

## Often Tested Mortality Models

**DeMoivre's Law (DML):**  $X \sim U(0, \omega) \Rightarrow T = T(x) \sim U(0, \omega - x)$

Usually characterized in problems by

$$\mu(x) = \frac{1}{\omega - x} \quad \text{or} \quad l_x = l_0 \cdot \left(1 - \frac{x}{\omega}\right) \quad \text{or} \quad s(x) = {}_x p_0 = \frac{\omega - x}{\omega} \quad \text{or} \quad F(x) = {}_x q_0 = \frac{x}{\omega}$$

Other Important Formulas

$${}_t p_x = \frac{\omega - x - t}{\omega - x} \qquad {}_t q_x = \frac{t}{\omega - x} = t \cdot q_x$$

$$f_x(x) = \frac{1}{\omega} \qquad f_T(t) = {}_t p_x \cdot \mu(x + t) = \frac{1}{\omega - x} = q_x$$

$${}_{k|} q_x = {}_t q_x = \frac{t}{\omega - x}$$

$$e_x = \frac{\omega - x}{2} = e_x + \frac{1}{2}$$

$$\text{Var}(T(x)) = \frac{(\omega - x)^2}{12}$$

$$e_{x:\overline{n}|} = n - \frac{n^2}{2(\omega - x)}$$

$$e_{xx} = \frac{\omega - x}{3} \quad (\text{Independent lives } (x) \text{ and } (x))$$

$$A_{x:\overline{n}|}^1 = \frac{1}{\omega - x} \cdot a_{\overline{n}|}$$

$$\overline{A}_{x:\overline{n}|}^1 = \frac{1}{\omega - x} \cdot \overline{a}_{\overline{n}|}$$

$${}^2 A_{x:\overline{n}|}^1 = \frac{1}{\omega - x} \cdot \frac{a_{\overline{2n}|}}{s_{\overline{2}|}}$$

$${}^2 \overline{A}_{x:\overline{n}|}^1 = \frac{1}{\omega - x} \cdot \overline{a}_{\overline{2n}|}$$

$$A_x = \frac{1}{\omega - x} \cdot a_{\overline{\omega-x}|}$$

$$\overline{A}_x = \frac{1}{\omega - x} \cdot \overline{a}_{\overline{\omega-x}|}$$

$${}^2 A_x = \frac{1}{\omega - x} \cdot \frac{a_{\overline{2(\omega-x)}|}}{s_{\overline{2}|}}$$

$${}^2 \overline{A}_x = \frac{1}{\omega - x} \cdot \overline{a}_{\overline{2(\omega-x)}|}$$

**Constant Force (CF):**  $X \sim EX(\text{mean} = \frac{1}{\mu}) \Rightarrow T = T(x) \sim EX(\text{mean} = \frac{1}{\mu})$

Usually characterized in problems by

$$\mu(x) = \mu \quad \text{or} \quad {}_x p_0 = e^{-\mu \cdot x}$$

Other Important Formulas

$${}_t p_x = e^{-\mu t} \quad {}_t q_x = 1 - e^{-\mu t} \quad (\text{Neither of these depend on } x)$$

$$f_x(x) = \mu \cdot e^{-\mu x} \quad f_T(t) = {}_t p_x \cdot \mu(x+t) = \mu \cdot e^{-\mu t}$$

$${}_{k|} q_x = e^{-\mu k} (1 - e^{-\mu t})$$

$${}^0 e_x = \frac{1}{\mu} \quad e_x = \frac{1}{e^{\mu} - 1} \quad \text{Var}(T(x)) = \frac{1}{\mu^2}$$

$${}^0 e_{x:\overline{n}|} = e_x \cdot {}_n q_x = \frac{1 - e^{-\mu n}}{\mu} \quad e_{x:\overline{n}|} = e_x \cdot {}_n q_x = \frac{1 - e^{-\mu n}}{e^{\mu} - 1}$$

$$A_{x:\overline{n}|}^1 = \frac{q(1 - v^n \cdot p^n)}{q + i} \quad \overline{A}_{x:\overline{n}|}^1 = \frac{\mu}{\mu + \delta} \cdot (1 - {}_n E_x) = \frac{\mu}{\mu + \delta} \cdot (1 - e^{-(\mu + \delta)n})$$

$${}^2 A_{x:\overline{n}|}^1 = \frac{q(1 - v^{2n} \cdot p^n)}{q + 2i + i^2} \quad {}^2 \overline{A}_{x:\overline{n}|}^1 = \frac{\mu}{\mu + 2\delta} \cdot (1 - {}_n E_x^2) = \frac{\mu}{\mu + 2\delta} \cdot (1 - e^{-(\mu + 2\delta)n})$$

$$A_x = \frac{q}{q + i} \quad \overline{A}_x = \frac{\mu}{\mu + \delta}$$

$${}^2 A_x = \frac{q}{q + 2i + i^2} \quad {}^2 \overline{A}_x = \frac{\mu}{\mu + 2\delta}$$

