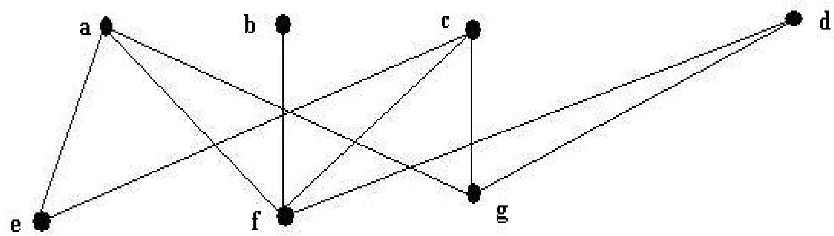
MAD 3105, Section 1 - Quiz #3 and 4, Solutions

1. (6 points) Give precise definitions of the following:
   (a) A <u>Hamilton circuit</u> in a graph G: It is a closed path in G that contains all the vertices of G and does not repeat any vertices except the first and the last.
   (b) A <u>bipartite graph</u>: A graph G such that $V(G) = V_1 \cup V_2, V_1 \cap V_2 = \emptyset, V_1 \neq \emptyset$ and $V_1 \neq \emptyset$ and such that each edge in G connects a vertex in V_1 to a vertex in V_2.
   (c) An <u>atom</u> of a Boolean algebra: a is an atom if it cannot be written as $a = x \vee y, x \neq a, y \neq a$.

2. (6 points) For the graph given below, give a Hamilton circuit or explain why none exists.



Solution: The graph is bipartite, with $V_1 = \{a, b, c, d\}, V_2 = \{e, f, g\}$. Since $|V_1| \neq |V_2|$, by theorem 4 of section 6.5, it follows that the graph cannot have a Hamilton circuit.

For all x, y and z in a Boolean algebra, the five axioms of a Boolean algebra are given below (their dual statements are not given. However, you are welcome to use them.):

1a. $x \vee y = y \vee x$.
2a. $(x \vee y) \vee z = x \vee (y \vee z)$.
3a. $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$.
4a. $x \vee 0 = x$.
5a. $x \vee x' = 1$.

3. (8 points) Using only the axioms given above, show that in a Boolean algebra, if $w \vee z = 1$ and $w \wedge z = 0$, then $z = w'$. Carefully state which of the axioms (or hypothesis) you have used in deducing your statements.

Solution:

$$\begin{aligned}
z &= z \vee 0 && \text{(4a)}\\
&= z \vee (w \wedge w') && \text{(5b)}\\
&= (z \vee w) \wedge (z \vee w') && \text{(3a)}\\
&= 1 \wedge (z \vee w') && \text{(Hypothesis)}\\
&= (w' \vee w) \wedge (w' \vee w) && \text{(5a and 1a)}\\
&= w' \vee (w \wedge z) && \text{(3a)}\\
&= w' \vee 0 && \text{(Hypothesis)}\\
&= w' && \text{(4a)}.
\end{aligned}$$