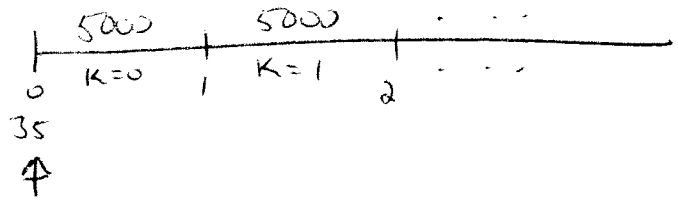
1)



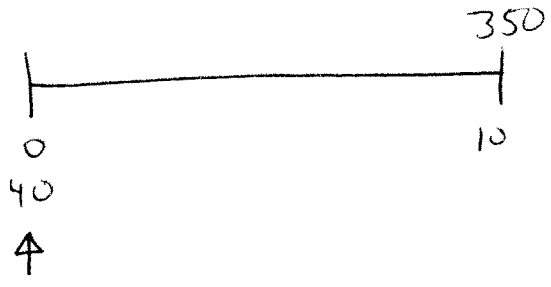
(a) $PVRV = Z = 5000\, Z_{35} = 5000\, v^{K+1} \qquad K = K_{35}$

(b) $EPV = 5000\, A_{35} \overset{ILT}{=} 5(128.72) = 643.60$

(c) $Var(Z) = 5000^2 \cdot Var(Z_{35}) = 5000^2 \left[{}^2A_{35} - (A_{35})^2 \right]$

$\overset{ILT}{=} 5000^2 \left[(.03488) - (.12872)^2 \right] \doteq 457779.04$

2)



$T = T_{40}$

(a) $PVRV = Z = 350 \cdot Z_{40:\overline{10}|} = \begin{cases} 0 & \text{if } T < 10 \\ 350 v^{10} & \text{if } T > 10 \end{cases}$

doesn't matter where we put the "=" part

(b) $APV = 350 A_{40:\overline{10}|} = 350 \cdot {}_{10}E_{40} \overset{ELT}{=} 350(.53667) = 187.83$

(c) $Var(Z) = 350^2 \left[{}^2A_{40:\overline{10}|} - \left(A_{40:\overline{10}|}\right)^2 \right]$

$= 350^2 \left[{}^2_{10}E_{40} - \left({}_{10}E_{40}\right)^2 \right]$

$= 350^2 \left[v^{10} \cdot {}_{10}E_{40} - \left({}_{10}E_{40}\right)^2 \right]$

$\overset{ELT}{=} 350^2 \left[(1.06)^{-10}(.53667) - (.53667)^2 \right] = 1428.23$

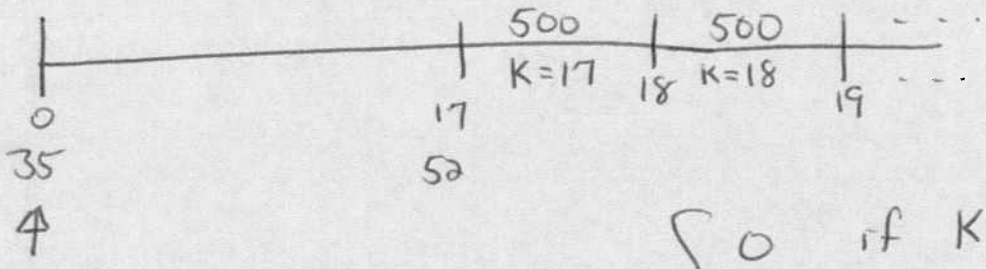
3) (See Video Solution)

(a) $Z = 10000\, Z_{35:\overline{17}|}$

(b) $EPV = 10000\, {}_{17}E_{35} = 3485.05$

(c) $Var(Z) = 796657.52$

4)



4

(a) $PVRV = Z = 500\ {}_{17|}Z_{35} = \begin{cases} 0 & \text{if } K < 17 \\ 500v^{K+1} & \text{if } K \geq 17 \end{cases}$

