Merton Jump-Diffusion Revisited: A Lévy Copula Approach

Pierre Garreau [1] - Craig Nolder

5th Annual Graduate Student Conference in Probability
April 29 - May 01, 2011

[1]Financial Mathematics, Florida State University

1. Issues in option pricing

2. Modeling dependence

3. Risk neutral pricing

4. Results

The Black-Scholes syndrom

- Black-scholes diffusion: $dS_t = S_t\left(rdt + \sigma dW_t\right)$

The Black-Scholes syndrom

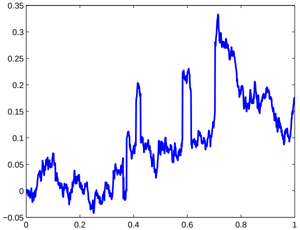
- Black-scholes diffusion: $dS_t = S_t\left(rdt + \sigma dW_t\right)$
- Pricing - Arbitrage Theory / Feynman-Kac:

$$P(t,x) = \mathbb{E}^{\mathbb{Q}}\left[f(\widetilde{S}_T^{t,x})|\mathcal{F}_t\right]$$

- Hedging - Martingale representation theorem:

$$V_t = c + \int_0^t \phi_s d\widetilde{S}_s, \quad \mathbb{Q} - a.s.$$

The Black-Scholes syndrom



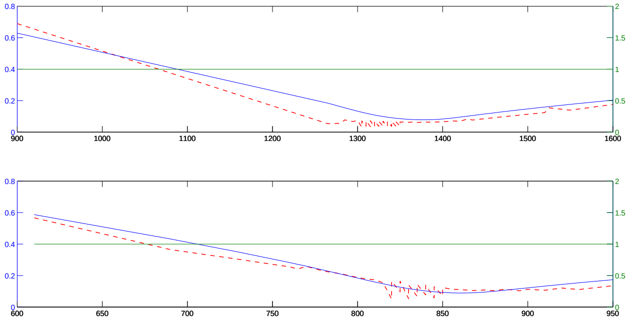
- Black-scholes diffusion: $dS_t = S_t \left(rdt + \sigma dW_t + dq_t \right)$
- Pricing - Arbitrage Theory / Feynman-Kac:

$$P(t,x) = \mathbb{E}^{\mathbb{Q}} \left[f(\widetilde{S}_T^{t,x}) | \mathcal{F}_t \right]$$

- Hedging - Martingale representation theorem:

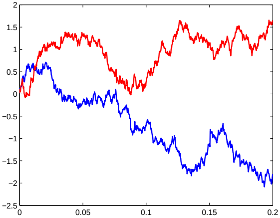
$$V_t = c + \int_0^t \phi_s d\widetilde{S}_s, \quad \mathbb{Q} - a.s.$$

The Black-Scholes syndrom



$$\sigma^* = C^{-1}(K, S, T, r), \quad C(K, S, T, r) = N(d_{t,1}) - e^{k_t} N(d_{t,2})$$

Risk neutral in two dimensions



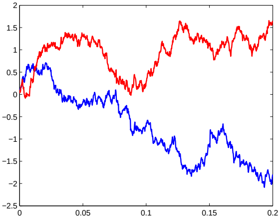
$\nearrow \mathbb{Q}_1$ $$d\tilde{S}^1_t/\tilde{S}^1_t = \sigma_1 dB^1_t$$

$\searrow \mathbb{Q}_2$ $$d\tilde{S}^2_t/\tilde{S}^2_t = \sigma_2 dB^2_t$$

$$P_1(t,x) = \mathbb{E}^{\mathbb{Q}_1}\left[f(\widetilde{S}^{1,x}_T)|\mathcal{F}_t\right]$$

$$P_2(t,x) = \mathbb{E}^{\mathbb{Q}_2}\left[f(\widetilde{S}^{2,x}_T)|\mathcal{F}_t\right]$$

Risk neutral in two dimensions



$\nearrow \mathbb{Q}_1$

$$d\tilde{S}^1_t/\tilde{S}^1_t = \sigma_1 dB^1_t + \sqrt{\rho\sigma_1\sigma_2} dB^2_t$$

$\searrow \mathbb{Q}_2$

$$d\tilde{S}^2_t/\tilde{S}^2_t = \sigma_2 dB^2_t + \sqrt{\rho\sigma_1\sigma_2} dB^1_t$$

$$P_1(t,x) = \mathbb{E}^{\mathbb{Q}_1}\left[f(\widetilde{S}^{1,x}_T)|\mathcal{F}_t\right]$$

$$P_2(t,x) = \mathbb{E}^{\mathbb{Q}_2}\left[f(\widetilde{S}^{2,x}_T)|\mathcal{F}_t\right]$$

and $\mathbb{Q}_i = \mathbb{E}\left[\frac{d\mathbb{Q}}{d\mathbb{P}}|_{\mathcal{F}^i_t}\right], \frac{d\mathbb{Q}}{d\mathbb{P}}|_{\mathcal{F}_t} = e^{\langle\theta,W_t\rangle - \|\theta\|^2 t}$.

