Introduction to Sets and Functions,

Overview: In the first chapter of the Course Lecture notes we will cover basic sets and functions. You will need to become familiar with the definitions and notation given in the lecture notes for these sections.

Introduction to Sets

How to Study: Before beginning each section, briefly look over the exercises assigned for a grade if available. Next read the Course lecture notes. As you read the course notes, work the exercises in the notes. Refer to the sections given below in the text for more examples and alternative explanations. As we cover the material in class, ask questions about concepts that you do not understand. Go to office hours to ask questions beyond what we have time for in class.

Read and use the practice exercises from the text listed here to supplement the areas that you are having difficulty with or not covered by exercises in the notes. This course will take time! To survive this course you will need to read the lecture notes, work all exercises in the lecture notes, and work practice exercises in the text on concepts you have difficulty with, in addition to the exercises you must do for a grade.

Reading:

- Course Lecture Notes Chapter 1 Section 1 Introduction to Sets
- Rosen Section 2.1 pages 111-121 Through Example 12, Cartesian Products through Example 18
- Rosen Section 2.2 pages 121-133 Through Example 9, Generalized Unions and Intersections through Example 15

Practice Exercises: The practice exercises will help you with the concepts needed to work the exercises you will turn in. **Work the exercises in the course notes!!!** The exercises listed here should be considered supplementary to the exercises in the notes and you should use them to work on the areas that you need more practice on.

The Student's Solution Manual has solutions to many of the practice exercises in the text. Beware, though, proofs in the solutions are often only outlines of what should really be written. You may bring up questions on these problems on our class discussion board or in class.

- Rosen Section 2.1 p. 119-121 # 1-9 odd, 17-25 odd, 28-29, 31
- Rosen Section 2.2 p. 130-133 # 1, 3, 14, 25, 33,

Introduction to Functions

Read:

- Course Lecture Notes Chapter 1 Section 2 Introduction to Functions

- Rosen Section 2.3 pages 133-149 Through Example 7, Some Important Functions through Example 26

Practice Exercises: Rosen Section 2.3 p. 146-149 # 1-9 odd, 27, 39

Logic

Overview: In this Chapter we cover formal logic. Here you will learn to solve problems that integrate the concepts and principles of logic. In Logic and Propositional Equivalences you will learn to solve problems that integrate the concepts and principles of logic. The material covered in this chapter lays the foundation for constructing a proof.

Read:

- Course Lecture Notes Chapter 2 Section 1 Logic
- Rosen Section 1.1 pages 1-21 All

Practice Exercises: Rosen Section 1.1 p. 16-21 # 1-5 odd, 9-17 every other odd, 19-25 odd, 29, 33, 47-53 odd

Propositional Equivalences

Read:

- Course Lecture Notes Chapter 2 Section 2 Propositional Equivalences
- Rosen Section 1.2 pages 21-30 All

Practice Exercises: Rosen Section 1.2 p. 28-30 # 5-15 odd, 17-33 every other odd, 41, 47, 49, 53

Predicates and Quantifiers

Overview: With this section we continue our discussion of logic by covering predicate functions, the universal quantifier, the existential quantifier, and the unique existential quantifiers. One important concept you should master is to translate statements in English into logical expressions using predicates, quantifiers, and logical connectives. Two more important concepts are to learn to recognize equivalent expressions and find the negations of statements.

Read:

- Course Lecture Notes Chapter 2 Section 3 Predicates and Quantifiers
- Rosen Section 1.3 pages 30-50 All
- Rosen Section 1.4 pages 50-62 All

Practice Exercises:

- Rosen Section 1.3 p. 46-50 # 1-41 every other odd, 51, 53, 59
- Rosen Section 1.4 p. 58-62 # 1-45 every other odd

- Rosen Section 2.1 p. 119-121 # 33, 35

Logical Arguments and Formal Proofs

Overview: The rules of inference and how they are applied to tell if an argument is valid of invalid is what we cover in this section.

Read:

- Course Lecture Notes Chapter 3 Section 1 Logical Arguments and Formal Proofs
- Rosen Section 1.5 pages 63-74 All

Practice Exercises: Rosen Section 1.5 p. 72-74 # 1, 3, 13, 15, 17, 19, 23, 27, 29,

Methods of Proof

Overview: The principles we learn in this section will be used for the remainder of the course and for many of your future courses. Some of the methods of proof we discuss are the direct proof, proof using the contrapositive, proof by contradiction, and proof by cases.