(b) $EPV = 500\ {}_{17|}A_{35} = 500 A_{52} \cdot {}_{17}E_{35}$

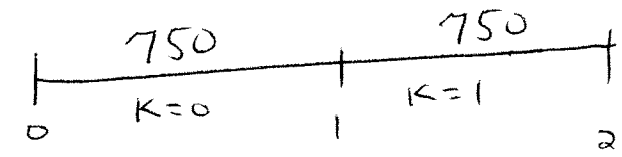
$\overset{ILT}{=} 500(.2705) \cdot (1.06)^{-17} \cdot \frac{l_{52}}{l_{35}} = 47.14$

(c) $Var(Z) = 500^2 \left[{}^{2}A_{52} \cdot {}^{2}{}_{17}E_{35} - (A_{52} \cdot {}_{17}E_{35})^2 \right]$

$\overset{ILT}{=} 500^2 \left[(.10792) \cdot (1.06)^{-34} \cdot \frac{l_{52}}{l_{35}} - \left((.2705)(1.06)^{-17} \cdot \frac{l_{52}}{l_{35}} \right)^2 \right]$

$= 1270.08$

5) (See Video Solution)



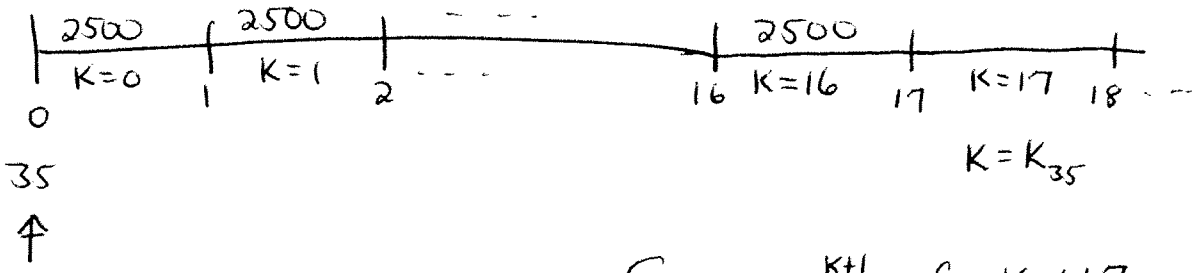
32

(a) $PVRV = Z = 750 \cdot Z_{\frac{1}{32}:\overline{2}|} = \begin{cases} 750v^{K+1} & \text{if } K < 2 \\ 0 & \text{if } K \geq 2 \end{cases}$

(b) $APV = 750 A_{\frac{1}{32}:\overline{2}|} = 2.40$

(c) $Var(Z) = 1641.50$

6)



(a) $PVRV = Z = 2500\, Z_{35:\overline{17}|}^{1} = \begin{cases} 2500\, v^{K+1} & \text{if } K < 17 \\ 0 & \text{if } K \geq 17 \end{cases}$

(b) $APV = 2500\, A_{35:\overline{17}|}^{1} = 2500\left(A_{35} - {}_{17|}A_{35}\right)$

$= 2500\left(A_{35} - {}_{17}E_{35} \cdot A_{52}\right)$

$\overset{ILT}{=} 2500\left(.12872 - (1.06)^{-17} \cdot \frac{\ell_{52}}{\ell_{35}} \cdot (.2705)\right) = 86.12$

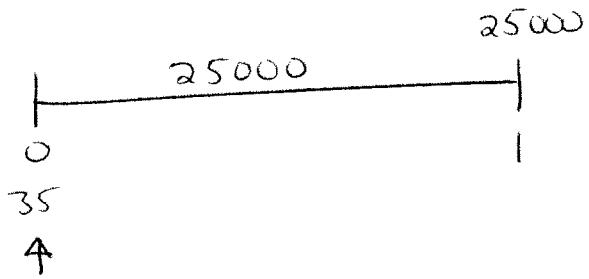
(c) $Var(Z) = 2500^2\left[{}^{2}A_{35:\overline{17}|}^{1} - \left(A_{35:\overline{17}|}^{1}\right)^2\right]$

$A_{35:\overline{17}|}^{1} = A_{35} - {}_{17}E_{35} \cdot A_{52} \overset{ILT}{=} .12872 - (1.06)^{-17} \cdot \frac{\ell_{52}}{\ell_{35}}(.2705) = \boxed{1}$