The problem

Problem (Conceptually)

Price two options simulatenously on underlyings which dependence structure is not trivial while respecting the term structure of the volatility on both assets.

The problem

Problem (Least Square Minimization)

Given a market $S^\xi = e^{rt+X^\xi}$ and a collection of option prices $\{C_{k,*}(K_i, T_j)\}_{ij}$ for different strikes K_i and maturities T_j, $i, j \in \mathcal{I}$ find ξ^* which minimizes

$$\sum_{k=1}^{d} \left\{ \sum_{i,j\in\mathcal{I}} \omega_{ij}^k \mid C_{k,*}(K_i^k, T_j^k) - C(K_i^k, T_j^k, S^{\xi,k}) \mid^2 \right\}.$$

The problem

Problem (Least Square Minimization)

Given a market $S^{\xi} = e^{rt+X^{\xi}}$ and a collection of option prices $\{C_{k,*}(K_i, T_j)\}_{ij}$ for different strikes K_i and maturities T_j, $i, j \in \mathcal{I}$ find ξ^* which minimizes

$$\sum_{k=1}^{d} \left\{ \sum_{i,j\in\mathcal{I}} \omega_{ij}^{k} \mid C_{k,*}(K_i^k, T_j^k) - C(K_i^k, T_j^k, S^{\xi,k}) \mid^2 \right\}.$$

- $S^{\xi,k}$ - Construct a multidimentional stochastic process - Lévy copulae.

The problem

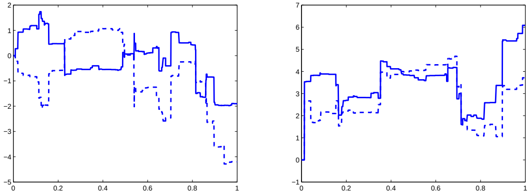
Problem (Least Square Minimization)

Given a market $S^{\xi} = e^{rt+X^{\xi}}$ and a collection of option prices $\{C_{k,*}(K_i, T_j)\}_{ij}$ for different strikes K_i and maturities T_j, $i,j \in \mathcal{I}$ find ξ^* which minimizes

$$\sum_{k=1}^{d} \left\{ \sum_{i,j \in \mathcal{I}} \omega_{ij}^{k} \mid C_{k,*}(K_i^k, T_j^k) - C(K_i^k, T_j^k, S^{\xi,k}) \mid^2 \right\}.$$

- $\mathbb{Q}$ - Find an arbitrage-free pricing measure which includes the dependence of assets - Esscher Transform.
- $C(K_i^1, T_j^1, S^{\xi,k})$ - Compute derivative prices - Fourier Transform.

Constructing a multidimensional Lévy process



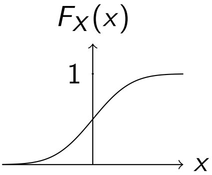
Definition (Copula)

A 2-dimensional **copula** is a distribution function on $[0,1]^2$ such that $C_k(u) = u, \; k = 1, 2$

Theorem (Sklar's Theorem)

Consider an 2-dimensional distribution function F with marginals $F_i, \; i = 1, 2$. Then there exist a Copula C such that

$$\mathbb{P}(X_1 \le x_1, X_2 \le x_2) = C(\mathbb{P}(X_1 \le x_1), \mathbb{P}(X_2 \le x_2)).$$



$$\mathbb{P}(X_1 \le F_1^{-1}(x_1), X_2 \le F_2^{-1}(x_2))$$

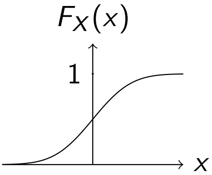
Definition (Copula)

A 2-dimensional **copula** is a distribution function on $[0,1]^2$ such that $C_k(u) = u, \;\; k = 1, 2$

Theorem (Sklar's Theorem)

Consider an 2-dimensional distribution function F with marginals $F_i, \;\; i = 1, 2$. Then there exist a Copula C such that

$$\mathbb{P}(X_1 \leq x_1, X_2 \leq x_2) = C(\mathbb{P}(X_1 \leq x_1), \mathbb{P}(X_2 \leq x_2)).$$



$$\mathbb{P}(X_1 \leq F_1^{-1}(x_1), X_2 \leq F_2^{-1}(x_2))$$

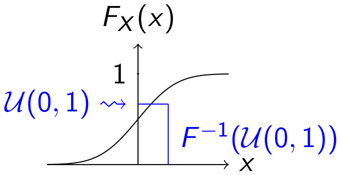
Definition (Copula)

A 2-dimensional **copula** is a distribution function on $[0,1]^2$ such that $C_k(u) = u, \;\; k = 1, 2$

Theorem (Sklar's Theorem)

Consider an 2-dimensional distribution function F with marginals $F_i, \;\; i = 1, 2$. Then there exist a Copula C such that

$$\mathbb{P}(X_1 \le x_1, X_2 \le x_2) = C(\mathbb{P}(X_1 \le x_1), \mathbb{P}(X_2 \le x_2)).$$



$$\mathbb{P}(X_1 \le F_1^{-1}(x_1), X_2 \le F_2^{-1}(x_2))$$

Lévy processes

Definition (Lévy processes)

A càdlàg stochastic process $\{X_t\}_{t\geq 0}$ on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with independent and stationary increments is called a **Lévy process**.

