

## UNIT 2 MODULE 3 (Alternative version)

### CATEGORICAL SYLLOGISMS AND DIAGRAMMING

Consider the following argument:

Some lawyers are judges. Some judges are politicians. Therefore, some lawyers are politicians.

Although the premises and conclusion of this argument sound reasonable, and although the structure of the argument looks similar to transitive reasoning, this argument is **invalid**.

In order to show that the argument is invalid, all we have to do is conceive of a situation in which the conclusion is false, while both premises are true. In order to do so, it helps if we imagine a world with a small population of lawyers, judges and politicians. Suppose there are only two lawyers, Alice and Bill, and that Bill is also a judge, but Alice isn't. Suppose that in addition to Bill there is only one other judge, Carla, and Carla is also a politician, but Bill isn't a politician. Finally, suppose there is one other politician, Don, who isn't a lawyer and isn't a politician.

In this conceivable world, some lawyers are judges (Bill), and some judges are politicians (Carla), but no lawyers are politicians. Since it is possible to conceive of a situation in which the conclusion is false while both premises are true, this argument is invalid.

The previous argument is an example of a **CATEGORICAL SYLLOGISM**, which is an argument involving two premises, both of which are **categorical statements**. Categorical statements are statements of the form "all are...", "none are..." or "some are..." A categorical statement of the form "all are..." is also called a **positive universal** statement. A categorical statement of the form "none are..." is also called a **negative universal** statement. A categorical statement of the form "some are..." or "some aren't" is also called an **existential statement**.

In this discussion we are primarily concerned with categorical syllogisms in which at least one premise is an existential statement, because such arguments cannot be analyzed using the methods of Unit 2 Module 1.

#### Existential statements

A statement of the form "Some are...", such as "Some lawyers are judges," is conceptually quite different from a universal statement, in that it cannot be restated

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positive universal statement such as "All cats are mammals" can be informally restated as "If \_\_\_ is a cat, then \_\_\_ is a mammal," and whereas a negative universal statement such as "No cats are dogs" can be restated as "If \_\_\_ is a cat, then \_\_\_ isn't a dog," it is not possible to make such a transition with an existential statement such as "Some mammals are predators."

This means that the techniques of Unit 2 Module 1, which are based on truth tables and logical connectives, are of no use for arguments involving the existential statement.

### **Diagramming categorical statements**

There is an extensive literature on the topic of categorical syllogisms, dating back to medieval scholarship and earlier. This includes an impressive body of special terminology, symbols, and characterizations of forms, which a student might encounter in a more intense study of the subject, such as in a history of philosophy course.

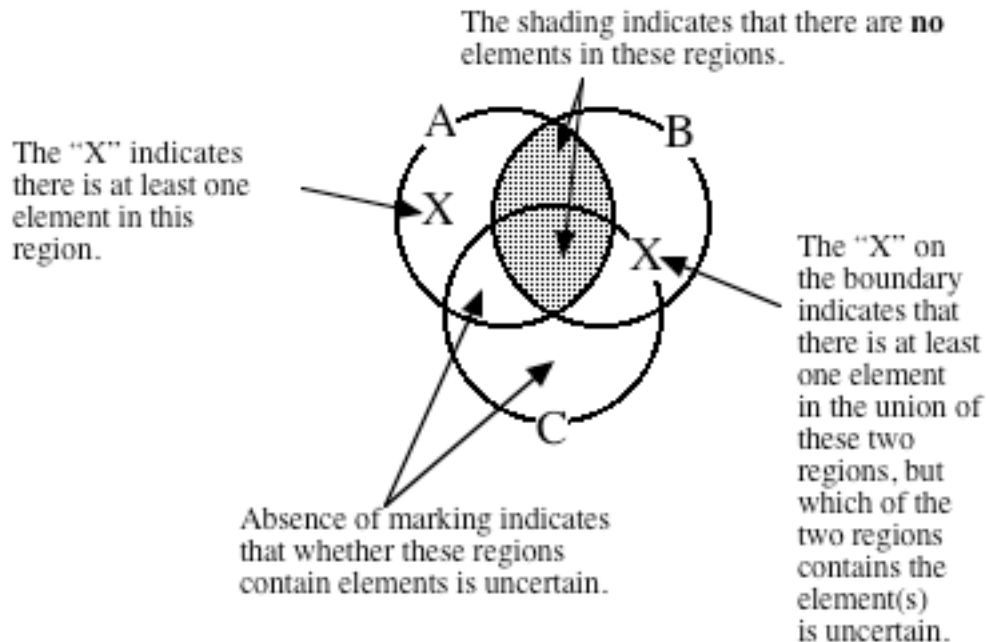
This discussion will be limited to the presentation of a method of analyzing categorical syllogisms through the use of three-circle Venn diagrams. This method is called **diagramming**.

