

**UNIT 2 MODULE 1**  
**ARGUMENTS AND PATTERNS OF REASONING**

*An **argument** is formed when we try to connect bits of evidence (**premises**) in a way that will force the audience to draw a desired **conclusion**.*

Consider these two arguments, such as a prosecutor might present to a jury:

1. The person who robbed the Mini-Mart drives a 1989 Toyota Tercel. Gomer drives a 1989 Toyota Tercel. Therefore, Gomer robbed the Mini-Mart.
2. The person who drank my coffee left this fingerprint on the cup. Gomer is the only person in the world who has this fingerprint. Therefore, Gomer is the person who drank my coffee.

One of these arguments is convincing, and one is not. Why?

*1. The person who robbed the Mini-Mart drives a 1989 Toyota Tercel. Gomer drives a 1989 Toyota Tercel. Therefore, Gomer robbed the Mini-Mart.*

Evidence (**premises**):

- A. The person who robbed the Mini-Mart drives a 1989 Toyota Tercel.
- B. Gomer drives a 1989 Toyota Tercel.

Desired **conclusion**:

Therefore, Gomer robbed the Mini-Mart.

*2. The person who drank my coffee left this fingerprint on the cup. Gomer is the only person in the world who has this fingerprint. Therefore, Gomer is the person who drank my coffee.*

Evidence (**premises**):

- A. The person who drank my coffee left this fingerprint on the cup.
- B. Gomer is the only person in the world who has this fingerprint.

Desired **conclusion**:

Therefore, Gomer is the person who drank my coffee.

### VALID ARGUMENTS

In a well-formulated argument, it should be *logically impossible to reject the conclusion* if we accept all of the evidence ("the truth of the premises **forces** the conclusion to be true;" or "the conclusion is an inescapable consequence of the premises").

Such an argument is called **VALID**.

On the other hand:

### INVALID ARGUMENTS

An argument is poorly-formed if it is logically *possible* for the audience to believe all of the evidence and yet reject the conclusion.

More formally:

An argument is said to be **INVALID** if it is *logically possible for the CONCLUSION to be FALSE even though EVERY PREMISE is assumed to be TRUE*.

The preceding statement may be referred to as the **Fundamental Principle of Argumentation**. It governs our entire discussion of arguments and reasoning.

Notice that in the first argument given above, even if the jury believes all of the evidence, they don't necessarily have to believe the conclusion (because there are many people besides Gomer who drive 1989 Toyota Tercels). That is what makes the first argument **invalid**.

Notice that in the second argument, however, if the jury believes all of the evidence, then they **must** accept the conclusion. That is what makes the second argument **valid**.

Based on the Fundamental Principle of Argumentation, we have the following procedure that can be used to analyze arguments whose statements can be symbolized with logical connectives:

### TECHNIQUE FOR USING TRUTH TABLES TO ANALYZE ARGUMENTS

1. Symbolize (consistently) all of the premises and the conclusion.
2. Make a truth table having a column for each premise and for the conclusion.
3. If there is a row in the truth table where every premise column is true but the conclusion column is false (a **bad row**) then the argument is **invalid**. If there are no bad rows, then the argument is valid.

## UNIT 2 MODULE 1

### EXAMPLE 2.1.1

Use a truth table to test the validity of the following argument.

If the apartment is damaged, then the deposit won't be refunded.

The apartment isn't damaged.

Therefore, the deposit will be refunded.

### EXAMPLE 2.1.1 Solution

*Step 1: Symbolize the argument.*

Let  $p$  be the statement "The apartment is damaged."

Let  $q$  be the statement "The deposit **won't** be refunded."

The argument has this form:

$p \rightarrow q$

$\sim p$

$\therefore \sim q$

Note: the triangular configuration of dots represents the word "therefore."

*Step 2: Make a truth table having a column for each premise and for the conclusion.*

		PREM	PREM	CONC
$p$	$q$	$p \rightarrow q$	$\sim p$	$\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	<b>T</b>	<b>T</b>	<b>F</b>
F	F	T	T	T

*Step 3: Look for the indication of an INVALID argument (a row where every premise is true while the conclusion is false).*

Notice that in the third row, both premises are true while the conclusion is false; this "bad row" tells us that the argument is INVALID.

## HACKING MATHEMATICS

### EXAMPLE 2.1.2

*Use a truth table to test the validity of this argument.*

If I had a hammer, I would hammer in the morning.