Theorem (Lévy-Khintchine, Compound Poisson)

Let X be a $2-$dimensional Lévy process in $\mathbb{R}^2$, with characteristic triplet $(0, \nu, 0)$, such that $\nu(\mathbb{R}) < \infty$. Then,

$$\phi_t(u) = \mathbb{E}[e^{i\langle u, X_t\rangle}] = e^{t\psi(u)} = \int_{\mathbb{R}^2} (e^{i\langle u,x\rangle} - 1)\nu(dx)$$

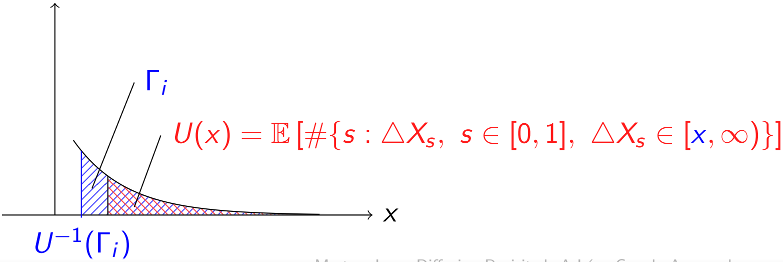
$$\nu(A) = \mathbb{E}\left[\#\{t \in [0,1] : (\triangle X_t^1, \triangle X_t^2) \in A\}\right], \; A \in \mathcal{B}(\mathbb{R}^2 \setminus \{0\})$$

Tail Integrals

Definition

The tail integral of a Lévy process on $(0,\infty)^2$ is defined as $U(x,y) = \nu([x,\infty)\times[y,\infty))$, with the convention that $U(0,0)=\infty$. The marginal tail integrals are $U_X(x) = U(x,0)$ and $U_Y(y) = U(0,y)$.

$\nu(x) = ce^{-\lambda x}1_{x>0}$

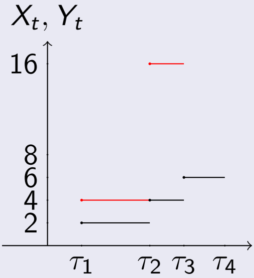


Tail Integrals

Definition

The tail integral of a Lévy process on $(0, \infty)^2$ is defined as $U(x, y) = \mathbb{E}\left[\#\left\{\triangle X_t \geq x, \triangle Y_t \geq y, t \in [0,1]\right\}\right].$

Example ($X_t = 2N_t$, $Y_t = X_t^2$.)



$$U(x, y) = \lambda.1_{x \leq 2, y \leq 4}$$

Definition (Levy copula)

A **Lévy Copula** is a function $L : \overline{\mathbb{R}}_+^2 \to \overline{\mathbb{R}}$ such that

(i) $L(u, v) \neq \infty$ for $(u, v) \neq (\infty, \infty)$

(ii) $L(u, v) = 0$ if $u_i = 0$ for at least one $i \in \{1, 2\}$

(iii) L is 2-increasing

(iv) $L^{\{i\}}(u) = u$ for any $i \in \{1, 2\}$, $u \in \mathbb{R}$.

Theorem (Sklar's theorem for Lévy Copulas [Tan04])

(1) *Given an $\mathbb{R}^2$-valued Lévy process (X, Y), there exists a Lévy copula C so that*

$$U(x, y) = C(U_X(x), U_Y(y)) \tag{2.1}$$

(2) *Conversly, for a 2-dimensional Lévy copula C and U_1, U_2 tail integrals of real-valued Lévy processes, there exists an $\mathbb{R}^2$-valued Lévy process (X, Y) whose components have tail integrals U_X, U_Y and whose marginal tail integrals satify (2.1). The Lévy measure ν of X is then uniquely determined by C and U_i for $i = 1, 2$.*

$$U(x, y) = \mathbb{E}\left[\#\left\{\triangle X_t \geq x, \triangle Y_t \geq y, t \in [0, 1]\right\}\right].$$

Theorem (Sklar's theorem for Lévy Copulas [Tan04])

