Solutions to MLC Module 1 Section 4 Exercises

1. (a) Since there is a uniform distribution of deaths between neighboring ages, and since there are 40 deaths between ages 50 and 51, then there must be 10 deaths for any 0.25 year period between ages 50 and 51. Therefore, there are 10 deaths between ages 50 and 50.25, and so $l_{50.25} = 600 - 10 = 590$.

   With a UDD assumption, instead of going through this process each time, from now on we will use linear interpolation between integer ages. That is, we'll determine the life table value at fractional ages by taking the weighted *arithmetic* mean of the life table values at corresponding neighboring ages. In this problem, we get

   $l_{50.25} \overset{UDD}{=} .75 \cdot l_{50} + .25 \cdot l_{51} = .75 \cdot 600 + .25 \cdot 560 = 590$ as before.

   Notice that since the age is 50.25, then the weights are 0.25 and 0.75. (Weights always sum to 1.) Since 50.25 is closer to age 50, the higher weight of 0.75 gets assigned to l_{50}. This sort of logic will apply in both UDD and CF problems.

   (b) When using a CF assumption, we determine the life table value at fractional ages by taking the weighted *geometric* mean of the life table values at corresponding neighboring ages. Assigning the weights as is part (a), we get

   $$l_{50.25} \overset{CF}{=} (l_{50})^{.75} \cdot (l_{51})^{.25} = (600)^{.75} \cdot (560)^{.25} \approx 589.73981$$

2. From the given information, we have $l_{47} = 1000$, $l_{48} = l_{47} \cdot p_{47} = 1000(.95) = 950$, and $l_{49} = l_{47} \cdot {}_2p_{47} = 1000(.90) = 900$. Therefore,

   (a) $l_{48.5} \overset{UDD}{=} .5 \cdot l_{48} + .5 \cdot l_{49} = .5 \cdot (950 + 900) = 925$

   (b) $l_{48.3} \overset{CF}{=} (l_{48})^{.7} \cdot (l_{49})^{.3} = (950)^{.7} \cdot (900)^{.3} \approx 934.71514$

   Note, again, how the weights are assigned.

3. (a) We have ${}_{0.2}q_{65.3} = 1 - {}_{0.2}p_{65.3} = 1 - \frac{l_{65.5}}{l_{65.3}}$.

   $l_{65.5} \overset{UDD}{=} .5 \cdot l_{65} + .5 \cdot l_{66} \overset{ILT}{=} .5 \cdot (7{,}533{,}964 + 7{,}373{,}338) = 7{,}453{,}651$, and
   $l_{65.3} \overset{UDD}{=} .7 \cdot l_{65} + .3 \cdot l_{66} \overset{ILT}{=} .7 \cdot (7{,}533{,}964) + .3 \cdot (7{,}373{,}338) = 7{,}485{,}776.2$

   Therefore, ${}_{0.2}q_{65.3} = 1 - \frac{7{,}453{,}651}{7{,}485{,}776.2} \approx 0.00429$

(b) ${}_{1.5}p_{72.2} = \frac{l_{73.7}}{l_{72.2}}$

$l_{73.7} \overset{CF}{=} (l_{73})^{.3} \cdot (l_{74})^{.7} = (5{,}920{,}394)^{.3} \cdot (5{,}664{,}051)^{.7} \approx 5{,}739{,}766.056$, and
$l_{72.2} \overset{CF}{=} (l_{72})^{.8} \cdot (l_{73})^{.2} = (6{,}164{,}663)^{.8} \cdot (5{,}920{,}394)^{.2} \approx 6{,}115{,}015.95$

Therefore ${}_{1.5}p_{72.2} = \frac{l_{73.7}}{l_{72.2}} \approx 0.93863$

4. For fractional age problems, if you're not given l_x values, then we can develop them by starting with an arbitrary value (like 1000) at the earliest age. Looking ahead at all our calculations, we notice that the earliest age for which we'll need an l_x value is for part (c), where we'll need an l_{30}. Likewise the latest age for which we'll need an l_x value is for parts (b) and (d), where we'll need an l_{34}. So let's start with $l_{30} = 1000$ and develop the other l_x values up to and including l_{34}. We're given $q_{30} = 0.1, q_{31} = 0.15, q_{32} = 0.2$, and $q_{33} = 0.25$. Then,

$l_{30} = 1000$
$l_{31} = l_{30} \cdot p_{30} = 1000(.9) = 900$
$l_{32} = l_{31} \cdot p_{31} = 900(.85) = 765$
$l_{33} = l_{32} \cdot p_{32} = 765(.8) = 612$
$l_{34} = l_{33} \cdot p_{33} = 612(.75) = 459$

(a) ${}_{0.2}q_{31} = 1 - {}_{0.2}p_{31} = 1 - \frac{l_{31.2}}{l_{31}}$

Since $l_{31.2} \overset{UDD}{=} .8 \cdot l_{31} + .2 \cdot l_{32} = .8 \cdot (900) + .2(765) = 873$, then

${}_{0.2}q_{31} \overset{UDD}{=} 1 - \frac{873}{900} = .03$

To illustrate that it won't matter what the starting l_x value is, if we would have started with $l_{31} = 1000$, we would get $l_{32} = 1000 \cdot p_{31} = 1000 \cdot (.85) = 850$. Then $l_{31.2} \overset{UDD}{=} .8 \cdot l_{31} + .2 \cdot l_{32} = .8 \cdot (1000) + .2(850) = 970 \Rightarrow {}_{0.2}q_{31} = 1 - \frac{970}{1000} = .03$ as above. The key idea is that for these probabilities, we're taking *ratios* of the l_x values, and so the result will not depend on the starting value. For the rest of this problem, we'll use the l_x values we computed before starting part (a).