${}^{2}A_{35:\overline{17}|}^{1} = {}^{2}A_{35} - {}^{2}_{17}E_{35} \cdot {}^{2}A_{52} \overset{ILT}{=} .03488 - (1.06)^{-34} \cdot \frac{\ell_{52}}{\ell_{35}}(.10792) = \boxed{2}$

$Var(Z) = 2500^2\left[\boxed{2} - (\boxed{1})^2\right] = 123287.35$

7) (See Video Solution)

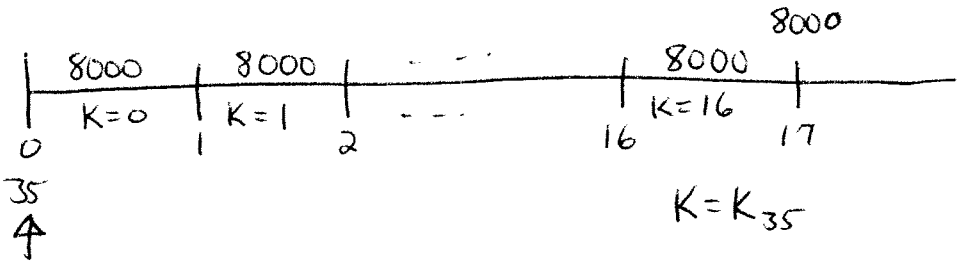


(a) $PVRV = Z = 25000\, Z_{35:\overline{1}|} = 25000\, v$ (constant, not random)

(b) $EPV = \frac{25000}{1.06} = 23584.91$

(c) $Var(Z) = 0$

8)



(a) $PVRV = Z = 8000\, Z_{35:\overline{17}|} = \begin{cases} 8000\, v^{K+1} & \text{if } K < 17 \\ 8000\, v^{17} & \text{if } K \geq 17 \end{cases}$

(b) $APV = 8000\, A_{35:\overline{17}|} = 8000\, A^{\frac{1}{}}_{35:\overline{17}|} + 8000\, A_{35:\overline{17}|}^{\frac{1}{}}$

$= 8000\left[A_{35} - {}_{17}E_{35} \cdot A_{52}\right] + 8000\, {}_{17}E_{35}$

$\overset{ILT}{=} 8000\left[.12872 - (1.06)^{-17} \cdot \frac{\ell_{52}}{\ell_{35}}(.2705)\right] + 8000(1.06)^{-17} \cdot \frac{\ell_{52}}{\ell_{35}}$

$= 3063.64$

(c) $Var(Z) = 8000^2\left[{}^2A_{35:\overline{17}|} - (A_{35:\overline{17}|})^2\right]$

$A_{35:\overline{17}|} = \left[A_{35} - {}_{17}E_{35} \cdot A_{52}\right] + {}_{17}E_{35}$

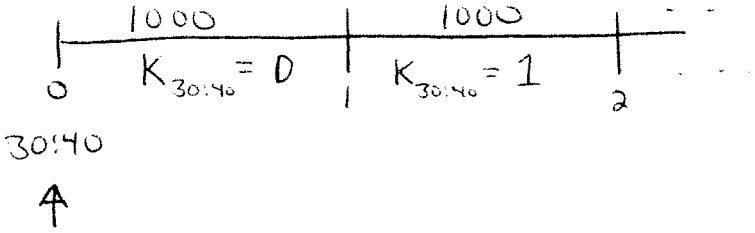
$\overset{ILT}{=} .12872 - (1.06)^{-17} \cdot \frac{\ell_{52}}{\ell_{35}}(.2705) + (1.06)^{-17} \cdot \frac{\ell_{52}}{\ell_{35}} = \boxed{1}$

${}^2A_{35:\overline{17}|} = \left[{}^2A_{35} - {}^{2}_{17}E_{35} \cdot {}^2A_{52}\right] + {}^{2}_{17}E_{35}$

