
& \vdots \\
59 & \quad 40000 (1.03)^{34} \\
& \vdots \\
64 & \quad 40000 (1.03)^{39}
\end{align*}
\]

\[
\Rightarrow \text{company match} = \frac{1}{2} (\text{ee contribution})
\]

\[
\text{ee contribution} > 6\% \\
\Rightarrow \text{company match} = 3\%
\]

(a) Sue deposits 7\% \Rightarrow \text{company match} = 3\% \Rightarrow \text{total deposit} = 10\%

DC deposits

\[
\begin{array}{cccc}
\text{yrs} & \text{age} & \text{salary} & \text{AV} \\
0 & 25 & 40000 & 1(40000)(1.03)^{34} \\
1 & 26 & 40000 (1.03) & 1(40000)(1.03)^{33} \\
\vdots & \vdots & \vdots & \vdots \\
35 & 59 & 40000 (1.03)^{34} \\
\end{array}
\]

\[
\text{Note: Sue's final year salary} = 40000 (1.03)^{34}
\]

Converting to a benefit we get

\[
\begin{align*}
AV & = B_{60} \cdot \ddot{a}_{60} \\
\Rightarrow B_{60} & = \frac{AV}{11.1454} = \frac{58,286.82}{11.1454} = 5,286.82 \\
\therefore RR & = \frac{B_{60}}{\text{final yr salary}} = 533 (53.3\%)
\end{align*}
\]
1) (b) total deposit = 10% of salary each year as in (a)

\[ \text{Note: Sue's final year salary} = 40000 \times (1.03)^{39} \]

\[ \text{AV} = 1(40000)(1.03)^{39} + 1(40000)(1.03)^{38}(1.06) + \ldots \ (40 \text{ terms}) \]

\[ = 1(40000)(1.03)^{39} \left[ 1 + \frac{1.06}{1.03} + \ldots \right] = 1(40000)(1.03)^{39} S_{901}(1.03\cdot 1) \]

Converting to a benefit, we get \( \text{AV} = B_{65} \cdot \bar{A}_{65} \)

\[ \Rightarrow B_{65} = \frac{\text{AV}}{9.8969} = 94,624.65 \]

\[ \therefore \text{RR} = \frac{B_{65} \text{ final yr salary}}{\text{final yr salary}} = .747 \ (74.7\%) \]

(c) Sue deposits 5% \( \Rightarrow \) company match = 2.5%

\[ \Rightarrow \text{total deposit} = 7.5\% \text{ of salary each year} \]

We could go through the calculations in part (b) again, but it is quicker to recognize that we'll get the answer here by multiplying the answer in part (b) by \( \frac{.075}{.1} \).

\[ \therefore \text{RR} = \frac{.075}{.1} \times (.747) = .560 \ (56.0\%) \]
2) \[
\begin{array}{c|c}
\text{Age} & \text{Salary} \\
45 & 50000 \\
46 & 50000(1.04) \\
64 & 50000(1.04)^{19} = \text{Tan's final year salary}
\end{array}
\]

DC Account:

<table>
<thead>
<tr>
<th>Year</th>
<th>X</th>
<th>X</th>
<th>X</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>45</td>
<td>46</td>
<td>47</td>
<td>65</td>
</tr>
</tbody>
</table>

\[AV = X \cdot S_{30|1.09}^{65}\]

\[RR = 0.35 = \frac{\text{Annual Ret. Benefit}}{50000 \cdot (1.04)^{19}}\]

\[\Rightarrow \text{Annual Ret. Benefit} = B = 0.35 \cdot 50000 \cdot (1.04)^{19}\]

\[\text{Also } AV = X \cdot S_{30|1.09}^{65} = B \cdot \dddot{a}_{65}\]

\[\Rightarrow X \cdot S_{30|1.09}^{65} = 0.35 \cdot 50000 \cdot (1.04)^{19} \cdot 10 = B \cdot \dddot{a}_{65}\]

\[\Rightarrow X = 7206.76\]

Since the company matches dollar for dollar up to 2500, Tan needs to deposit \[X - 2500 = 4706.76\] each year.
3) As in #2, \( X = 7206.76 \). For Tom to get the maximum company match, he would have to deposit 5000, but then the total deposit would be \( 5000 + 2500 = 7500 > X \).