(1) *Given an $\mathbb{R}^2$-valued Lévy process (X, Y), there exists a Lévy copula C so that*

$$U(x, y) = C(U_X(x), U_Y(y)) \tag{2.1}$$

(2) *Conversly, for a 2-dimensional Lévy copula C and U_1, U_2 tail integrals of real-valued Lévy processes, there exists an $\mathbb{R}^2$-valued Lévy process (X, Y) whose components have tail integrals U_X, U_Y and whose marginal tail integrals satify (2.1). The Lévy measure ν of X is then uniquely determined by C and U_i for $i = 1, 2$.*

$$\begin{aligned} &U(U_X^{-1}(x), U_Y^{-1}(y)) \\ &= \mathbb{E}\left[\#\left\{\triangle X_t \geq U_X^{-1}(x), \triangle Y_t \geq U_Y^{-1}(y), t \in [0,1]\right\}\right]. \end{aligned}$$

Theorem (Sklar's theorem for Lévy Copulas [Tan04])

(1) *Given an $\mathbb{R}^2$-valued Lévy process (X, Y), there exists a Lévy copula C so that*

$$U(x, y) = C(U_X(x), U_Y(y)) \tag{2.1}$$

(2) *Conversly, for a 2-dimensional Lévy copula C and U_1, U_2 tail integrals of real-valued Lévy processes, there exists an $\mathbb{R}^2$-valued Lévy process (X, Y) whose components have tail integrals U_X, U_Y and whose marginal tail integrals satify (2.1). The Lévy measure ν of X is then uniquely determined by C and U_i for $i = 1, 2$.*

$$C(x, y) = \mathbb{E}\left[\#\left\{U_X(\triangle X_t) \leq x, U_Y(\triangle Y_t) \leq y, t \in [0, 1]\right\}\right].$$

Theorem (Sklar's theorem for Lévy Copulas [Tan04])

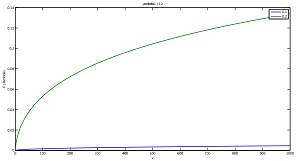
(1) *Given an $\mathbb{R}^2$-valued Lévy process (X, Y), there exists a Lévy copula C so that*

$$U(x, y) = C\left(U_X(x), U_Y(y)\right) \tag{2.1}$$

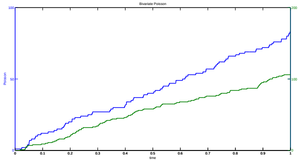
(2) *Conversly, for a 2-dimensional Lévy copula C and U_1, U_2 tail integrals of real-valued Lévy processes, there exists an $\mathbb{R}^2$-valued Lévy process (X, Y) whose components have tail integrals U_X, U_Y and whose marginal tail integrals satify (2.1). The Lévy measure ν of X is then uniquely determined by C and U_i for $i = 1, 2$.*

$$C(x, y) = \mathbb{E}\left[\#i : \{\Gamma_i \leq x, \Upsilon_i \leq y, t \in [0, 1]\}\right].$$

$C(u,v) = (u^{-\eta} + v^{-\eta})^{-1/\eta}$



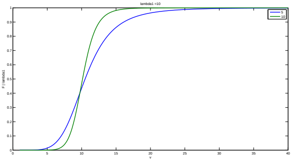
(c) $\eta = 0.1, \lambda_1 = 10$



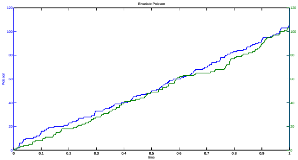
(d) $\eta = 0.1, \lambda_1 = 100$

Figure: Conditional distribution of the number of jumps in Y, given $\mathbb{E}[\#\triangle X_t] = \lambda_1$ and simulations of the corresponding Bidimensional Poisson Process.

$$C(u, v) = (u^{-\eta} + v^{-\eta})^{-1/\eta}$$



(a) $\eta = 10, \lambda_1 = 10$



(b) $\eta = 10, \lambda_1 = 100$

Figure: Conditional distribution of the number of jumps in Y (1(c) and **??**) given $\mathbb{E}[\#\triangle X_t] = \lambda_1$ and Simulations of the corresponding Bidimensional Poisson Process (1(d) and **??**).

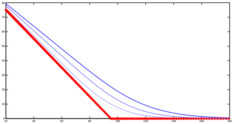
The model

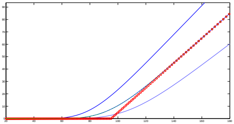
The aim of this section is to price 2 vanilla options on securities displaying a jump dependence. We are considering a 2-dimensional market $(S_t)_{t\geq 0} = (S_t^1, S_t^2)_{t\geq 0}$ such that

$$\begin{aligned} S_t^i &= e^{rt+X_t^i} \\ X_t^i &= G_t^i + Y_{N_t^i} \end{aligned} \qquad (2.2)$$

where $(Y_t)_{t\geq 0}$ is an 2-dimensional Lévy process with indendent components and $(N_t)_{t\geq 0}$ a 2-dimensional Poisson process with parameters λ_1, λ_2 coupled with a Lévy copula C so that $Y_{N_t^i}$ denotes the i^{th} component of the jump process.