Read:

- Course Lecture Notes Chapter 3 Section 2 Methods of Proofs
- Rosen Section 1.6 pages 75-86 All
- Rosen Section 1.7 pages 86-104 through Proof Strategy in Action
- Rosen Section 5.2 pages 347-354 through Example 10

Practice Exercises:

- Rosen Section 1.6 p. 85-86 # 1-21 every other odd, 23, 27, 31, 33, 35, 39, 41
- Rosen Section 1.7 p. 102-104 # 1-13 every other odd, 15, 17, 21, 27, 31
- Rosen Section 5.2 p. 353-354 # 1-17 odd

Induction

Overview: Many of the proofs you will be asked to do this semester will use the principles of induction. This method of proof is also an important concept in many computer science application. An understanding of induction is critical to the computer science major.

Summation and product notation will be used in this course. This is covered in precalculus and will not be explicitly covered, but you should be familiar with this notation.

Read:

- Rosen Section 2.4 pages 149-163 Review as needed from this section. You should be able to read and use sequence, summation, and product notation. The sum formula from this section you will be expected to have memorized for this course is

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

- Course Lecture Notes Chapter 3 Section 3 Induction
- Rosen Section 4.1 pages 263-283 Through Example 9, Example 12, Why Mathematical Induction is valid, Errors in Proofs Using Mathematical Induction.
- Rosen Section 4.2 pages 283-294, Through Example 4

Practice Exercises:

- Rosen Section 4.1 p. 279-283 # 3-37 odd, 45, 47, 49, 57
- Rosen Section 4.2 p. 291-294 # 1-7 odd, 25, 29

Set Operations

Overview: We now revisit sets and induction. This time we cover operations on sets and learn methods of proving set identities.

Read:

- Course Lecture Notes Chapter 4 Section 1 Set Operations
- Rosen Section 2.1 pages 111-121 All
- Rosen Section 2.2 pages 121-133 All
- Rosen Section 4.1 pages 263-283 Example 10

Practice Exercises:

- Rosen Section 2.1 p. 119-121 # 11, 13, 15, 27
- Rosen Section 2.2 p. 94 # 5-23 odd, 31, 37, 41, 45, 47, 49
- Rosen Section 4.1 p. 279-283 # 39, 41, 63

Properties of Functions

Overview: We now revisit functions. This time we cover the properties injective, surjective, and bijective, as well as several other properties of functions.

Read:

- Course Lecture Notes Chapter 4 Section 2 Properties of Functions
- Rosen Section 2.3 pages 133-149 All

Practice Exercises: Rosen Section 2.3 p. 146-149 # 11-15 all, 17, 19, 29, 31, 37, 41, 66, 68

Recurrence

Overview: Recurrence is a method of defining sets, functions, and a variety of other objects that uses an iterative approach. The three main skillsl you will work on in this section are: defining an object recursively, determining what recursively defined object is, and proving assertions about recursively defined objects. The main technique used to prove facts about recursively defined objects is induction.

Read:

- Course Lecture Notes Chapter 4 Section 3 Recurrence
- Rosen Section 4.3 pages 294-311 Through Example 6, Recursively Defined Sets and Structures through Example 9, Structural Induction through Example 12, Example 14

Practice Exercises: Rosen Section 4.3 p. 308-311 # 1, 5-17 odd, 23, 25, 27, 37, 39, 41, 49-55 odd

Growth

Overview: Chapter 4 Section 4 of the Course Lecture notes covers Growth of Functions. We cover the definitions of big-O, big-Ω, and big-Θ and use the definitions to prove properties involving these concepts. We use some of the properties we will prove to find big-O estimates of functions.

Growth of functions is an important concept in computer science. One area in which it is used is to estimate the complexity of an algorithm. There are 2 important skills to work on in this section. The first is the ability to recognize the "optimal" big-O estimate. This is a problem solving skill. The second skill you should work on is proving big-O estimates.

Read:

- Course Lecture Notes Chapter 4 Section 4 Growth
- Section 3.2 pages 180-193 All

Practice Exercises: Rosen Section p. 191-193 # 1, 5, 7, 9, 13, 15, 17, 21, 25, 27, 31, 44

Integers and Division

Overview: We rediscover the properties of integers and division that we have probably known and used since elementary school. We will learn precise definitions of division and related topics. Among the topics we cover this week are greatest common divisors, modular arithmetic, and the Euclidean Algorithm. Note that many of these concepts will be very familiar to you, but the emphasis here is on carefully *proving* the results using the definitions and the techniques of proof we have covered. You should memorize the definitions given here and use these definitions. Note that in this section we are only working with integers. Do *not* bring non-integer numbers into your proofs.

Read:

- Course Lecture Notes Chapter 5 Section 1 Integers and Division
- Rosen Section 3.4 pages 200-210 Through Corollary 2

- Rosen Section 3.5 pages 210-218 Through Example 5, Greatest Common Divisors and Least Common Multiples through Theorem 5
- Rosen Section 3.4 pages 266-267, examples 12

Practice Exercises:

- Rosen Section 3.4 p. 208-210 # 3, 7-25 odd
- Rosen Section 3.5 p. 217-218 # 5, 11-23 odd, 27, 31, 35
- Rosen Section 4.1 p. 279-283 # 55

Integers and Algorithms

Overview: In this section we continue our discussion of integers and focus on the Euclidean Algorithm and a few other Theorems.