If the life of (x) is CF( $\mu_1$ ) and the life of (y) is CF( $\mu_2$ ) and is independent of the life of (x), then replace  $\mu$  by  $\mu_1 + \mu_2$  in the respective formulas to get

formulas for  ${}^0 e_{xy}$ ,  $e_{xy}$ ,  ${}^0 e_{xy:\overline{n}|}$ ,  $e_{xy:\overline{n}|}$ ,  $\overline{A}_{xy:\overline{n}|}^1$ ,  ${}^2 \overline{A}_{xy:\overline{n}|}^1$ ,  $\overline{A}_{xy}$ , and  ${}^2 \overline{A}_{xy}$ .

**Generalized DeMoivre's Law (GDML):**

Usually characterized in problems by

$$\mu(x) = \frac{\alpha}{\omega - x} \quad \text{or} \quad l_x = l_0 \cdot \left(1 - \frac{x}{\omega}\right)^\alpha \quad \text{or} \quad s(x) = {}_x p_0 = \left(\frac{\omega - x}{\omega}\right)^\alpha$$

**Other Important Formulas**

$${}_t p_x = \left(\frac{\omega - x - t}{\omega - x}\right)^\alpha$$

$$e_x = \frac{\omega - x}{\alpha + 1} \quad \text{Var}(T(x)) = \frac{\alpha \cdot (\omega - x)^2}{(\alpha + 1)^2 (\alpha + 2)}$$

**UDD (Uniform Distribution of Deaths:  ${}_t d_x = t \cdot d_x$  where  $0 \leq t \leq 1$ )**

(Usually told to use this assumption, or told to linearly interpolate between integer ages.)

### Important Formulas

$$\mu(x+t) = \frac{q_x}{1-t \cdot q_x}$$

$$l_{x+t} = l_x - t \cdot d_x$$

$${}_t p_x = 1 - t \cdot q_x \qquad {}_t q_x = t \cdot q_x$$

$$f_T(t) = {}_t p_x \cdot \mu(x+t) = \frac{1}{\omega - x} = q_x$$

$${}^0 e_x = e_x + 0.5 \qquad {}^0 e_{x:\overline{n}|} = e_{x:\overline{n}|} + 0.5({}_n q_x) \qquad {}^0 e_{x:\overline{1}|} = 1 - 0.5 \cdot q_x$$

$$\text{Var}(T(x)) = \text{Var}(K(x)) + \frac{1}{12}$$

$$\text{For } 0 \leq s+t \leq 1, \quad {}_t q_{x+s} = \frac{t \cdot q_x}{1-s \cdot q_x}$$

$$m_x = \frac{q_x}{1-0.5 \cdot q_x} \quad (\text{central death rate at age } x)$$

$$\overline{A}_x = \frac{i}{\delta} A_x$$

$$\overline{A}_{x:\overline{n}|} = \frac{i}{\delta} A_{x:\overline{n}|}^1$$

$$\text{So, } \overline{A}_{x:\overline{n}|} = \overline{A}_{x:\overline{n}|}^1 + A_{x:\overline{n}|}^{\frac{1}{2}} = \frac{i}{\delta} A_{x:\overline{n}|}^1 + A_{x:\overline{n}|}^{\frac{1}{2}}$$

$${}_k V(\overline{A}_x) = \frac{i}{\delta} \cdot {}_k V_x$$

$${}_k V(\overline{A}_{x:\overline{n}|}) = \frac{i}{\delta} \cdot {}_k V_{x:\overline{n}|}^1$$

$$\text{So, } {}_k V(\overline{A}_{x:\overline{n}|}) = \frac{i}{\delta} \cdot {}_k V_{x:\overline{n}|}^1 + {}_k V_{x:\overline{n}|}^{\frac{1}{2}}$$