Integration Practice Problems Solutions

MATH 142 Fall '04

1. $f(x) = \int(4(3x-5)^7)dx = \int(4u^7)\frac{1}{3}du = \int \frac{4}{3}u^7du = \frac{4}{3}\frac{1}{8}u^8 + C = \frac{4}{24}(3x-5)^8 + C = \frac{1}{6}(3x-5)^8 + C$
$u = 3x-5$
$\frac{du}{dx} = 3$
$du = 3dx$
$\frac{1}{3}du = dx$

2. $f(x) = \int((5x^2-6x^3+4x^4)^{99}(30x-54x^2+48x^3))dx = \int u^{99}(3du) = \int 3u^{99}du = 3\frac{1}{100}u^{100} + C = \frac{3}{100}(5x^2-6x^3+4x^4)^{100} + C$
$u = 5x^2-6x^3+4x^4$
$\frac{du}{dx} = (10x-18x^2+16x^3)$
$du = (10x-18x^2+16x^3)dx$
$3du = (30x-54x^2+48x^3)dx$

3. $f(x) = \int(\frac{30x^4-16}{3x^5-8x+10})dx = \int \frac{1}{u}2du = 2\ln|u| + C = 2\ln|3x^5-8x+10| + C$
$u = 3x^5-8x+10$
$\frac{du}{dx} = (15x^4-8)$
$du = (15x^4-8)dx$
$2du = (30x^4-16)dx$

4. $f(x) = \int((-16x+6)e^{(4x^2-3x+7)})dx = \int e^u(-2)du = \int -2e^udu = -2e^u + C = -2e^{4x^2-3x+7} + C$
$u = 4x^2-3x+7$
$\frac{du}{dx} = 8x-3$
$du = (8x-3)dx$
$-2du = (-16x+6)dx$

5. $f(x) = \int(\frac{1}{2x\ln(x^5)})dx = \int \frac{1}{2}\frac{1}{u}(\frac{1}{5}du) = \int \frac{1}{10}\frac{1}{u}du = \frac{1}{10}\ln|u| + C = \frac{1}{10}\ln|\ln(x^5)| + C$
$u = \ln(x^5)$
$\frac{du}{dx} = \frac{5x^4}{x^5}$
$du = \frac{5}{x}dx$
$\frac{1}{5}du = \frac{1}{x}dx$
or
$f(x) = \int(\frac{1}{2x\ln(x^5)})dx = \int \frac{1}{2}\frac{1}{5}\frac{1}{u}du = \int \frac{1}{10}\frac{1}{u}du = \frac{1}{10}\ln|u| + C = \frac{1}{10}\ln|\ln(x)| + C$
$u = \ln(x)$
$\frac{du}{dx} = \frac{1}{x}$
$du = \frac{1}{x}dx$

6. $f(x) = \int((8x^2-4x+16)^4(4x-1))dx = \int u^4\frac{1}{4}du = \int \frac{1}{4}u^4du = \frac{1}{4}\frac{1}{5}u^5 + C = \frac{1}{20}(8x^2-4x+16)^5 + C$
$u = 8x^2-4x+16$
$\frac{du}{dx} = 16x-4$
$du = (16x-4)dx$
$\frac{1}{4}du = (4x-1)dx$

7. $f(x) = \int((15x^4-9x^2-3)e^{(x^5-x^3-x+10)})dx = \int e^u(3du) = \int 3e^udu = 3e^u + C = 3e^{x^5-x^3-x+10} + C$
$u = x^5-x^3-x+10$
$\frac{du}{dx} = 5x^4-3x^2-1$
$du = (5x^4-3x^2-1)dx$
$3du = (15x^4-9x^2-3)dx$

8. $f(x) = \int(\frac{8e^x-56+96x}{e^x-7x+6x^2})dx = \int \frac{1}{u}8du = \int 8\frac{1}{u}du = 8\ln|u| + C = 8\ln|e^x-7x+6x^2| + C$
$u = e^x-7x+6x^2$
$\frac{du}{dx} = e^x-7+12x$
$du = (e^x-7+12x)dx$
$8du = (8e^x-56+96x)dx$

9. $f(x) = \int(9x^2-5)dx = \frac{9}{3}x^3 - 5x + C = 3x^3 - 5x + C$

10. $f(x) = \int(e^x + \frac{1}{x})dx = e^x + \ln|x| + C$

11. $f(x)=\int(\frac{1}{x^2}+\frac{4}{x^3}-\frac{9}{x^4})dx=\int(x^{-2}+4x^{-3}-9x^{-4})dx=-\frac{1}{1}x^{-1}+\frac{4}{-2}x^{-2}-\frac{9}{-3}x^{-3}+C=-x^{-1}-2x^{-2}+3x^{-3}+C=-\frac{1}{x}-\frac{2}{x^2}+\frac{3}{x^3}+C$

