Often Tested Statistics Facts

1. Given the conditional distribution $X \mid Y$ and the marginal distribution for Y

   $\Pr(X < c) = \int_0^\infty \Pr(X < c \mid Y = y) \cdot f_Y(c)dy$ (Change inequality as necessary)

   $E[f(X)] = \int_0^\infty E[f(X) \mid Y = y] \cdot f_Y(y)dy$

   Double Expectation Theorem

   $E[X] = E\big[E[X \mid Y]\big]$

   $Var(X) = E\big[Var(X \mid Y)\big] + Var\big(E[X \mid Y]\big)$

2. If $N \mid \Lambda \sim P(\Lambda)$ and $\Lambda \sim \Gamma(\alpha, \theta)$ then $N \sim NB(r, \beta)$ where $r = \alpha$ and $\beta = \theta$.

   Special Case: $\Gamma(1, \theta) = EX(\theta)$ and $NB(1, \beta) = G(\beta)$, and so

   If $N \mid \Lambda \sim P(\Lambda)$ and $\Lambda \sim EX(\theta)$ then $N \sim G(\beta)$

3. If $M \mid N \sim B(N, q)$ and $N \sim NB(r, \beta)$ then $M \sim NB(r, \beta \cdot q)$.

   Context: *M* = the number of claims that exceed a deductible

4. If $X \sim Par(\alpha, \theta)$ then $r \cdot X \sim Par(\alpha, r \cdot \theta)$.

   θ is a scale parameter in the Pareto distribution

5. $\int_0^\infty t^k \cdot e^{-at} dt = \frac{\Gamma(k+1)}{a^{k+1}} = \frac{k!}{a^{k+1}}$ (the last equality if *k* is a non-negative integer)

6. If $\{X_i\}$, $i = 1, \ldots, n$ is a mutually independent collection of random variables, with $X_i \sim EX(\theta)$ for each *i*, then $\sum_{i=1}^{n} X_i \sim \Gamma(n, \theta)$

7. If $Y = g(X)$ and g is invertible, then the pdf of Y is given by

$$f_Y(y) = f_X(g^{-1}(y)) \cdot \left| \frac{d}{dy} g^{-1}(y) \right|$$

8. Moment Generating Function $M_X(t) = E[e^{t \cdot X}]$

$$M_X^{(n)}(0) = E[X^n]$$

9. Probability Generating Function $P_N(t) = E[t^N] = M_N(\ln(t))$

$$P_N^{(n)}(0) = n! \Pr(N = n)$$

$$P_N'(1) = E[N] \qquad P_N''(1) = E[N(N-1)] = E[N^2] - E[N]$$

$$P_N'''(1) = E[N(N-1)(N-2)] \quad \ldots$$