Read:

- Course Lecture Notes Chapter 5 Section 2 Integers and Algorithms
- Rosen Section 3.6 pages 219-231 The Euclidean Algorithm through end of section
- Rosen Section 3.7 pages 231-246 Through Theorem 2

Practice Exercises:

- Rosen Section 3.6 p. 229-231 # 23, 25
- Rosen Section 3.7 p. 244-246 # 1

Applications of Number Theory

Overview: Among the applications of the material covered in the material that we will cover are base b expansions, inverses of integers modulo m, linear congruences, and Fermat's Little Theorem. If you read through the text you will find many more applications, but we will not covers all of these. The suggested homework and the examples listed below will guide you to the material we will concentrate on in this course.

Read:

- Course Lecture Notes Chapter 5 Section 3 Applications of Number Theory
- Rosen Section 3.6 pages 219-231 Through Example 6
- Rosen Section 3.7 pages 231-246 Linear Congruences through Example 4, Pseudoprimes through Example 10

Practice Exercises:

- Rosen Section 3.6 p. 229-231 # 1, 5, 9, 11, 15
- Rosen Section 3.7 p. 244-246 # 3-15 odd, 27

Matrices

Overview: It will be assumed that you know the basic matrix operations: addition, subtraction, scalar multiplication, and matrix multiplication. It will also be assumed that you are familiar with the identity matrix, the transpose of a matrix, and the inverse of a matrix. You should review any of these topics that you are not familiar with. We will cover zero-one matrices and the Boolean operations on these matrices: join, meet, and the Boolean product. The other area we will concentrate on in when covering matrices is *proving* properties of matrices and matrix operations.

Read:

- Course Lecture Notes Chapter 5 Section 4 Matrices,
- Rosen Section 3.8 pages 246-156 All

Practice Exercises:

- Rosen Section 3.8 p. 204 # 9, 21, 29-37 odd

Introduction to Graphs

Overview: This week we cover the basics of graph theory. Our focus this week is in defining a variety of types of graphs, some specific examples of graphs, and many of the important characteristics of graphs. The first section on graphs focuses on the definitions of a graph and the algebraic way one can represent graphs.

Read:

- Course Lecture Notes Chapter 6 Section 1 Introduction to Graphs
- Rosen Section 9.1 pages 589-597 Up to Graph models

Practice Exercises:

- Rosen Section 9.1 p. 595-597 # 1-9 odd
- Rosen Section 9.1 p. 595-597 Define the graphs in exercises 3-9 by giving the vertex set, an edge set, and where necessary a function from the edge set to an appropriate form of pairs of vertices.

Graph Terminology

Overview: The second section of graphs focuses on terminology involving the various characteristics of graphs and some of the basic properties of a graph.

Read:

- Course Lecture Notes Chapter 6 Section 2 Graph Terminology
- Rosen Section 9.2 pages 597-611 Through Example 1, Theorem 1 through Example 13, New Graphs from Old through Example 18

Practice Exercises:

- Rosen Section 9.2 p. 608-611 # 1, 3, 4, 5-9 odd, 21-25 odd, 26, 29-37 odd, 45, 51, 53, 55, 57,

Representing Graphs and Graph Isomorphism

Overview: In the last section on graphs we cover the use of matrices to represent graphs. We also revisit isomorphism and learn to use invariants to recognize when two graphs are not isomorphic and to find an isomorphism when two graphs are isomorphic. We will not spend time this semester on trying to rigorously prove or disprove two graphs are isomorphic. At this point you should concentrate on learning to recognize when 2 particular graphs are or are not isomorphic using a combination of graph invariants and visualization.

In MAD 3105 this topic will be revisited with more emphasis on the theory.

Read:

- Course Lecture Notes Chapter 6 Section 3 Representing Graphs and Graph Isomorphism
- Rosen Section 9.3 pages 611-621 All

Practice Exercises: Rosen Section 9.3 p. 618-621 # 1-23 odd, 27, 35-43 odd

Introduction to Relations

Overview: We end our semester by introducing relations. MAD 3105 will continue with this material. Our goals for this semester is to learn the definition of a relation, become comfortable with the notation, and to learn about the properties a relation may have. The material covered in the lecture slides and the text is beyond what will be expected this term, but if you plan to go on to MAD 3105 you will eventually have to learn all the material presented.

Read:

- Online Lecture Slides Module 7.1 Introduction to Relations
- Section 8.1 pages 519-529 All

Practice Exercises: Section 8.1 p. 527-529 # 1-43 odd