I don't hammer in the morning.

Therefore, I don't have a hammer.

### EXAMPLE 2.1.2 Solution

*Step 1: Symbolize the argument.*

Let  $p$  be the statement "I have a hammer."

Let  $q$  be the statement "I hammer in the morning."

Then the argument has this form:

$$p \rightarrow q$$

$$\frac{\sim q}{\quad}$$

$$\therefore \sim p$$

*Step 2: Make a truth table having a column for each premise and for the conclusion.*

		PREM	PREM	CONC
$p$	$q$	$p \rightarrow q$	$\sim q$	$\sim p$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

*Step 3: Look for the indication of an INVALID argument (a row where every premise is true while the conclusion is false).*

Notice that there is **no row** where the conclusion column is false while both premise columns are true; the absence of a "bad row" tells us that the argument is VALID.

**COMMON PATTERNS OF REASONING: CONTRAPOSITIVE REASONING**

From the result in EXAMPLE 2.1.2 we have the following general fact

*Any argument that can be reduced to the form*

$p \rightarrow q$

$\sim q$

$\therefore \sim p$

*will be a valid argument.*

This is a common form of valid reasoning known as **Contrapositive Reasoning** or **Modus Tollens**.

**EXAMPLE 2.1.3**

*Without making a truth table, we know automatically that this is a valid argument:*

If it rains, then I won't go out.

I went out.

Therefore, it didn't rain.

*Here is another example of **Contrapositive Reasoning** (in set language):*

All cats have rodent breath.

Whiskers doesn't have rodent breath.

Thus, Whiskers isn't a cat.

Note: the previous argument agrees with the form **Contrapositive Reasoning** because it can be rephrased in "if...then" language:

*If one is a cat, then one has rodent breath.*

Whiskers doesn't have rodent breath.

Therefore, Whiskers isn't a cat

(where the individual "Whiskers" is taking the place of the general subject "one..." in the first premise).

**COMMON PATTERNS OF REASONING: FALLACY OF THE INVERSE**

Generalizing from the result of Example 2.1.1 above, we see that any argument that can be reduced to the form

$$p \rightarrow q$$

$$\frac{\sim p}{\therefore \sim q}$$

will be an **invalid** argument.

This is a common form of **invalid** reasoning known as **Fallacy of the Inverse**.

**EXAMPLE 2.1.4**

*Without having to make a truth table, we automatically know that the following argument is INVALID:*

If you drink Pepsi, then you are happy.

You don't drink Pepsi.

Therefore, you aren't happy.

*Here is another example of **Fallacy of the Inverse** (in set language):*

All firefighters are courageous.

Gomer the Bold isn't a firefighter.

Thus, Gomer the Bold isn't courageous.

**EXAMPLE 2.1.5**

*"There's a fine line between clever and stupid."*

Nigel Tufnel, lead guitarist, *Spinal Tap*

Can you discern the "fine line between clever and stupid" in these two arguments?

**Argument 1:**

If I get a huge tax refund, then I'll buy a Yugo. I didn't buy a Yugo.

Therefore, I didn't get a huge tax refund.

**Argument 2:**

If I get a huge tax refund, then I'll buy a Yugo. I didn't get a huge tax refund.

Therefore, I didn't buy a Yugo.

**EXAMPLE 2.1.5 Solution**

Argument 1 is VALID (it is Contrapositive Reasoning), whereas Argument 2 is INVALID (it is Fallacy of the Inverse).

**EXAMPLE 2.1.6**

*Use a truth table to test the validity of this argument.*

If one grows vegetables, then one is a gardener.

Gomer is a gardener.

Therefore, Gomer grows vegetables.

**COMMON PATTERNS OF REASONING: FALLACY OF THE CONVERSE**

Generalizing from the result of EXAMPLE 2.1.6, we have this fact:

*Any argument that can be reduced to the form*

$$p \rightarrow q$$

$$\frac{q}{\quad}$$

$$\therefore p$$

*will be an **invalid** argument.*

This is a common form of invalid reasoning known as **Fallacy of the Converse**.

**EXAMPLE 2.1.7**

*Test the validity of the following arguments.*

If I eat Wheaties, then I am healthy. I am healthy.

Therefore, I eat Wheaties.

All great writers are philosophical. Thoreau was philosophical.

