

**New Technologies on Mathematical
Research**

**Round Table, EMC, Barcelona, July 13,
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**Mika Seppälä
Florida State University**

Changing research environment and topics

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- Need to find algorithmic proofs for theorems.
- Implement the algorithmic proofs.
- Use these implementations to discover new properties of your favorite mathematical objects.

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- A genus 2 compact Riemann surface is defined by giving its uniformizing Fuchsian group $G = \langle g_1, g_2, h_1, h_2 \rangle$.
- A genus 2 algebraic curve is defined by its equation $y^2 = P(x) = (x - \lambda_1) \dots (x - \lambda_6)$ with distinct values for the roots λ_j .

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- Can be (partially) solved by computational methods too complex for computations without computers.

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- **Automatic integration complemented that.**
- **Other fields, like complex analysis, come next.**
- **Some areas will resist.**

Publishing mathematics

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- Everything about publishing educational and research mathematics is going to change.

OpenMath and MathML

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- OpenMath extends MathML to so that any area of mathematics can be covered.

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- Center of OpenMath activities is currently in Oxford at NAG.

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- Computational methods flourish and slowly penetrate new areas of mathematics.

Problems in mathematical education

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- Every calculus student needs to be able to compute $\int \sin x dx$ also in the future.
- How about $\int \arcsin x dx$?

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- Need to stress *understanding* of mathematical concepts.

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- Implementation of algorithms is a new kind of creative activity in mathematics and needs to be acknowledged as such.
- What is a proof?