$\overset{ILT}{=} \left[.03488 - (1.06)^{-34} \cdot \frac{\ell_{52}}{\ell_{35}}(.10792)\right] + (1.06)^{-34} \cdot \frac{\ell_{52}}{\ell_{35}} = \boxed{2}$

$Var(Z) = 8000^2\left[\boxed{2} - (\boxed{1})^2\right] = 235583.55$

9)



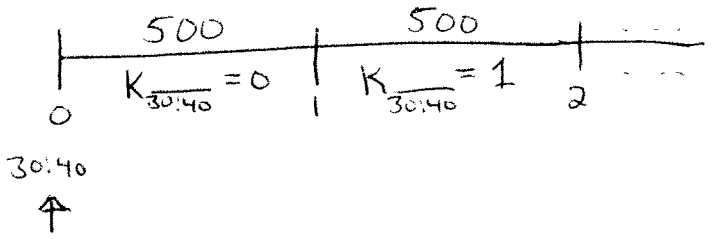
(a) $PVRV = Z = 1000\, Z_{30:40}$

(b) $EPV = 1000\, A_{30:40} \overset{ILT}{=} 195.84$

(c) $Var(Z) = 1000^2 \left[{}^2A_{30:40} - (A_{30:40})^2 \right]$

$\overset{ILT}{=} 1000^2 \left[.06672 - (.19584)^2 \right] = 28366.69$

10)



(a) $PVRV = Z = 500\, Z_{\overline{30:40}}$

(b) $EPV = 500\, A_{\overline{30:40}} = 500\left(A_{30} + A_{40} - A_{30:40}\right)$

$\overset{ILT}{=} 500(.10248 + .16132 - .19584) = 33.98$

(c) $Var(Z)$:

$A_{\overline{30:40}} = A_{30} + A_{40} - A_{30:40}$ $\left(\overset{\text{part b}}{=} \frac{33.98}{500} = .06796\right)$

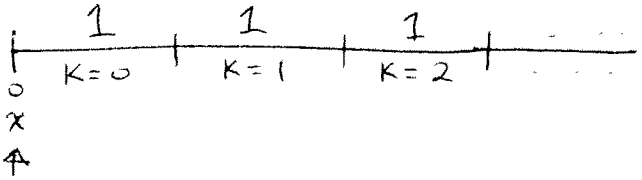
$^2A_{\overline{30:40}} = {}^2A_{30} + {}^2A_{40} - {}^2A_{30:40}$

$\overset{ILT}{=} .02531 + .04863 - .06672 = .00722$

$\therefore Var(Z) = 500^2\left[{}^2A_{\overline{30:40}} - \left(A_{\overline{30:40}}\right)^2\right]$

$= 500^2\left[.00722 - (.06796)^2\right] \approx 650$

11) A_x:



$$A_x \overset{VEP}{=} v\,{}_1q_x + v^2\,{}_{11}q_x + v^3\,{}_{21}q_x + \cdots$$

$$CF \Rightarrow {}_1q_x = q$$

$${}_{11}q_x = p \cdot q$$

$${}_{21}q_x = p^2 \cdot q$$

$$\therefore A_x \overset{CF}{=} vq + v^2pq + v^3p^2q + \cdots \quad (\text{geometric } w/\ r = vp)$$

$$= \frac{vq}{1 - vp} \left(\text{multiply by } \frac{1+i}{1+i}\right) = \frac{q}{(1+i) - p} = \frac{q}{q+i}$$

$$A_x \overset{CF}{=} \frac{q}{q+i}$$

(a) $A_x = \frac{q}{q+i}$; $i = .08$ $\quad p = e^{-\mu} = e^{-(-\ln(.9))} = .9$ $\quad q = 1-p = .1$

$$\therefore EPV = 3000\,A_x = 3000\,\frac{.1}{.1 + .08} = 1666.67$$

(b) $${}^2A_x \overset{CF}{=} \frac{q}{q + (2i + i^2)} = \frac{.1}{.1 + (.16 + .0064)} = \frac{.1}{.2664}$$

$$\therefore E[Z^2] = 3000^2\,{}^2A_x = 3000^2\,\frac{.1}{.2664} = 3\,378\,378.38$$

