

Solutions to MLC457 Exercises

$$1) (a) \quad {}_0V \stackrel{EP}{=} 0$$

For (b) - (e), we'll be using:

$$\text{TDE: } {}_t\dot{V} = {}_tV \cdot \delta + \pi - \mu_{x+t} (S - {}_tV)$$

$$\text{EM: } y(t+h) \approx y(t) + h \cdot \dot{y}(t) \Rightarrow {}_{t+h}V \approx {}_tV + h \cdot {}_t\dot{V}$$

$$(b) \quad {}_0\dot{V} = 0 \cdot \delta + 5 - 0(100 - 0) = 5$$

$$(c) \quad \left. \begin{array}{l} t=0 \\ h=.1 \end{array} \right\} \quad {}_{.1}V \approx {}_0V + .1({}_0\dot{V}) = 0 + .1(5) = .5$$

$$(d) \quad \begin{aligned} {}_{.1}\dot{V} &= {}_{.1}V(.05) + 5 - .002(100 - {}_{.1}V) \\ &\approx .5(.05) + 5 - .002(100 - .5) = 4.826 \end{aligned}$$

$$(e) \quad \left. \begin{array}{l} t=.1 \\ h=.1 \end{array} \right\} \quad \begin{aligned} {}_{.2}V &\approx {}_{.1}V + .1({}_{.1}\dot{V}) = .5 + .1(4.826) \\ &\Rightarrow {}_{.2}V \approx .9826 \end{aligned}$$

2) (See Video Solution)

$$(a) \quad {}_0V \stackrel{EP}{=} 0$$

$$(b) \quad {}_1\dot{V} = 4.8 + .052({}_1V)$$

$$(c) \quad {}_1V \approx .482509$$

$$(d) \quad {}_2\dot{V} = 4.6 + .054({}_2V)$$

$$(e) \quad {}_2V \approx .947626$$

$$3) (a) {}_5V = 500$$

For (b) - (e), we'll be using

$$\text{TDE: } {}_t\dot{V} = {}_tV \cdot \delta_t + \pi_t - e_t - \mu_{x+t} (S_t + E_t - {}_tV)$$

$$\text{EM: } \gamma(t+h) \approx \gamma(t) + h \cdot \dot{\gamma}(t) \Rightarrow {}_{t+h}V \approx {}_tV + h \cdot {}_t\dot{V}$$

$$(b) {}_5\dot{V} = 500(.05) + 7 - 3.5 - .001(1.1)^{45} (500 + 50 - 500)$$

$$\Rightarrow {}_5\dot{V} \approx 24.855476$$

$$(c) {}_{4.5}V = {}_5V - .5({}_5\dot{V}) = 500 - .5(24.855476)$$

$$\left. \begin{array}{l} t=5 \\ h=-.5 \end{array} \right\} \curvearrowright$$

$$\Rightarrow {}_{4.5}V \approx 487.572262$$

$$(d) {}_{4.5}\dot{V} = {}_{4.5}V \cdot (.045) + 6.5 - 3.25 - .001(1.1)^{44.5} (450 + 45 - {}_{4.5}V)$$

$$\Rightarrow {}_{4.5}\dot{V} \approx 24.674536$$

$$(e) {}_4V \approx {}_{4.5}V - .5({}_{4.5}\dot{V}) \stackrel{(c) \dot{V}(d)}{=} 475.235$$

$$\left. \begin{array}{l} t=4.5 \\ h=-.5 \end{array} \right\} \curvearrowright$$

$$4) (a) {}_5V = 500$$

For (b) - (e), we'll be using

$$\text{TDE: } \dot{V}_t = V_t \cdot \delta_t + \Pi_t - e_t - \mu_{x+t} (S_t + E_t - V_t)$$

$$\text{EM: } \gamma(t+h) \approx \gamma(t) + h \cdot \dot{\gamma}(t) \implies {}_{t+h}V \approx V_t + h \cdot \dot{V}_t$$

$$(b) {}_{4.5}\dot{V} = {}_{4.5}V \cdot (.045) + 6.5 - 3.25 - .001(1.1)^{44.5} (450 + 45 - {}_{4.5}V)$$

$$\implies {}_{4.5}\dot{V} = {}_{4.5}V \cdot (.045) + 3.25 - .001(1.1)^{44.5} (495 - {}_{4.5}V)$$

$$(c) \left. \begin{array}{l} t=4.5 \\ h=.5 \end{array} \right\} {}_5V \approx {}_{4.5}V + .5 \cdot ({}_{4.5}\dot{V})$$

Let ${}_{4.5}V = x$ and note ${}_5V = 500$

$$\therefore 500 \approx x + .5 \left(.045x + 3.25 - .001(1.1)^{44.5} (495 - x) \right)$$

$$\implies x = {}_{4.5}V \approx 487.657833$$

$$(d) {}_4\dot{V} = {}_4V \cdot (.04) + 6 - 3 - .001(1.1)^{44} (400 + 40 - {}_4V)$$

$$\implies {}_4\dot{V} = {}_4V \cdot (.04) + 3 - .001(1.1)^{44} (440 - {}_4V)$$

$$(e) \left. \begin{array}{l} t=4 \\ h=.5 \end{array} \right\} {}_{4.5}V \approx {}_4V + .5 \cdot ({}_4\dot{V})$$

Again, let $x = {}_4V$.

$$\therefore 487.657833 \approx x + .5 \left(.04x + 3 - .001(1.1)^{44} (440 - x) \right)$$

$$\implies x = {}_4V \approx 475.473$$