Individual statements are diagrammed as follows.

1. Use shading to diagram universal statements, by shading out any region that is known to contain no elements.
2. Use an "X" to diagram an existential statement. If a region is known to contain at least one element, place an "X" in that region. If it is uncertain which of two regions must contain the element(s), then place the "X" on the boundary between those two regions.
3. If a region contains no marking, then it is uncertain whether or not that region contains any elements.

The marked Venn diagram below illustrates these ideas.

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### Diagramming categorical syllogisms

To test the validity of a categorical syllogism, follow these steps.

1. In order to be valid, a categorical syllogism must have at least one premise that is a universal statement. If none of the premises is a universal statement, then the argument is invalid, and we are done. The following steps assume that at least one premise is a universal statement.
2. Begin by diagramming the universal premise(s). A universal statement will have the effect of shading (blotting out, so to speak) some region of the diagram, because a universal statement will always assert, directly or otherwise, that some region of the diagram has no elements.
3. Confining your attention to the part of the diagram that is **unshaded**, diagram an existential premise by placing an "X" in a region of the diagram that is known to contain at least one element. If it is uncertain which of two regions should contain the element(s), place the "X" on the boundary between those two regions.
4. After diagramming the premises, if the diagram shows the conclusion of the argument to be true, then the argument is valid. If the diagram shows the conclusion to be uncertain or false, then the argument is invalid.
5. If all the statements in the argument are universal statements, then the argument can be analyzed in terms of transitive reasoning or false chains (see Unit 2 Module

6. If both premises are universal statements but the conclusion is an existential statement, then the argument is invalid. No diagram is necessary. You cannot deduce “some” from “all” or “none.”

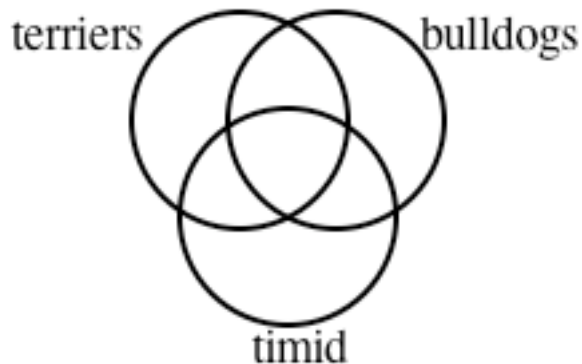
**EXAMPLE A**

Use diagramming to test the validity of this argument:

No terriers are timid. Some bulldogs are terriers. Therefore, some bulldogs are not timid.

**SOLUTION**

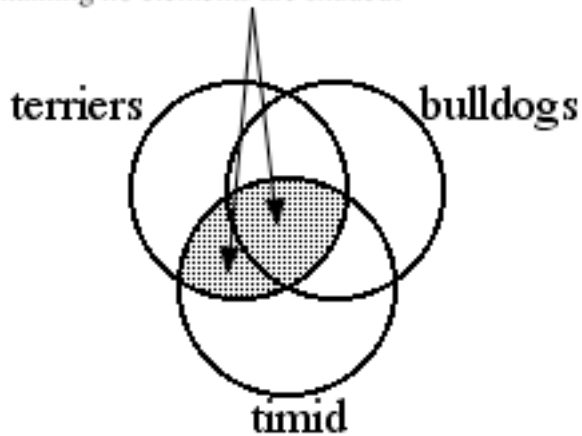
We will mark this three-circle Venn diagram, which shows the sets "terriers," "bulldogs" and "timid (things):"



First, diagram the negative universal premise "No terriers are timid." According to this premise, the overlap of those two sets contains no elements, so that part of the diagram is shaded, or "blotted out."