(c) $$Var(Z) = 3000^2\left[{}^2A_x - (A_x)^2\right]$$

$$\overset{CF}{=} 3000^2\left[\frac{q}{q + (2i + i^2)} - \left(\frac{q}{q+i}\right)^2\right] = 600{,}600$$

12)

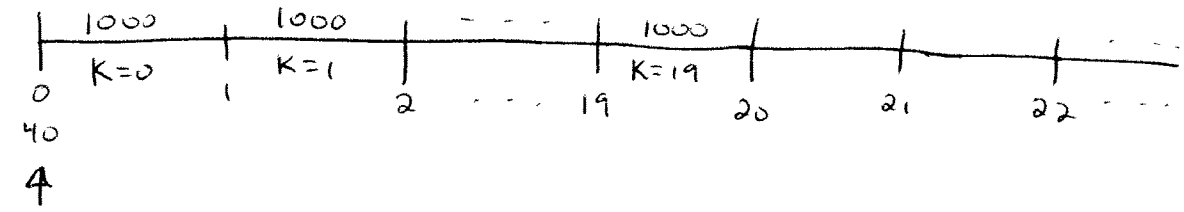
$$APV = 100\, A_{x:\overline{12}|}^{1} = 100\, {}_{12}E_x = 100\, v^{12} \cdot {}_{12}p_x$$

$${}_np_x \overset{CF}{=\!=} p^n \quad \text{where} \quad p = e^{-\mu}$$

$$\mu = -\ln(.9) \implies p = .9$$

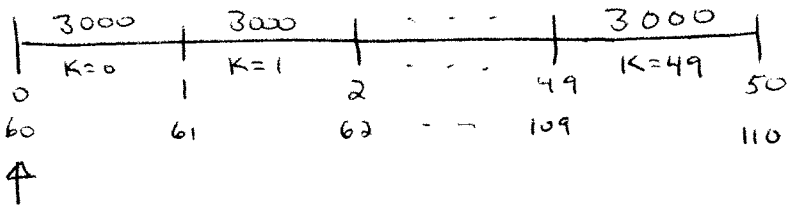
$$\therefore APV = 100(1.08)^{-12}(.9)^{12} = 100\left(\frac{.9}{1.08}\right)^{12} = 11.22$$

13) (See Video Solution)



$$EPV = 1000\, A^{1}_{40:\overline{20}|} = 541.06$$

14) (See Video Solution)



$PVRV = Z = 3000\, Z_{60}$

(a) $EPV = E[Z] = 3000\, A_{60} = 734.01$

(b) $E[Z^2] = 3000^2 \cdot {}^2A_{60} = 1081239.02$

(c) $Var(Z) = E[Z^2] - (E[Z])^2 = 3000^2[{}^2A_{60} - (A_{60})^2]$

$= 542\,468.34$

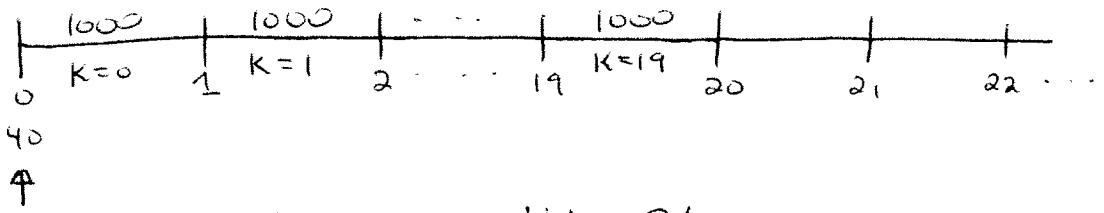
15)

$$APV = 100\, A_{50:\overline{12}|} = 100\, {}_{12}E_{50} = 100\, v^{12}\, {}_{12}P_{50}$$

$${}_{12}P_{50} \underset{\omega=110}{\overset{DML}{=}} \frac{110-50-12}{110-50} = \frac{48}{60} = .8$$