12. $f(x)=\int(\frac{\frac{1}{2}}{\sqrt{x}}+\frac{\frac{1}{3}}{x^{2/3}}-\frac{\frac{1}{4}}{x^{3/4}})dx=\int(\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{3}x^{-\frac{2}{3}}-\frac{1}{4}x^{-\frac{3}{4}})dx=\frac{\frac{1}{2}}{\frac{1}{2}}x^{\frac{1}{2}}+\frac{\frac{1}{3}}{\frac{1}{3}}x^{\frac{1}{3}}-\frac{\frac{1}{4}}{\frac{1}{4}}x^{\frac{1}{4}}=x^{1/2}+x^{1/3}-x^{1/4}+C$

13. $f(x)=\int(42x^6-120x^4+18x^5+4)dx=\frac{42}{7}x^7-\frac{120}{6}x^5+\frac{18}{6}x^6+\frac{4}{1}x^1+C=6x^7-24x^5+3x^6+4x+C$

14. $f(x)=\int(\frac{-32}{(4x-6)^3})dx=\int-32u^{-3}(\frac{1}{4}du)=\int-8u^{-3}du=-8\frac{1}{-2}u^{-2}+C=4\frac{1}{u^2}+C=\frac{4}{(4x-6)^2}+C$
$u=4x-6$
$\frac{du}{dx}=4$
$du=4dx$
$\frac{1}{4}du=dx$

15. $f(x)=\int(4x^3-10x)dx=\frac{4}{4}x^4-\frac{10}{2}x^2+C=x^4-5x^2+C$

16. $f(x)=\int(\frac{6}{x^3}-\frac{6}{x^4})dx=\int(6x^{-3}-6x^{-4})dx=\frac{6}{-2}x^{-2}-\frac{6}{-3}x^{-3}+C=-3x^{-2}+2x^{-3}+C$

17. $f(x)=\int((22x^2-11x^3)^6(4x-3x^2))dx=\int u^6(\frac{1}{11}du)=\int\frac{1}{11}u^6du=\frac{1}{11}\frac{1}{7}u^7+C=\frac{1}{77}(22x^2-11x^3)^7+C$
$u=22x^2-11x^3$
$\frac{du}{dx}=44x-33x^2$
$du=(44x-33x^2)dx$
$\frac{1}{11}du=(4x-3x^2)dx$

18. $f(x)=\int(-2e^{-2x+4})dx=\int e^u du=e^u+C=e^{-2x+4}+C$
$u=-2x+4$
$\frac{du}{dx}=-2$
$du=-2dx$

19. $f(x)=\int(\frac{-4x+1}{8x^2-4x+2})dx=\int\frac{1}{u}(-\frac{1}{4}du)=\int-\frac{1}{4}\frac{1}{u}du=-\frac{1}{4}\ln|u|+C=-\frac{1}{4}\ln|8x^2-4x+2|+C$
$u=8x^2-4x+2$
$\frac{du}{dx}=16x-4$
$du=(16x-4)dx$
$-\frac{1}{4}du=(-4x+1)dx$

20. $f(x)=\int(\frac{x^2-4}{\sqrt{6x^3-72x+10}})dx=\int u^{-\frac{1}{2}}\frac{1}{18}du=\int\frac{1}{18}u^{-\frac{1}{2}}du=\frac{1}{18}\frac{1}{\frac{1}{2}}u^{\frac{1}{2}}+C=\frac{1}{9}(6x^3-72x+10)^{1/2}+C$
$u=6x^3-72x+10$
$\frac{du}{dx}=18x^2-72$
$du=(18x^2-72)dx$
$\frac{1}{18}du=(x^2-4)dx$

21. $f(x)=\int((80x^3-120x)(x^4-3x^2+8)^3)dx=\int u^3(20du)=\int 20u^3du=20\frac{1}{4}u^4+C=5(x^4-3x^2+8)^4+C$
$u=x^4-3x^2+8$
$\frac{du}{dx}=4x^3-6x$
$du=(4x^3-6x)dx$
$20du=(80x^3-120x)dx$

22. $f(x)=\int((-x+2)e^{x^2-4x})dx=\int e^u(-\frac{1}{2}du)=\int-\frac{1}{2}e^u du=-\frac{1}{2}e^u+C=-\frac{1}{2}e^{x^2-4x}+C$
$u=x^2-4x$
$\frac{du}{dx}=2x-4$
$du=(2x-4)dx$
$-\frac{1}{2}du=(-x+2)dx$