Thus, Thoreau was a great writer.

**EXAMPLE 2.1.7 Solution**

Both arguments are INVALID, because they are examples of **Fallacy of the Converse**. It is not necessary to make the truth tables, although the truth tables will verify the claims that these arguments are invalid.

**EXAMPLE 2.1.8**

*Test the validity of this argument (from Aristotle):*

All men are mortal. Socrates is a man. Therefore, Socrates is mortal.



**COMMON PATTERNS OF REASONING: DIRECT REASONING**

*Any argument that can be reduced to the form*

$p \rightarrow q$

p

$\therefore q$

*is a **valid** argument.*

This common form of valid reasoning is called **Direct Reasoning** or *Modus Ponens*.

**EXAMPLES**

The following arguments are automatically valid (because they are examples of **Direct Reasoning**):

If I quit school, I'll sell apples on the corner.

I did quit school.

Therefore, I sell apples on the corner.

In set language:

All Gators are obnoxious.

Steve is a Gator.

Thus, Steve is obnoxious.

No beggars are choosers.

Diogenes is a beggar.

Hence, Diogenes is not a chooser.

Note: This last argument conforms to the pattern of Direct Reasoning because the statement "No beggars are choosers" can be rephrased as "If one is a beggar, then one isn't a chooser."

## HACKING MATHEMATICS

### EXAMPLE 2.1.8A

Test the validity of each argument.

- |   |  |
|---|--|
| 1. All men are mortal.<br>Socrates is a man.<br>Therefore, Socrates is mortal.      | 2. All women are normal.<br>Socrates isn't a woman.<br>Therefore, Socrates isn't normal.                 |
| 3. All toads are wartful.<br>Socrates is wartful.<br>Therefore, Socrates is a toad. | 4. All edifices have portals.<br>Socrates doesn't have portals.<br>Therefore, Socrates isn't an edifice. |

### EXAMPLE 2.1.9

Test the validity of this argument:

I have my keys or I'm locked out.

I'm not locked out.

Therefore, I have my keys.

**COMMON PATTERNS OF REASONING: DISJUNCTIVE SYLLOGISM**

*Any argument that can be reduced to the form*

$p \vee q$

$\sim q$

$\therefore p$

*will be a valid argument.*

This is a common form of valid reasoning known as **disjunctive syllogism**.

Note: because the "or" connective is symmetric, this pattern can also be written as

$p \vee q$

$\sim p$

$\therefore q$

**EXAMPLE**

This argument is automatically valid:

Socrates is in Athens or Socrates is in Sparta.

Socrates isn't in Sparta.

Thus, Socrates is in Athens.

**EXAMPLE 2.1.9A**

*Test the validity of this argument:*

I walk or I chew gum.

I'm walking.

Therefore, I'm not chewing gum.

**COMMON PATTERNS OF REASONING: DISJUNCTIVE FALLACY**

*Any argument that can be reduced to one of these forms:*

$$\begin{array}{cc} p \vee q & p \vee q \\ \underline{p} & \underline{q} \\ \therefore \sim q & \therefore \sim p \end{array}$$

*is automatically **INVALID**.*

This incorrect attempt to use the disjunctive syllogism is called **DISJUNCTIVE FALLACY**.

**EXAMPLES**

The following arguments are **INVALID**, because they are examples of **Disjunctive Fallacy**;

Today isn't Sunday or I can stay home.  
I can stay home.  
Therefore, today is Sunday.

Fido is a poodle or has brown fur.  
Fido is a poodle.  
Therefore, Fido doesn't have brown fur.

**WORLD WIDE WEB NOTE**

For lots of practice in the art of using truth tables to analyze arguments, visit the companion website and try **THE ARGUE-MENTOR**

**EXAMPLE 2.1.10**

*Test the validity of the argument.*

If I get elected, I'll reduce taxes.

If I reduce taxes, the economy will prosper.

Thus, if I get elected, the economy will prosper.

**COMMON PATTERNS OF REASONING: TRANSITIVE REASONING**

*Any argument that can be reduced to the form*

$$p \rightarrow q$$

$$\underline{q \rightarrow r}$$

$$\therefore p \rightarrow r$$

*will be a valid argument.*

This is a common form of valid reasoning known as **Transitive Reasoning**.

**EXAMPLES**

The following arguments are **valid** because they are examples of **Transitive Reasoning**.