Pricing options simultaneously on dependent underlyings





Essher Transform

Given a probability measure $\mathbb{P}$, we define $\mathbb{Q}$ by

$$\left.\frac{d\mathbb{Q}}{d\mathbb{P}}\right|_{\mathcal{F}_t} = \frac{e^{\langle\theta, X_t\rangle}}{\mathbb{E}^{\mathbb{P}}\left[e^{\langle\theta, X_t\rangle}\right]}, \quad \theta \in \mathbb{R}^d. \tag{3.1}$$

Note that we need to assume $\mathbb{E}^{\mathbb{P}}\left[e^{\theta X_t}\right] < \infty$, which is equivalent to $\int_{|x|\geq 1} e^{\langle\theta, X_t\rangle}\nu(dx) < \infty$.

Proposition (Lévy triplet under Esscher Transform)

A d-dimensional Lévy process $(X_t)_{\{t\geq 0\}}$, with triplet (γ, Σ, ν) under $\mathbb{P}$, admits the representation $(\tilde{\gamma}, \tilde{\Sigma}, \tilde{\nu})$ under $\mathbb{Q}$ by Esscher Transform with :

$$\begin{cases} \tilde{\Sigma} & = & \Sigma \\ \tilde{\nu}(dx) & = & e^{\langle\theta, x\rangle}\nu(dx) \\ \tilde{\gamma} & = & \gamma + \Sigma\theta + \int_{|x|<1}\left(e^{\langle\theta, x\rangle} - 1\right) x\nu(dx) \end{cases}$$

Fourier Transform

Proposition ([Tan04])

Define the truncated time value of the option by

$$\tilde{\kappa}_T(k) = \kappa_T(k) - C^{BS}(k),$$

where $C^{BS}(k)$ is the corresponding normalized Black-Scholes option price, then for an exponential martingale e^{X_T} such that $\exists \alpha > 0,\ \mathbb{E}\left[e^{(1+\alpha)X_T}\right] < \infty$, the Fourier transform of the time value of the option defined by $\zeta_T(\upsilon) = \int_{-\infty}^{\infty} e^{i\upsilon k}\tilde{\kappa}_T(k)dk$, is

$$\zeta_T(\upsilon) = \frac{\Phi_T(\upsilon - i) - \Phi_T^{BS}(\upsilon - i)}{i\upsilon(1 + i\upsilon)}$$

$$C(s,t) = s\left[C^{BS}(k) + \frac{1}{2\pi}\int_{\mathbb{R}} \zeta_T(\upsilon)e^{-i\upsilon k}d\upsilon\right] \quad (3.2)$$

Key Result

Proposition

Let $(N_t)_{t\geq 0}$ be a d-dimensional subordinator with dependence structure given a Lévy Copula C, so that $N_t^i \sim \mathcal{P}(\lambda_i)$, and let $(Y_t)_{t\geq 0}$ be a d-dimensional Lévy process with independent components. Then the characteristic function $\phi_{Y_{N_t}}$ of the subordinated process is given by

$$\phi_{Y_{N_t}}(u) = \exp t\left[\sum_{k=1}^{d} \lambda_k^{\perp}(e^{\psi_{Y_t^k(u_k)}} - 1) + \lambda(e^{\sum_{k=1}^{d} \psi_{Y_t^k(u_k)}} - 1)\right],$$

with

$$\lambda = C(\lambda_1, \ldots, \lambda_d) \text{ and } \lambda_k^{\perp} = \lambda_k - \lambda.$$

In particular, this allows to compute the marginals $\phi_{Y_{N_t}}(u_1, 0)$ and $\phi_{Y_{N_t}}(0, u_2)$, and the martingale conditions $\phi(-i) = 0$.

The model

The model

Lévy Copula $C(u,v) = (u^{-\eta} + v^{-\eta})^{-1/\eta}$ → $\Phi^{\eta,\xi}$

Subordination $X_t^{\xi} = G_t + Y_{N_t}$ → $\Phi^{\eta,\xi}$

The model

Lévy Copula $C(u,v) = (u^{-\eta} + v^{-\eta})^{-1/\eta}$

Subordination $X_t^{\xi} = G_t + Y_{N_t}$

$\Phi^{\eta,\xi}$

$\mathbb{Q}$

$\Phi^{\eta,\xi,\theta}$

The model

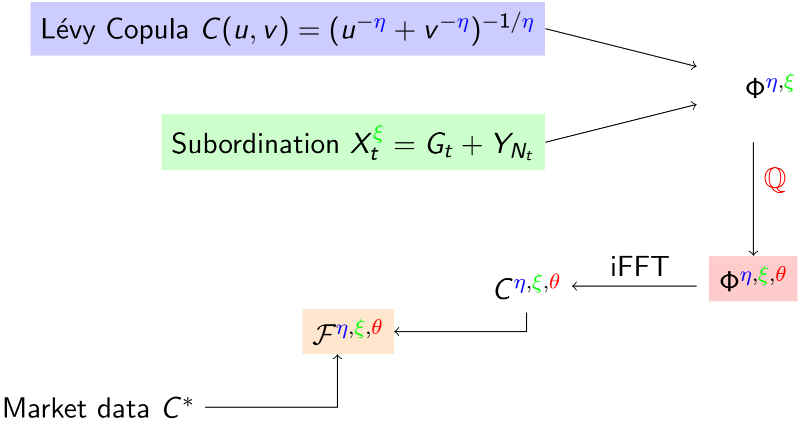
Lévy Copula $C(u, v) = (u^{-\eta} + v^{-\eta})^{-1/\eta}$ → $\Phi^{\eta,\xi}$