Since for every 3 total dollars deposited, Tom contributes 2 and the company contributes 1, the company match accounts for \( \frac{1}{3} \) of the total deposit and Tom’s contribution accounts for \( \frac{2}{3} \) of the total deposit.

\[ \therefore \text{Tom needs to deposit } \frac{2X}{3} = \frac{2 	imes 4804.51}{3} \] each year.

4) Let \( X = \) Omar’s monthly benefit

\[ \therefore 50000 \ \dd{a}_{65} = 12X \cdot \dd{a}_{65} \Rightarrow \dd{a}_{65} = \frac{9.8969}{X} \]

(a) \[ \dd{a}_{65} \overset{UPP}{=} \alpha \left( \dd{a}_{65} - \beta \right) \overset{ILT}{=} (1.00028)(9.8969) - 0.46812 \]

\[ \Rightarrow X = 4372.25 \]

(b) \[ \dd{a}_{65} \overset{2-term}{=} \frac{\dd{a}_{65} - \frac{I}{24}}{w} \Rightarrow X = 4369.00 \]

(c) \[ \dd{a}_{65} \overset{3-term}{=} \frac{\dd{a}_{65} - \frac{I}{24} - \frac{143}{1728} (\mu_{65} + s)}{w} \Rightarrow \frac{X}{2} = \frac{e^{-2 \mu_{65}}}{l_{66}} \]

\[ s = \ln(1.06) = \ln(1.06) \]

\[ \Rightarrow X = 4372.02 \]
5) Kim has 35 complete years of service.

(a) \( NR_B = 1800(35) = 63000 \) (payable annually in advance for life)

(b) \( LS = 63000 \cdot \ddot{a}_{65} = 63000 \cdot (9.8969) = 623,504.70 \)

(c) For 10-year C.f.L, \( 63000 \cdot \ddot{a}_{65} = X \cdot \ddot{a}_{65:101} = X(\ddot{a}_{101} + 10 \ddot{a}_{65}) \)

\[
\ddot{a}_{65} = 9.8969 \quad \ddot{a}_{75} = 7.217
\]

\[
10 \ddot{E}_{65} = 7.9994 \quad \ddot{a}_{101} \text{ TVA} = 7.8017
\]

\[ \Rightarrow X = 58336.53 \]

For Joint and 100% Survivor, \( 63000 \ddot{a}_{65} = X \ddot{a}_{65:65} \)

\[
\ddot{a}_{65:65} = \ddot{a}_{65} + \ddot{a}_{65} - \ddot{a}_{65:65} = 2(9.8969) - 7.8552
\]

\[ \Rightarrow X = 52225.95 \]

For Joint \( \frac{1}{2} \) 50% Survivor, \( 63000 \ddot{a}_{65} = X \ddot{a}_{65:65} + \frac{1}{2}X(\ddot{a}_{65} - \ddot{a}_{65:65}) \)

\[
\ddot{a}_{65:65} = 7.8552
\]

\[
\ddot{a}_{65} = 9.8969
\]

\[ \Rightarrow X = 57109.27 \]

For Joint \( \frac{1}{3} \) 33 1/3% Contingent, \( 63000 \ddot{a}_{65} = X \ddot{a}_{65:65} + \frac{1}{3}X(\ddot{a}_{65} - \ddot{a}_{65:65}) \)

\[ \Rightarrow X = 63000 \]

\( \because \) since both are same age, 65.

Remark: The Joint \( \frac{1}{2} \) 50% Survivor/Contingent terminology is not universal and not on the exam. They will explain the terminology as I did in this problem.
6) They both terminated employment with 3 yrs of service. According to Jim's plan, he is 0% vested. So for Jim, \( APV = 0 \).

According to Tim's plan, he is 60% vested. His vested benefit is \((0.6)(5000) = 3000\), and

\[
APV_{43}^{\text{Tim}} = 3000 \cdot E_{43}^{\text{Tim}} = 3000 \cdot 22 \cdot E_{43} \cdot \hat{a}_{65}^{\text{Tim}}
\]

\[
\hat{a}_{65}^{\text{LT}} = 9.8969 \quad \text{and} \quad q_{22}E_{43} = \nu_{22} \cdot P_{\nu 3} = \nu_{22} \cdot \frac{l_{65}}{l_{43}}
\]

\[
\Rightarrow APV_{43}^{\text{Tim}} = 6725.38
\]