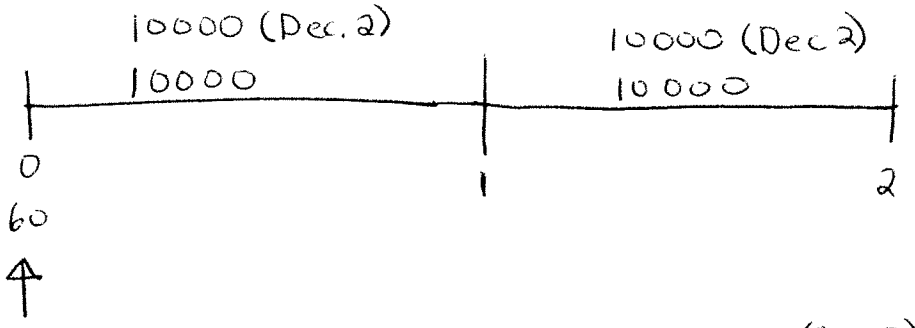
$$\therefore APV = 100\,(1.08)^{-12}\,(.8) = 31.77$$

16) (See Video Solution)



$$EPV = 1000\, A_{40:\overline{20}|}^{1} = 140.26$$

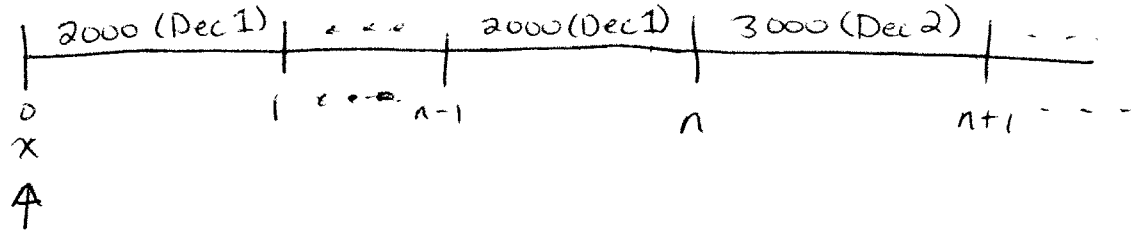
17) (See Video Solution)



$$APV = 10000\,A^{(1)}_{60:\overline{2}|} + 10000\,A^{(1)}_{60:\overline{2}|}$$

$$= 339.41 + 119.09 = 458.50$$

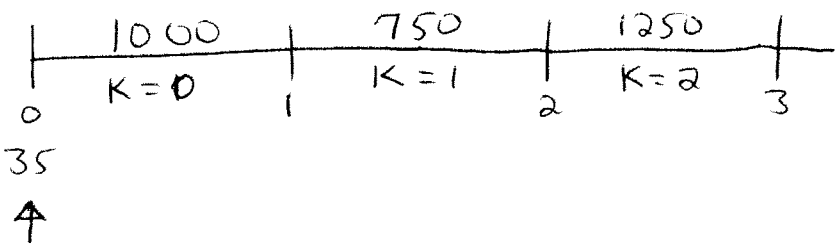
18) (See Video Solution)



$$EPV = 2000\, A^{(Dec\,1)}_{x:\overline{n}|} + 3000 \cdot {}_{n|}A^{(Dec\,2)}_{x}$$

$$= 232.10 + 550.68 = 782.78$$

19)



K	PVRV = Z	Pr
0	$1000v$	$q_{35} = .02$
1	$750v^2$	${}_{11}q_{35} = p_{35}\, q_{36} = .98(.025) = .0245$
2	$1250v^3$	${}_{21}q_{35} = {}_2p_{35}\, q_{37} = p_{35}\, p_{36}\, q_{37} = (.98)(.975)(.03) = .028665$
3+	0	${}_3p_{35} = 1 - .02 - .0245 - .028665 = .926835$

$v = 1 - d = .95$

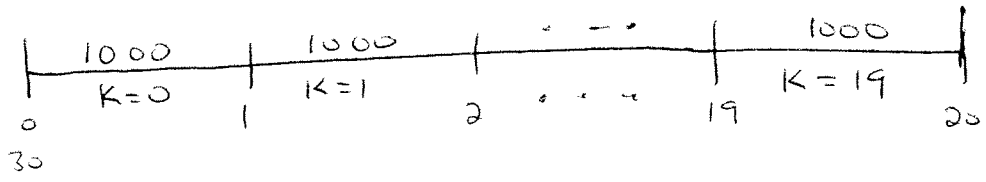
$\therefore E[Z] = 1000(.95)(.02) + 750(.95)^2(.0245) + 1250(.95)^3(.028665)$