23. $f(x)=\int(\frac{\ln(x^3-4x+5)(3x^2-4)}{x^3-4x+5})dx=\int udu=\frac{1}{2}u^2+C=\frac{1}{2}(\ln(x^3-4x+5))^2+C$
$u=\ln(x^3-4x+5)$
$\frac{du}{dx}=\frac{3x^2-4}{x^3-4x+5}$
$du=\frac{3x^2-4}{x^3-4x+5}dx$

24. $f(x)=\int(\frac{3x^2-4}{3x^3-12x+15})dx=\int\frac{1}{u}\frac{1}{3}du=\int\frac{1}{3}\frac{1}{u}du=\frac{1}{3}\ln|u|+C=\frac{1}{3}\ln|3x^3-12x+15|+C$
$u=3x^3-12x+15$
$\frac{du}{dx}=9x^2-12$
$du=(9x^2-12)dx$
$\frac{1}{3}du=(3x^2-4)dx$

25. $f(x)=\int(\frac{12x}{(x^2+4)\ln(x^2+4)})dx=\int\frac{1}{u}6du=\int 6\frac{1}{u}du=6\ln|u|+C=6\ln|\ln(x^2+4)|+C$
$u=\ln(x^2+4)$
$\frac{du}{dx}=\frac{2x}{x^2+4}$
$du=\frac{2x}{x^2+4}dx$
$6du=\frac{12x}{x^2+4}dx$

26. $f(x)=\int(\frac{4}{x\ln(x^3)})dx=\int 4\frac{1}{u}(\frac{1}{3}du)=\int\frac{4}{3}\frac{1}{u}du=\frac{4}{3}\ln|u|+C==\frac{4}{3}\ln(\ln|x^3|)+C$
$u=\ln(x^3)$
$\frac{du}{dx}=\frac{3x^2}{x^3}$
$du=\frac{3}{x}dx$
$\frac{1}{3}du=\frac{1}{x}dx$
or
$f(x)=\int(\frac{4}{x\ln(x^3)})dx=\int 4\frac{1}{3}\frac{1}{u}du=\int\frac{4}{3}\frac{1}{u}du=\frac{4}{3}\ln|u|+C=\frac{4}{3}\ln|\ln(x)|+C$
$u=\ln(x)$
$\frac{du}{dx}=\frac{1}{x}$
$du=\frac{1}{x}dx$

27. $f(x)=\int((3x-10x^4)(3x^2-4x^5+4)^{99})dx=\int u^{99}\frac{1}{2}du=\int\frac{1}{2}u^{99}du=\frac{1}{2}\frac{1}{100}u^{100}+C=\frac{1}{200}(3x^2-4x^5+4)^{100}+C$
$u=3x^2-4x^5+4$
$\frac{du}{dx}=(6x-20x^4)$
$du=(6x-20x^4)dx$
$\frac{1}{2}du=(3x-10x^4)dx$

28. $f(x)=\int((5-18x)e^{5x-9x^2})dx=\int e^u du=e^u+C=e^{5x-9x^2}+C$
$u=5x-9x^2$
$\frac{du}{dx}=5-18x$
$du=(5-18x)dx$

29. $f(x)=\int(\frac{x^6+\frac{5}{x^6}}{(x^7-\frac{7}{x^5})^2})dx=\int u^{-2}\frac{1}{7}du=\int\frac{1}{7}u^{-2}du=\frac{1}{7}\frac{1}{-1}u^{-1}+C=-\frac{1}{7}\frac{1}{u}+C=-\frac{1}{7u}+C=-\frac{1}{7(x^7-\frac{7}{x^5})}+C$
$u=x^7-\frac{7}{x^5}=x^7-7x^{-5}$
$\frac{du}{dx}=7x^6+35x^{-6}$
$du=(7x^6+35x^{-6})dx$
$\frac{1}{7}du=(x^6+5x^{-6})dx$

30. $f(x)=\int(\frac{16e^{4x}-9e^{3x}}{8e^{4x}-6e^{3x}})dx=\int\frac{1}{u}\frac{1}{2}du=\frac{1}{2}\frac{1}{u}du=\frac{1}{2}\ln|u|+C=\frac{1}{2}\ln|8e^{4x}-6e^{3x}|+C$
$u=8e^{4x}-6e^{3x}$
$\frac{du}{dx}=32e^{4x}-18e^{3x}$
$du=(32e^{4x}-18e^{3x})dx$
$\frac{1}{2}du=(16e^{4x}-9e^{3x})dx$