Subordination $X_t^{\xi} = G_t + Y_{N_t}$ → $\Phi^{\eta,\xi}$

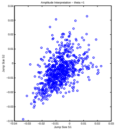
$\Phi^{\eta,\xi}$ —$\mathbb{Q}$→ $\Phi^{\eta,\xi,\theta}$

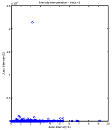
$C^{\eta,\xi,\theta}$ ←iFFT— $\Phi^{\eta,\xi,\theta}$

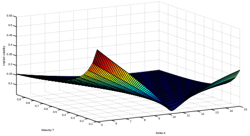
The model


Lévy Copula $C(u,v) = (u^{-\eta} + v^{-\eta})^{-1/\eta}$
Subordination $X_t^{\xi} = G_t + Y_{N_t}$
$\Phi^{\eta,\xi}$
$\mathbb{Q}$
$\Phi^{\eta,\xi,\theta}$
iFFT
$C^{\eta,\xi,\theta}$
$\mathcal{F}^{\eta,\xi,\theta}$
Market data C^*

Results


Amplitude Interpretation – theta =1
Jump Size S2
Jump Size S1


Intensity interpretation – theta =1
x 10^4
Jump Intensity S2
Jump Intensity S1


Implied volatility
Maturity T
Strike K

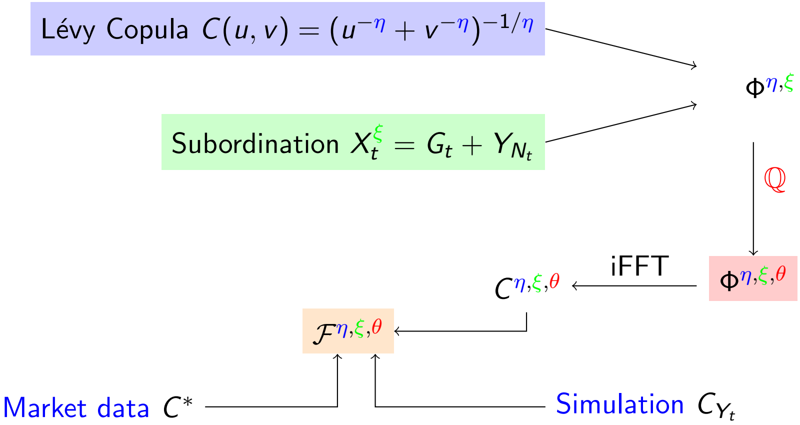
2-d compound Poisson coupled with a Lévy Copula

We address here a generalized version of Merton Jump-Diffusion model $Y \sim (\mu, \delta)$.

$$\phi^{\mathbb{Q}}_{X^i_t}(u) = \exp t\left[iu\left(-\frac{\sigma_i^2}{2} - \bar{\lambda}_i(e^{\mu+\delta^2+\theta_i\delta^2} - 1)\right) - \frac{u^2\sigma_i^2}{2} + \bar{\lambda}_i\left(e^{iu(\mu+\delta^2\theta_i)-\frac{u^2\delta^2}{2}} - 1\right)\right],$$

with $\bar{\lambda}_i = \left(\lambda_i + \lambda\left(e^{\mu\theta_j+\delta^2\theta_j^2} - 1\right)\right)e^{\theta_i(\mu+\delta^2/2)}$ and $\lambda = C(\lambda_1, \lambda_2)$.
We specify the dependence with a Clayton Copula,
$C(u, v) = (u^{-\eta} + v^{-\eta})^{-1/\eta}$.

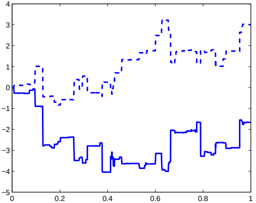
The model


Lévy Copula $C(u,v) = (u^{-\eta} + v^{-\eta})^{-1/\eta}$
Subordination $X_t^{\xi} = G_t + Y_{N_t}$
$\Phi^{\eta,\xi}$
$\mathbb{Q}$
$\Phi^{\eta,\xi,\theta}$
iFFT
$C^{\eta,\xi,\theta}$
$\mathcal{F}^{\eta,\xi,\theta}$
Market data C^*
Simulation C_{Y_t}