7) William's annual benefit amount is

\[
B = 0.02S_1 + 0.02S_2 + \cdots + 0.02S_{30}
\]

where

\[
S_k = \text{salary earned during the } k^{\text{th}} \text{ year working}
\]

\[
\therefore B = 0.02(S_1 + S_2 + \cdots + S_{30}) = 0.02 \cdot (\text{total salary earned over his working lifetime})
\]

\[
= 0.02(1,500,000) = 30,000
\]

William is paid \(30,000@\ BOY\) for life, starting at NRA.
8) (a) Cindy's accrued benefit at age 45
\[ B_{45} = 0.015 \times 600,000 = 9,000 \]
If she terminates employment now, she would receive \( \frac{9,000}{12} = 750 \) at the beginning of each month for her lifetime, starting at age 65.

(b) \[ \text{APV}^{AB}_{45} = 12(750) \cdot 200 \overline{a}_{45}^{(12)} = 9,000 \cdot 200 \overline{a}_{65}^{(12)} \]
\[ = 9,000 \cdot 200 \overline{a}_{65}^{(12)} \cdot 200 \overline{p}_{45}^{(12)} \cdot \overline{a}_{65} = 9,000(1.04)^{20} (0.9) (11.4) \]
\[ \therefore \text{APV}^{AB}_{45} = 42,142.77 \]

(c) If Cindy works to age 65, then her total salary is
\[ S = 600,000 + S_{45} + S_{46} + \cdots + S_{64} \]
\[ = S_{45}(1.03) + \cdots + S_{45}(1.03)^{19} \]
\[ \therefore S = 600,000 + S_{45} \left(1 + 1.03 + \cdots + (1.03)^{19}\right) \]
\[ = 70,000 \]
\[ \therefore S = 248,092,26.21 \]
\[ \Rightarrow B_{65} = 0.02 (S) = 49,618.52 \]

\[ \text{APV}^{RB}_{45} = 49,618.52 \cdot 200 \overline{a}_{45}^{(12)} \text{ see above} \] 232,334.021

Note \( \text{APV}^{RB}_{45} = \frac{\text{APV}^{AB}_{45}}{9,000} (49,618.52) \)

(d) Cindy's final year salary is \( 70,000(1.03)^{19} \).
\[ \therefore \text{RR} = \frac{49,618.52}{70,000(1.03)^{19}} = 0.404 \ (40.4\% \)
9) 

<table>
<thead>
<tr>
<th>Age</th>
<th>Salary</th>
</tr>
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<tbody>
<tr>
<td>35</td>
<td>40000</td>
</tr>
<tr>
<td>36</td>
<td>40000(1.05))</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>64</td>
<td>40000(1.05)^{29})</td>
</tr>
</tbody>
</table>

\[ B_{65} = 0.015 \cdot S_{64} \cdot (30) \]

\[ = 0.015 \cdot [40000(1.05)^{29}] \cdot (30) \]

\[ \therefore B_{65} = 74090.44 \]

10) Donna's final year salary is \( Y(1 + 0.01x)^{29} \)

\[ B_{65} = 0.015 \cdot [Y(1+0.01x)^{29}] \cdot (30) \]

\[ \therefore RR = \frac{0.015 \cdot [Y(1+0.01x)^{29}] \cdot (30)}{Y(1+0.01x)^{29}} = 0.015 \cdot (30) = 0.45 \]

Remark: Since this is a final year salary plan, we didn't need \( Y \) or \( x \) to get an answer. Note this would be the RR for Jamie in problem \# 9.
\( x = \text{Age}\)
\[
\begin{array}{c|c}
45 & 80000 = S_{45} \\
46 & 80000(1.03) = S_{46} \\
\vdots & \\
57 & 80000(1.03)^{12} \\
58 & 80000(1.03)^{13} \\
59 & 80000(1.03)^{14} \\
\vdots & \\
62 & 80000(1.03)^{17} \\
63 & 80000(1.03)^{18} \\
64 & 80000(1.03)^{19}
\end{array}
\]

Remark: There are several ways to sum \( S_{k}, S_{k+1}, \ldots, S_{k+2}\).
E.g. \( S_{57} + S_{58} + S_{59} = ? \)

Method 1: (Just do it; it's only 3 yrs)

Method 2: \( S_{57} + S_{58} + S_{59} \\
= S_{57}(1 + 1.03 + (1.03)^2) \\
= S_{57} \cdot \frac{1}{1 - 1.03}
\]

