MAD 3105, Section 1 - Quiz #2 Solutions

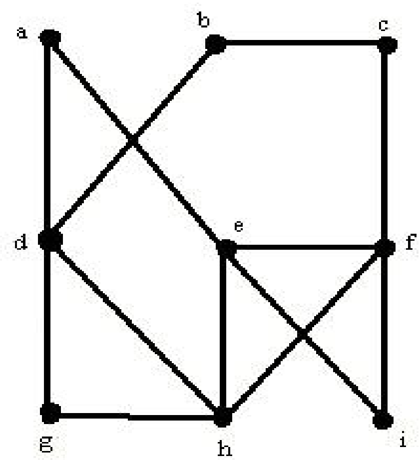
1. (4 points) If a tree has 3 vertices of degree 5, 2 vertices of degree 4, 3 vertices of degree 3 and the rest of the vertices of degree 1, how many vertices does the tree have?

   Solution: Suppose that the tree has n vertices. Then we know that it has $n-1$ edges. Moreover, the tree has $n - 3 - 2 - 3 = n - 8$ vertices of degree 1. Thus,

   $$2(n - 1) = 1 \cdot (n - 8) + 3 \cdot 3 + 4 \cdot 2 + 5 \cdot 3$$

   Solving for n we get that the number of vertices in tree is 26.

2. (6 points) Does the graph shown below have an Euler circuit? If so, then apply the EulerCircuit algorithm done in class and write down the vertex sequence of an Euler circuit.



   Choose any vertex in the graph, say h. Choose a cycle that contains h. the most obvious one has vertex sequence $hfeh$. Call this cycle C_1. We now remove the edges C_1 from the graph (but not the vertices) to obtain a graph that looks like the one shown in figure 1.

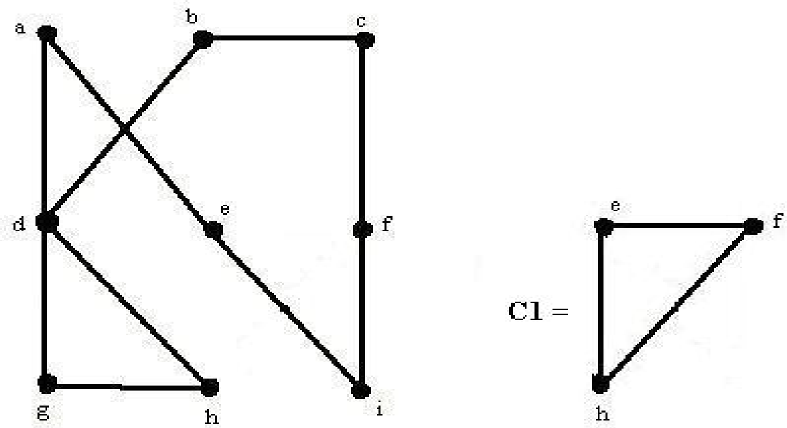


Figure 1

Now choose any vertex in C_1 that has non-zero degree. Again, say we choose h. Now find a cycle of which h is a vertex. Again, the most obvious one is the cycle with vertex sequence $hdgh$. Attach this new cycle to C_1 at h to obtain an extended cycle C_2 with vertex sequence $hfehdgh$, shown in figure 2.

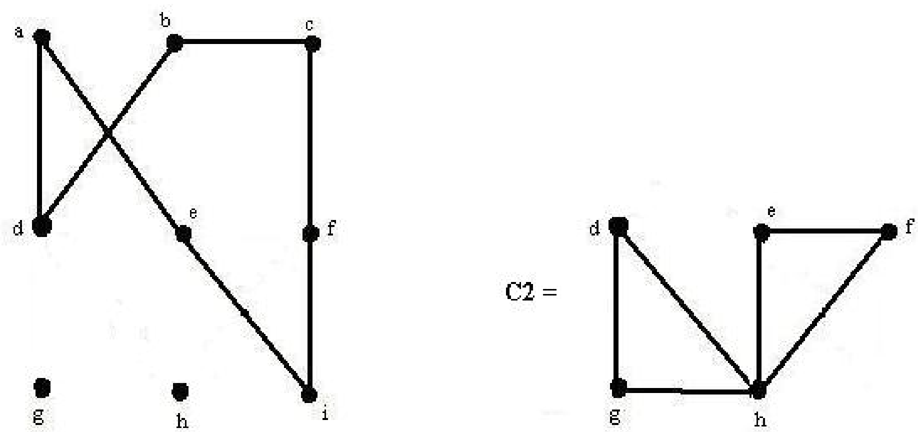


Figure 2

Now pick a vertex of non-zero degree in C_2, say d and get the cycle $dbcfiead$. Now we can always write the cycle C_2 as $dghfehd$. Attach the cycle $dbcfiead$ to C_2 at d to obtain the Euler circuit $dghfehdbcfiead$. Note that this cycle has the same number of edges as the original graph, and this is when we stop the Euler circuit algorithm. Of course, there are many other ways to get an Euler circuit. Can you write a code to get an Euler circuit?