

## Contents

Chapter 1. Relations	8
1. Relations and Their Properties	8
1.1. Definition of a Relation	8
1.2. Directed Graphs	9
1.3. Representing Relations with Matrices	10
1.4. Example 1.4.1	10
1.5. Inverse Relation	11
1.6. Special Properties of Binary Relations	11
1.7. Examples of Relations and their Properties	12
1.8. Theorem 1.8.1: Connection Matrices v.s. Properties	14
1.9. Combining Relations	15
1.10. Example 1.10.1	15
1.11. Definition of Composition	17
1.12. Example 1.12.1	18
1.13. Theorem 1.13.1: Characterization of Transitive Relations	19
1.14. Connection Matrices v.s. Composition	20
2. Closure of Relations	21
2.1. Definition of the Closure of Relations	21
2.2. Reflexive Closure	21
2.3. Symmetric Closure	22
2.4. Examples	22
2.5. Relation Identities	24
2.6. Characterization of Symmetric Relations	25
2.7. Paths	26
2.8. Paths v.s Composition	26
2.9. Characterization of a Transitive Relation	27
2.10. Connectivity Relation	28
2.11. Characterization of the Transitive Closure	28
2.12. Example	29
2.13. Cycles	30
2.14. Corollaries to Theorem 2.13.1	31
2.15. Example 10	31
2.16. Properties v.s Closure	32
3. Equivalence Relations	33
3.1. Definition of an Equivalence Relations	33
3.2. Example	33

3.3. Equivalence Classes	34
3.4. Partition	35
3.5. Intersection of Equivalence Relations	36
3.6. Example	36
3.7. Example	38
3.8. Isomorphism is an Equivalence Relation	39
3.9. Equivalence Relation Generated by a Relation $R$	40
3.10. Using Closures to find an Equivalence Relation	41
4. Partial Orderings	43
4.1. Definition of a Partial Order	43
4.2. Examples	43
4.3. Pseudo-Orderings	44
4.4. Well-Ordered Relation	44
4.5. Examples	45
4.6. Lexicographic Order	46
4.7. Examples 4.7.1 and 4.7.2	46
4.8. Strings	47
4.9. Hasse or Poset Diagrams	47
4.10. Example 4.10.1	48
4.11. Maximal and Minimal Elements	50
4.12. Least and Greatest Elements	51
4.13. Upper and Lower Bounds	51
4.14. Least Upper and Greatest Lower Bounds	52
4.15. Lattices	52
4.16. Example 4.16.1	53
4.17. Topological Sorting	53
4.18. Topological Sorting Algorithm	54
4.19. Existence of a Minimal Element	54
Chapter 2. Graphs	58
1. Introduction to Graphs and Graph Isomorphism	58
1.1. The Graph Menagerie	58
1.2. Representing Graphs and Graph Isomorphism	58
1.3. Incidence Matrices	60
1.4. Example 1.4.1	60
1.5. Degree	61
1.6. The Handshaking Theorem	62
1.7. Example 1.7.1	62
1.8. Theorem 1.8.1	63
1.9. Handshaking Theorem for Directed Graphs	64
1.10. Graph Invariants	64
1.11. Example 1.11.1	66
1.12. Proof of Section 1.10 Part 3 for simple graphs	68
2. Connectivity	70

2.1. Connectivity	70
2.2. Example 2.2.1	70
2.3. Connectedness	72
2.4. Examples	72
2.5. Theorem 2.5.1	73
2.6. Example 2.6.1	74
2.7. Connected Component	74
2.8. Example 2.8.1	75
2.9. Cut Vertex and Edge	76
2.10. Examples	77
2.11. Counting Edges	78
2.12. Connectedness in Directed Graphs	78
2.13. Paths and Isomorphism	79
2.14. Example 2.14.1	80
2.15. Theorem 2.15.1	81
3. Euler and Hamilton Paths	82
3.1. Euler and Hamilton Paths	82
3.2. Examples	82
3.3. Necessary and Sufficient Conditions for an Euler Circuit	83
3.4. Necessary and Sufficient Conditions for an Euler Path	85
3.5. Hamilton Circuits	86
3.6. Examples	86
3.7. Sufficient Condition for a Hamilton Circuit	87
4. Introduction to Trees	88
4.1. Definition of a Tree	88
4.2. Examples	88
4.3. Roots	90
4.4. Example 4.4.1	90
4.5. Isomorphism of Directed Graphs	90
4.6. Isomorphism of Rooted Trees	91
4.7. Terminology for Rooted Trees	91
4.8. $m$ -ary Tree	92
4.9. Counting the Elements in a Tree	92
4.10. Level	93
4.11. Number of Leaves	93
4.12. Characterizations of a Tree	94
5. Spanning Trees	96
5.1. Spanning Trees	96
5.2. Example 5.2.1	96
5.3. Example 5.3.1	97
5.4. Existence	97
5.5. Spanning Forest	98
5.6. Distance	99
6. Search and Decision Trees	101

6.1. Binary Tree	101
6.2. Example 6.2.1	102
6.3. Decision Tree	103
6.4. Example 6.4.1	103
7. Tree Traversal	106
7.1. Ordered Trees	106
7.2. Universal Address System	106
7.3. Tree Traversal	107
7.4. Preorder Traversal	107
7.5. Inorder Traversal	109
7.6. Postorder Traversal	110
7.7. Infix Form	111
Chapter 3. Boolean Algebra	115
1. Boolean Functions	115
1.1. Boolean Functions	115
1.2. Example 1.2.1	115
1.3. Binary Operations	116
1.4. Example 1.4.1	117
1.5. Boolean Identities	117
1.6. Dual	118
2. Representing Boolean Functions	119
2.1. Representing Boolean Functions	119
2.2. Example 2.2.1	119
2.3. Example 2.3.1	120
2.4. Functionally Complete	121
2.5. Example 2.5.1	121
2.6. NAND and NOR	122
3. Abstract Boolean Algebras	123
3.1. Abstract Boolean Algebra	123
3.2. Examples of Boolean Algebras	123
3.3. Duality	126
3.4. More Properties of a Boolean Algebra	126
3.5. Proof of Idempotent Laws	127
3.6. Proof of Dominance Laws	127
3.7. Proof of Theorem 3.4.1 Property 4	128
3.8. Proof of DeMorgan's Law	129
3.9. Isomorphism	130
3.10. Atoms	131
3.11. Theorem 3.11.1	133
3.12. Theorem 3.12.1	133
3.13. Basis	134
3.14. Theorem 3.14.1	135
4. Logic Gates	139

4.1. Logic Gates	139
4.2. Example 4.2.1	139
4.3. NOR and NAND gates	141
4.4. Example 4.4.1	142
4.5. Half Adder	143
4.6. Full Adder	143
5. Minimizing Circuits	146
5.1. Minimizing Circuits	146
5.2. Example 5.2.1	146
5.3. Karnaugh Maps	146
5.4. Two Variables	146
5.5. Three Variables	148
5.6. Four Variables	149
5.7. Quine-McCluskey Method	151