Module 2 Section 4 Exercises:

These are discrete "annuity" problems that are similar to the discrete "insurance" problems of Section 2. Note that the relationships between whole life, *n*-year deferred, and *n*-year temporary annuities are the same as their whole life, *n*-year deferred, and *n*-year term insurance counterparts.

For each of the following annuities in Numbers 1 through 12, draw an appropriate timeline and use ILT actuarial assumptions (mortality and interest) to determine

(a) an expression for the present value random variable, and
(b) the expected (actuarial) present value of the annuity.

1. an annuity of 5000, issued to a 35 year old, with the payment made at the beginning of each year for as long as the 35-year old is alive

2. a whole life annuity immediate of 5000 issued to (35)

3. a 17-year deferred whole life annuity due of 500 issued to (35)

4. a 17-year deferred whole life annuity immediate of 500 issued to (35)

5. a 2-year temporary annuity due of 750 issued to (32) (Also, determine the variance of the present value random variable for this annuity.)

6. a 17-year temporary annuity due issued to (35), with annual benefit of 8000 payable contingent on the life of (35)

7. a 17-year temporary life annuity immediate of 8000 issued to (35), payable annually

8. a 17-year certain-and-life annuity due issued to (35) with annual payments of 2000

9. a 17-year certain-and-life annuity immediate issued to (35) with annual payments of 2000

10. an annual payment annuity due of 1000 issued to independent lives aged 30 and 40, with payments lasting until the first death

11. an annual payment annuity due of 500 issued to independent lives aged 30 and 40, with payments continuing until the last death

12. an annual payment annuity due issued to independent lives aged 30 and 40, with payments of 1000 while both are alive, 750 continuing to the current 30-year old if the current 40-year old dies first, and 500 continuing to the current 40-year old if the current 30-year old dies first.

At this point in the "discrete insurance framework" we did a few DML and CF problems. In the "discrete annuity framework" we'll wait until a later section to address these types of problems.

13. In a homogeneous Markov model with 3 states: Healthy (H), Sick (S), and Dead (D), you are given:

    (i) the annual transition probabilities are

	H	S	D
H	.80	.15	.05
S	.40	.50	.10
D	0	0	1

    (ii) (x) is currently healthy

    A 3-year temporary annuity due with annual payments issued to (x) pays 100 if (x) is healthy and 25 if (x) is sick. Determine the actuarial present value of this annuity using an annual discount factor of 0.95.

14. A 4-year temporary annuity due with annual payments issued to (35) pays 1000 for the first year, 750 for the second year, 1250 for the third year, and 1500 for the fourth year. Given $q_{35+k} = 0.02 + 0.005k$, for k = 0, 1, and 2, determine the variance of the present value random variable for this annuity using d = 0.05.

15. A 20-year temporary life annuity due with annual payments issued to (30) has a first payment of 1000. Subsequent payments are 1% larger than their previous payments. Determine the actuarial present value of this annuity using ILT mortality and i = 7.06%.

16. A 30-year temporary life annuity due with annual payments, issued to (x), has an initial payment of 1000. Subsequent payments are 5% more than their preceding payment. Determine its APV, given ${}_{30}p_x = 0.4$, $e_{x:\overline{30|}} = 20.7$, and i = .05.

17. Draw the timeline, include the valuation date, that corresponds to the symbol $100(I\ddot{a})_x$.

18. An annual payment whole life annuity due issued to (25) pays 5 for the first year, 7 for the second year, and so on, where each year's payment is 2 more than the preceding year's benefit. Write an expression using actuarial notation for the EPV of this annuity.

19. A 10-year temporary annuity issued to (50), with benefit paid at the beginning of the year, has a payment of 1500 for the first year. For subsequent years, the payment is 100 less than the previous year's benefit. Write an expression using actuarial notation for the APV of this annuity.

For Numbers 20 and 21, determine the EPV of the annuity described, using
(a) ILT actuarial assumptions and the UDD assumption between integer ages
(b) ILT actuarial assumptions and the three term Woolhouse approximation

20. A whole life annuity issued to (40) with a benefit of 1000 payable at the beginning of each quarter

21. A 20-year deferred whole life annuity issued to (40) with a benefit of 1000 payable at the beginning of each quarter

22. Determine the APV of a 20-year temporary life annuity, issued to (40), with a benefit of 1000 paid at the beginning of each quarter, using ILT actuarial assumptions and the UDD assumption between integer ages.