$E[Z^2] = [1000(.95)]^2(.02) + [750(.95)^2]^2(.0245) + [1250(.95)^3]^2(.028665)$

$Var(Z) = E[Z^2] - (E[Z])^2 \approx 57800$

20) (See Video Solutions)

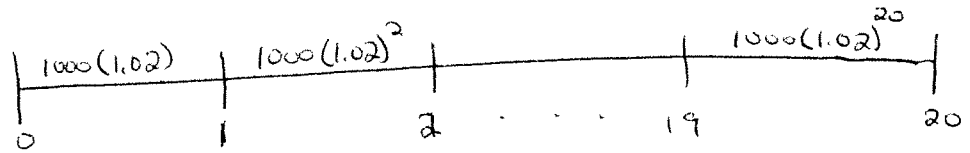
A:



30

$$APV^{A}_{.03} = 1000 A_{\overline{30:20|}} \overset{i=.03}{=\!=} 585$$

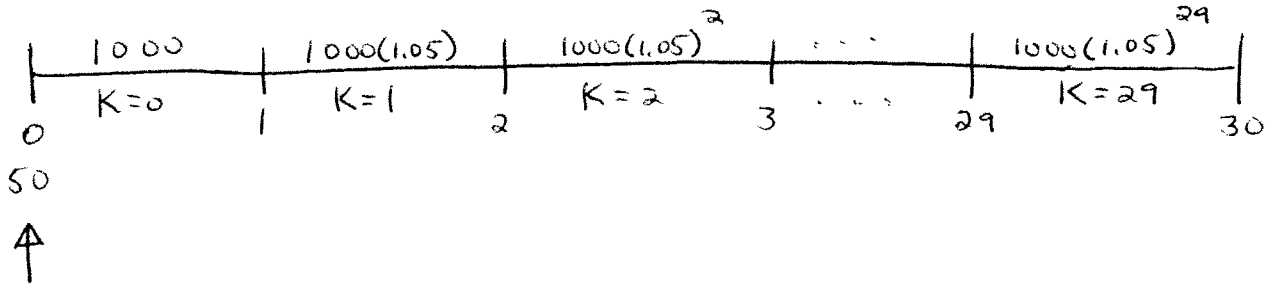
B:



30

$$APV^{B}_{.0506} = 585$$

21) (See Video Solution)

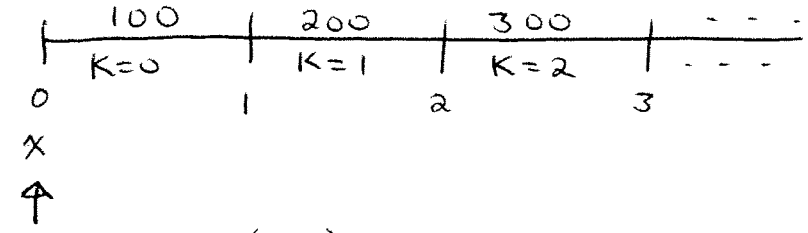


50

↑

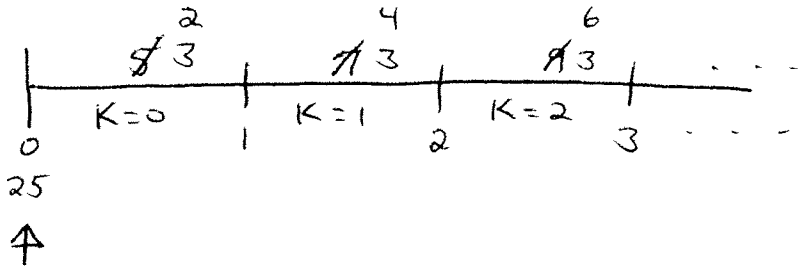
$APV_{.05} = 571.43$

22)