Method 3: \( S_{57} + S_{58} + S_{59} = S_{57} + S_{58} + S_{59} \\
= S_{59}(1 + \frac{1}{1.03} + \frac{1}{(1.03)^2}) = S_{59}(1 + 2 + 3) \\
= S_{59} \cdot \frac{1}{1 - 1.03}
\]

\( \text{(a)} \) Termination at age 60 \( \Rightarrow \) 15 yrs of service

\[ \text{final 3-year average salary} = \frac{1}{3}(S_{57} + S_{58} + S_{59}) \]

\[ = 117516.92 \]

\[ \therefore B_{60} = 0.02(100000)(15) + 0.03(17516.92)(15) = 37882.61 \]

\( \text{Note: This is an annual \#, Lou actually would receive} \]

\[ \frac{37882.61}{12} \text{ at the beginning of each month, starting at age 65}. \]

\( \text{(b)} \) Retirement at age 65 \( \Rightarrow \) 20 yrs of service

\[ \text{final 3-year average salary} = \frac{1}{3}(S_{62} + S_{63} + S_{64}) = 136234.31 \]

\[ \therefore B_{65} = 0.02(100000)(20) + 0.03(36234.31)(20) = 61740.59 \]

\( \text{(C)} \) \( APV_{65} = 61740.59 \cdot \ddot{a}_{65}^{(12)} = 753235.20 \)
12) Since Lou retires at age 60, his age 65 benefit is $B_{60} \text{ see } #11(a) = 37882.61$. We must reduce this amount by the ERF if he starts his benefit at age 60. Since ERB's are determined by actuarial equivalence, we have $B_{60} \times \ddot{A}_{60}^{(12)} \overset{11}{=} X \times \ddot{A}_{60}$ where $X =$ annual retirement benefit starting at age 60 (still payable monthly) but both sides are $APV_{60}$ of retirement benefit

\[ \text{LHS} = APV_{60}^{AB} \]
\[ \text{RHS} = APV_{60}^{ERB} \]

\[ \therefore (37882.61) \cdot 5E_{60} \cdot \ddot{A}_{65}^{(12)} = X \ddot{A}_{60} \]
\[ 5E_{60} = 2^{5} \cdot 5P_{60} = (1.04)^{5} \cdot (95) \]
\[ \ddot{A}_{65}^{(12)} \overset{11}{=} 12.2 \]
\[ \ddot{A}_{60} = 13.6 \]

\[ \implies X = 26534.92 \text{ (annual ERF)} \]

\[ \implies \text{Lou's monthly ERF} = \frac{26534.92}{12} = 2211.24 \]
\[
\begin{array}{c|c}
\text{Age} & \text{Salary} \\
35 & 60000 \\
36 & 60000(1.04) \\
\vdots & \vdots \\
60 & 60000(1.04)^{25} \\
61 & 60000(1.04)^{26} \\
62 & 60000(1.04)^{27} \\
63 & 60000(1.04)^{28} \\
64 & 60000(1.04)^{29} \\
\end{array}
\]
\[
\{ \text{Avg} = 173,268.35 \} = \text{Don's final 5-year average salary}
\]

Note: Don's final 5-year average salary

\[
= (1.04)^{-5} \cdot \text{Ron's final 5-year average salary}
\]

\[
= 142,413.95
\]

(a) \( B_{60}^{Ron} = .02 (142,413.95) (25) = 71,206.98 \) (payable at age 65)

\[
\text{Pension reduction factor} = .06(5) = .3 \Rightarrow \text{ERF} = .7
\]

\[
\therefore \text{ERB}_{60}^{Ron} = .7 (71,206.98) = 49,844.89 \text{ annually},
\]

or \( \frac{49,844.89}{12} = 4,153.74 \text{ monthly} \)

(b) \( B_{65}^{Don} = .02 (173,268.35) (30) = 10,396.01 \text{ annually},
\]

or \( \frac{10,396.01}{12} = 866.33 \text{ monthly} \)

(c) \( APV_{60}^{Ron \ RB} = 49,844.89 \cdot a^{12}_{60} = 6,778,90.50 \)

(d) \( APV_{65}^{Don \ RB} = 10,396.01 \cdot a^{12}_{65} = 1,268,324.32 \)