Variance Gamma

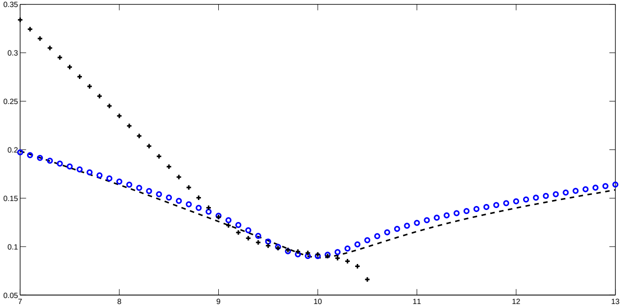
We first generate option prices from two variance gamma processes. Recall that the Variance gamma process is defined as a Gaussian process, the time scale of which is changed to a Gamma process $Y_t = \mu X_t + \sigma W_{X_t}$:

$$\begin{aligned} \mathbb{E}\left[e^{iuY_t}\right] &= \phi_{X_t}\left(uc + i\frac{u^2\sigma^2}{2}\right) \\ &= \left(1 + \frac{\sigma^2 u^2}{2\gamma} - i\frac{uc}{\gamma}\right)^{-\frac{t}{\gamma}} \end{aligned}$$



It is an example of Lévy process with infinite intensity of jumps.

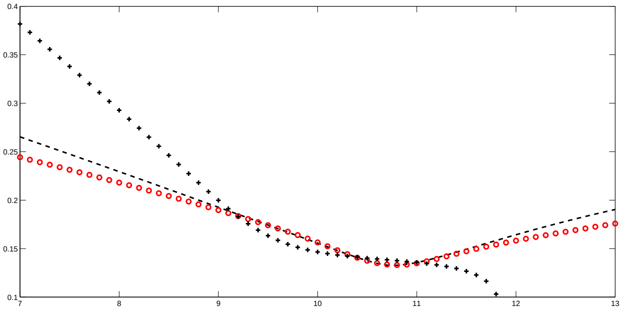
Calibration results (1)



(e) $S_1 \mid c = 0.05, \gamma_1 = 5$

Figure: Parameters : $S_1 = 9.75,\ S_2 = 10.5,\ r = 5\%,\ T = 0.2$

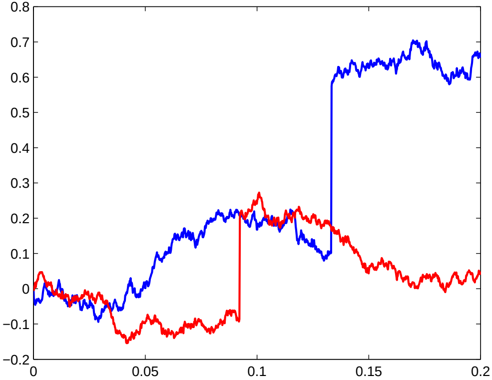
Calibration results (1)



(a) $S_2 \mid c = 0.05, \gamma_2 = 7$

Figure: Parameters : $S_1 = 9.75,\ S_2 = 10.5,\ r = 5\%,\ T = 0.2$

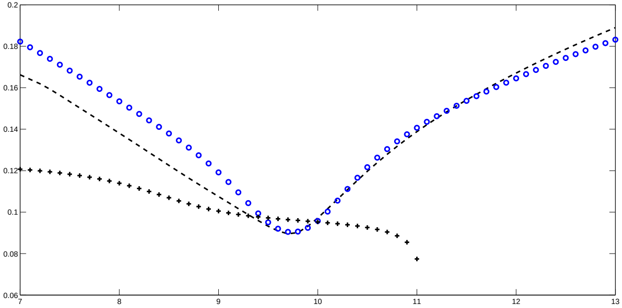
Calibration results (1)



(a) $\eta = 0.98$

Figure: Parameters : $S_1 = 9.75,\ S_2 = 10.5,\ r = 5\%,\ T = 0.2$

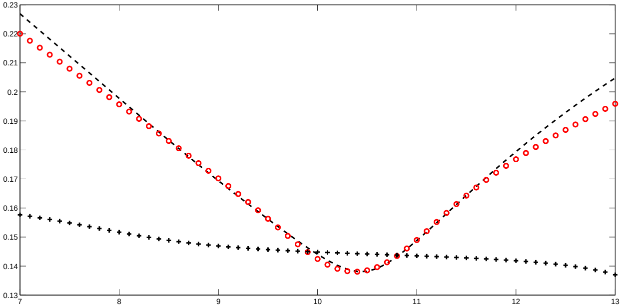
Calibration results(2)



(a) $S_1 \mid c = -0.05, \gamma_1 = 7$

Figure: Parameters : $S_1 = 9.75,\ S_2 = 10.5,\ r = 5\%,\ T = 0.2$

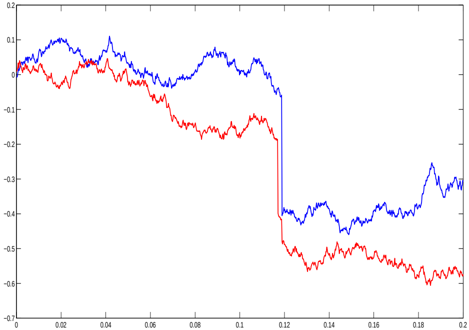
Calibration results(2)