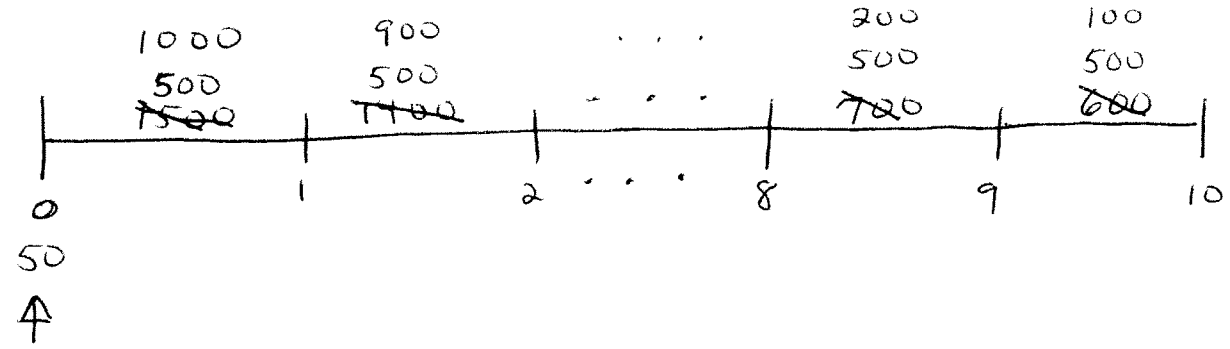
$EPV = 100\,(IA)_{\overline{x}|}$

23)



$$EPV = 3A_{25} + 2(IA)_{25}$$

24)



$$APV = 100(DA)_{\overline{50:10|}}^{1} + 500\,A_{\overline{50:10|}}^{1}$$

25) $EPV = 1000\, A_{40}^{(4)}$

(a) $A_{40}^{(4)} \overset{UDD}{=} \frac{i}{i^{(4)}} A_{40} \overset{\text{see ILT last page}}{=} (1.02223)(.16132)$

$\Rightarrow EPV = 1000(1.02223)(.16132) = 164.91$

(b) $A_{40}^{(4)} \overset{CAA}{=} (1+i)^{\frac{4-1}{2(4)}} A_{40} \overset{ILT}{=} (1.06)^{3/8}(.16132) \Rightarrow$

$EPV = 1000\,(1.06)^{3/8}(.16132) = 164.88$

26) $EPV = 1000\, A^{(4)}_{\overline{40:20}|}$

Note $A^{1}_{\overline{40:20}|} = A_{40} - {}_{20|}A_{40}$

$= A_{40} - {}_{20}E_{40} \cdot A_{60}$

$\overset{ILT}{=} .16132 - (.27414)(.36913) = .06013$

(a) $A^{(4)}_{\overline{40:20}|} \overset{UDD}{=} \frac{i}{i^{(4)}} A^{1}_{\overline{40:20}|} \overset{ILT}{=} (1.02223)(.06013)$

$\Rightarrow EPV = 1000\,(1.02223)(.06013) = 61.47$

(b) $A^{(4)}_{\overline{40:20}|} \overset{CAA}{=} (1+i)^{\frac{4-1}{2(4)}} A^{1}_{\overline{40:20}|} \overset{ILT}{=} (1.06)^{3/8}(.06013)$

$\Rightarrow EPV = 1000\,(1.06)^{3/8}(.06013) = 61.46$

27) (See Video Solution)

$$EPV = 1000 \cdot {}_{20|}A_{40}^{(4)}$$

(a) $EPV = 103.44$

(b) $EPV = 103.42$

28) $EPV = 1000 A^{(4)}_{40:\overline{20}|} = 1000 A^{1\,(4)}_{40:\overline{20}|} + 1000\,{}_{20}E_{40}$

Use #26 and $1000\,{}_{20}E_{40} \overset{ILT}{=} 274.14$

(a) $EPV \overset{VDD}{=} 61.47 + 274.14 = 335.61$

(b) $EPV \overset{CAA}{=} 61.46 + 274.14 = 335.60$