If today is Monday, then tomorrow is Tuesday.

If tomorrow is Tuesday, then the day after tomorrow is Wednesday.

Therefore, if today is Monday, then the day after tomorrow is Wednesday.

In set language:

All bulldogs are mean-looking dogs.

All mean-looking dogs are good watchdogs.

Therefore, all bulldogs are good watchdogs.

Another example:

If I eat my spinach, then I'll become well-muscled.

If I become well-muscled, then I'll become a professional wrestler.

If I become I professional wrestler, then I'll bleach my hair.

If I bleach my hair, then I'll wear sequined tights.

If I wear sequined tights, then I'll be ridiculous.

Therefore, if I eat my spinach, then I'll be ridiculous.

The previous example illustrates an important property of **Transitive Reasoning**: *This method of reasoning extends indefinitely.*

We easily can construct valid arguments that have as many "if...then" premises as we wish, as long as the fundamental pattern continues: namely, the **antecedent** of each new premise agrees with the **consequent** of the previous premise.

**EXAMPLE 2.1.11**

Test the validity of this argument:

If I get elected, I'll take lots of bribes.

If I get elected, I'll reduce taxes.

Thus, if I take lots of bribes, then I'll reduce taxes.

**Common patterns of reasoning FALSE CHAINS**

*Any argument that can be reduced to one of these forms*

$$p \rightarrow q$$

$$p \rightarrow q$$

$$\underline{p \rightarrow r}$$

$$\underline{r \rightarrow q}$$

$$\therefore q \rightarrow r$$

$$\therefore p \rightarrow r$$

*will be **invalid**.*

These common forms of invalid reasoning are called **False Chains**.

The following arguments are **INVALID** because they are examples of **False Chains**.

If today is a state holiday, then school is closed.

If today is Sunday, then school is closed.

Therefore, if today is a state holiday, then today is Sunday.

All cats are mammals.

All cats are predators.

Therefore, all mammals are predators.



**ARGUMENTS THAT DON'T CONFORM TO COMMON PATTERNS**

Common patterns of reasoning are useful in that they allow us to analyze arguments without having to construct truth tables. Of course, not every argument will conform to one of the familiar patterns that we have identified.

**EXAMPLE 2.1.12**

*Test the validity of the following argument.*

You have jumper cables or our date is cancelled.

You have a credit card or our date is cancelled.

Our date is cancelled.

Therefore, you don't have jumper cables or you don't have a credit card.

**EXAMPLE 2.1.13**

*Test the validity of this argument:*

I got a scholarship and I got an "A" in math.

I'm not good at logic or I got an "A" in math.

Therefore, I'm good at logic or I don't get a scholarship.

**EXAMPLE 2.1.14**

I will hire Gomer or I will hire Homer.

If I don't hire Homer then I'm not having a bad hair day.

I don't hire Gomer.

Therefore I'm having a bad hair day.

**EXAMPLE 2.1.15**

*Test the validity of the following argument.*

If I want to be a lawyer, then I want to study logic.

If I don't want to be a lawyer, then I don't like to argue.

Therefore, if I like to argue, then I want to study logic.

**EXAMPLE 2.1.16**

*Test the validity of the following argument:*

If I buy cheap gasoline, then my car runs badly.

If I don't change the oil, then my car runs badly.

Therefore, if I buy cheap gasoline, then I don't change the oil.

## HACKING MATHEMATICS

### PRACTICE EXERCISES

**1 – 27:** Test the validity of each argument.

- 1.** If I plant a tree, then I will get dirt under my nails. I didn't get dirt under my nails. Therefore, I didn't plant a tree.
- 2.** If I don't change my oil regularly, my engine will die. My engine died. Thus, I didn't change my oil regularly.
- 3.** All frogs are amphibians. All frogs have gills. Therefore, all amphibians have gills.
- 4.** You will meet a tall, handsome stranger or you will stay home and pick fleas off of your cat. You didn't meet and tall, handsome stranger. Therefore, you stayed home and picked fleas off of your cat.
- 5.** If I don't tie my shoes, then I trip. I didn't tie my shoes. Hence, I tripped.
- 6.** All racers live dangerously. Gomer is a racer. Therefore, Gomer lives dangerously.
- 7.** If you aren't polite, you won't be treated with respect. You aren't treated with respect. Therefore, you aren't polite.
- 8.** If you are kind to a puppy, then he will be your friend. You weren't kind to that puppy. Hence, he isn't your friend.
- 9.** If you drink Surge, then you won't fall off of your skateboard. You fell off of your skateboard. Therefore, you didn't drink Surge.
- 10.** If I don't pay my income taxes, then I file for an extension or I am a felon. I'm not a felon and I didn't file for an extension. Therefore, I paid my income taxes.
- 11.** I wash the dishes or I don't eat. I eat. Thus, I wash the dishes.
- 12.** All protons are subatomic particles. All neutrons are subatomic particles. Hence, all protons are neutrons.
- 13.** All sneaks are devious. All swindlers are sneaks. Therefore, all swindlers are devious.