(a) $S_2 \mid c = -0.05, \gamma_1 = 7$

Figure: Parameters : $S_1 = 9.75$, $S_2 = 10.5$, $r = 5\%$, $T = 0.2$

Calibration results(2)



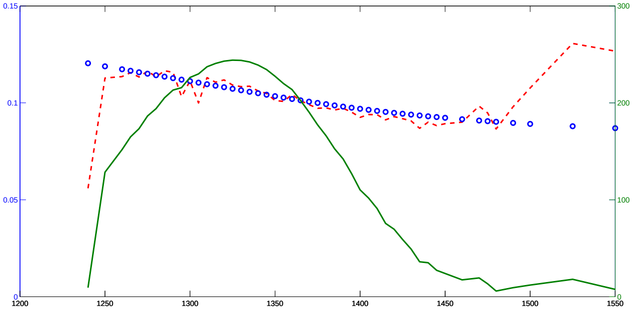
(a) $\eta = 7.03$

Figure: Parameters : $S_1 = 9.75,\ S_2 = 10.5,\ r = 5\%,\ T = 0.2$

Calibration on market data

We consider European Options on the American indexes Standard & Poor's 500 Index and Rusell 2000 Index, the former focusing on a wide variety of industries and the latter reflecting the behavior of the largest stocks in the U.S. Two maturities are considered: April (1 month) and June(3 months).

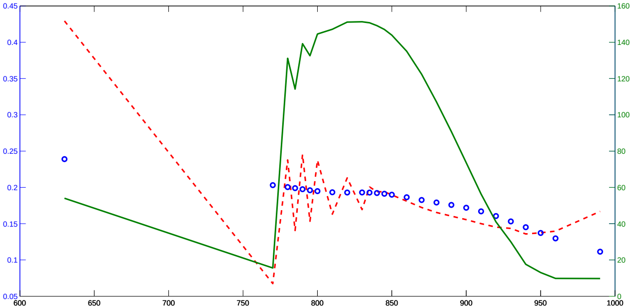
Market data - May



(a) S&P

Figure: $\rho = 0.89 \mid \eta = 12.56, \mu = -0.065, \delta = 0.002$

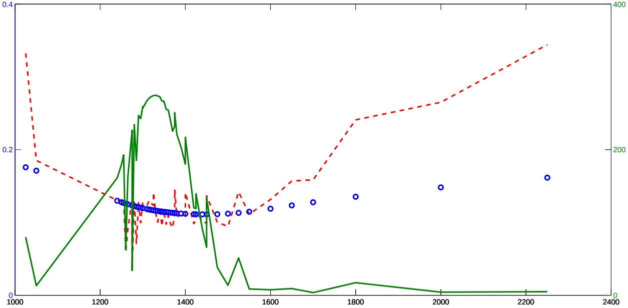
Market data - May



(a) Russel

Figure: $\rho = 0.89 \mid \eta = 12.56, \mu = -0.065, \delta = 0.002$

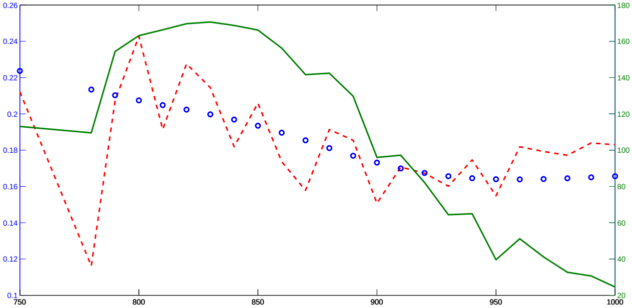
Market data - June



(a) S&P

Figure: $\rho = 0.96 \mid \eta = 13.50, \mu = -0.052, \delta = 0.063$

Market data - June



(a) Russel

Figure: $\rho = 0.96 \mid \eta = 13.50, \mu = -0.052, \delta = 0.063$

Conclusions

- Key result: It is possible to calibrate a multidimensional Lévy process on a basket of vanilla option prices while retrieving dependence structure.

- Non-parametric ?

$$\mathbb{E}\left[e^{iuX_t}\right] \sim \exp t\left(i\gamma u - \frac{\sigma^2 u^2}{2} + \sum_{-L}^{L}(e^{iux_i} - 1)\nu_i\right)$$

- Esmaeli H. and Klüpelberg (2010), Parameter estimation of a bivariate compound Poisson process, Insurance: Mathematics and Economics.
- El-Bachir N. (2008), Dependent jump processes with coupled Lévy measures, ICMA Centre, University of reading.
- Tankov P. (2004), Lévy processes in finance: Inverse problems and dependence modelling, PhD thesis, Ecole Polytechnique, France.