(b) We need $l_{33.5} \overset{CF}{=} (l_{33})^{.5} \cdot (l_{34})^{.5} = \sqrt{l_{33} \cdot l_{34}} = \sqrt{(612)(459)} \approx 530.00755$. Then ${}_{1.5}p_{32} = \frac{l_{33.5}}{l_{32}} \overset{CF}{\approx} .69282$

(c) ${}_{0.4|0.6}q_{30.8} = \frac{l_{31.2} - l_{31.8}}{l_{30.8}}$ From part (a), $l_{31.2} \overset{UDD}{=} 873$.
Also, $l_{31.8} \overset{UDD}{=} .2(900) + .8(765) = 792$ and $l_{30.8} \overset{UDD}{=} .2(1000) + .8(900) = 920$.
Therefore ${}_{0.4|0.6}q_{30.8} \overset{UDD}{=} \frac{873-792}{920} = \frac{81}{920}$

(d) ${}_{1.7|0.8}q_{31} = \frac{l_{32.7} - l_{33.5}}{l_{31}}$ From part (b), $l_{33.5} \overset{CF}{\approx} 530.00755$.

Also, $l_{32.7} \overset{CF}{=} (765)^{.3} \cdot (612)^{.7} \approx 654.37158$ and so ${}_{1.7|0.8}q_{31} \overset{CF}{\approx} .13818$

5. Other than how mortality is given, this problem is very similar to the previous problem. Once we get the l_x values, we proceed as above. Looking ahead, notice in parts (a) and (c) that we'll need l_{50} and that's the youngest age for which we'll need an l_x value. Instead of 1000, let's start with an arbitrary $l_{50} = 100$, to change it up a little. We're given $q_{50} = 0.01$, ${}_{1|}q_{50} = 0.02$, ${}_{2|}q_{50} = 0.03$, and ${}_{3|}q_{50} = 0.04$. Then,

$$d_{50} = l_{50} \cdot q_{50} = 100(0.01) = 1$$
$$d_{51} = l_{50} \cdot {}_{1|}q_{50} = 100(0.02) = 2$$
$$d_{52} = l_{50} \cdot {}_{2|}q_{50} = 100(0.03) = 3$$
$$d_{53} = l_{50} \cdot {}_{3|}q_{50} = 100(0.04) = 4$$

Therefore,

$$l_{50} = 100$$
$$l_{51} = l_{50} - d_{50} = 100 - 1 = 99$$
$$l_{52} = l_{51} - d_{51} = 99 - 2 = 97$$
$$l_{53} = l_{52} - d_{52} = 97 - 3 = 94$$
$$l_{54} = l_{53} - d_{53} = 94 - 4 = 90$$

(a) ${}_{0.2}q_{50} = 1 - {}_{0.2}p_{50} = 1 - \frac{l_{50.2}}{l_{50}}$

Since $l_{50.2} \overset{UDD}{=} .8 \cdot l_{50} + .2 \cdot l_{51} = .8 \cdot (100) + .2(99) = 99.8$, then

$${}_{0.2}q_{50} \overset{UDD}{=} 1 - \frac{99.8}{100} = .002$$

(b) We need $l_{52.5} \overset{CF}{=} (l_{52})^{.5} \cdot (l_{53})^{.5} = \sqrt{l_{52} \cdot l_{53}} = \sqrt{(97)(94)} \approx 95.48822$. Then ${}_{1.5}p_{51} = \frac{l_{52.5}}{l_{51}} \overset{CF}{\approx} .96453$

(c) ${}_{0.6|0.4}q_{50.8} = \frac{l_{51.4} - l_{51.8}}{l_{50.8}}$

We get $l_{51.4} \overset{UDD}{=} .6(99) + .4(97) = 98.2$ and $l_{51.8} \overset{UDD}{=} .2(99) + .8(97) = 97.4$ and $l_{50.8} \overset{UDD}{=} .2(100) + .8(99) = 99.2$. Therefore ${}_{0.6|0.4}q_{50.8} \overset{UDD}{=} \frac{98.2 - 97.4}{99.2} = \frac{8}{992}$

(d) ${}_{1.8|0.7}q_{51} = \frac{l_{52.8} - l_{53.5}}{l_{51}}$

We get $l_{52.8} \overset{CF}{=} (97)^{.2} \cdot (94)^{.8} \approx 94.59248$ and $l_{53.5} \overset{CF}{=} \sqrt{(94)(90)} \approx 91.97826$ and so ${}_{1.8|0.7}q_{51} \overset{CF}{\approx} .02641$

6. (See Video Solution)
   (a) ${}_tp_1 \overset{UDD}{=} 1 - .1t$

   (b) ${}_tp_2$ for $0 \leq t \leq 1$ using the UDD assumption

7. (See Video Solution)
   ${}_{30}p_{35} = \frac{1}{6}$

8. (See Video Solution)
   ${}_{30}p_{35} = (.72)^{15} \approx .00724$