## UNIT 2 MODULE 1

- 14.** All superheroes wear capes. The Masked Gomer wears a cape. Hence, The Masked Gomer is a superhero.
- 15.** All wolverines are cuddly. No weasels are wolverines. Thus, no weasels are cuddly.
- 16.** If you want to be a used-car salesman, then you have to be a flashy dresser. You don't want to be a used-car salesman. Thus, you don't have to be a flashy dresser.
- 17.** If an animal is cute, then it isn't a squid. This animal isn't a squid. Therefore, this animal is cute.
- 18.** If you play golf during a thunderstorm, you'll get hit by lightning. You didn't get hit by lightning. Therefore, you didn't play golf during a thunderstorm.
- 19.** I will run for office or I will shut my mouth. I ran for office. Thus, I didn't shut my mouth.
- 20.** If I am literate, then I can read and write. I can read but I can't write. Thus, I am not literate.
- 21.** If it rains or snows, then my roof leaks. My roof is leaking. Thus, it is raining and snowing.
- 22.** All cyclists wear helmets. Gomer doesn't wear a helmet. Therefore, Gomer isn't a cyclist.
- 23.** All firefighters wear red suspenders. Gomer wears red suspenders. Therefore, Gomer is a firefighter.
- 24.** All Yugo-owners are used to hitchhiking. Gomer isn't a Yugo-owner. Therefore, Gomer isn't used to hitchhiking.
- 25.** If I lose my keys, then I can't start my car. If I lose my keys, then I can't get in my house. Therefore, if I can't start my car, then I can't get in my house.
- 26.** If an animal is a squid, then it has tentacles. If an animal is an octopus, then it has tentacles. Therefore, if an animal is a squid, then it is an octopus.

## HACKING MATHEMATICS

27. If you are a fire-eater, then you work in the circus. If you don't like cotton candy, then you don't work in the circus. Therefore, if you are a fire-eater, then you like cotton candy.

### ANSWERS TO LINKED EXAMPLES

**EXAMPLE 2.1.6** Invalid

**EXAMPLE 2.1.8.A** 1. Valid      2. Invalid      3. Invalid      4. Valid

**EXAMPLE 2.1.9** Valid

**EXAMPLE 2.1.9A** Invalid

**EXAMPLE 2.1.10** Valid

**EXAMPLE 2.1.11** Invalid

**EXAMPLE 2.1.12** Invalid

**EXAMPLE 2.1.13** Invalid

**EXAMPLE 2.1.14** Invalid

**EXAMPLE 2.1.15** Valid

**EXAMPLE 2.1.16** Invalid

### ANSWERS TO PRACTICE EXERCISES

1. Valid (contrapositive reasoning)

3. Invalid (false chain)

5. Valid (direct reasoning)

7. Invalid (fallacy of converse)

9. Valid (contrapositive reasoning)

11. Valid (disjunctive syllogism)

13. Valid (transitive reasoning)

15. Invalid (false chain)

17. Invalid (fallacy of converse)

19. Invalid (not disjunctive syllogism)

21. Invalid (use truth table)

23. Invalid (fallacy of converse)

25. Invalid (false chain)

27. Valid (transitive reasoning)

2. Invalid (fallacy of converse)

4. Valid (disjunctive syllogism)

6. Valid (direct reasoning)

8. Invalid (fallacy of inverse)

10. Valid (use truth table)

12. Invalid (false chain)

14. Invalid (fallacy of converse)

16. Invalid (fallacy of inverse)

18. Valid (contrapositive reasoning)

20. Valid (use truth table)

22. Valid (contrapositive reasoning)

24. Invalid (fallacy of inverse)

26. Invalid (false chain)