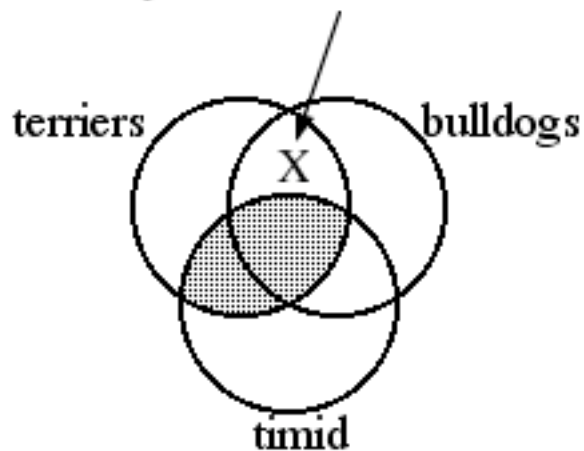
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Because "No terriers are timid," the intersection of those two circles contains no elements. Regions containing no elements are shaded.



Next, diagram the existential premise "Some bulldogs are terriers" by placing an "X" in the appropriate location in the unshaded portion of the diagram.

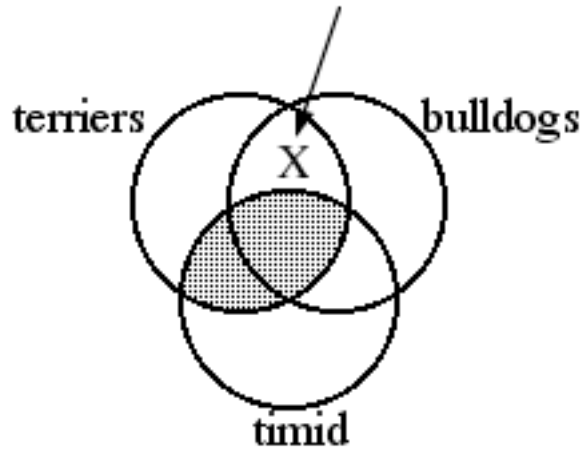
Because "Some bulldogs are terriers,:" there must be at least one element in the unshaded region where those two circles intersect..



Now that both premises have been diagrammed, check to see if the marked diagram shows the conclusion to be true.

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The "X" in this region shows that there must be at least one bulldog who is not timid. The diagram shows that the conclusion "Some bulldogs are not timid" must be true.



Because the marked diagram shows that the conclusion is true, the argument is **valid**.

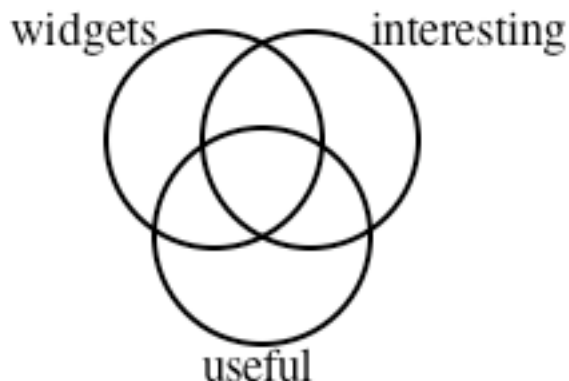
### EXAMPLE B

Use diagramming to test the validity of this argument.

Some useful things are interesting. All widgets are interesting. Therefore, some widgets are useful.

### SOLUTION

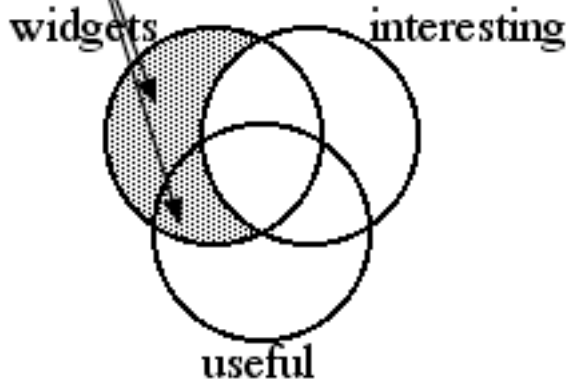
We can use this three-circle Venn diagram, shows the sets of widgets, interesting (things) and useful (things):



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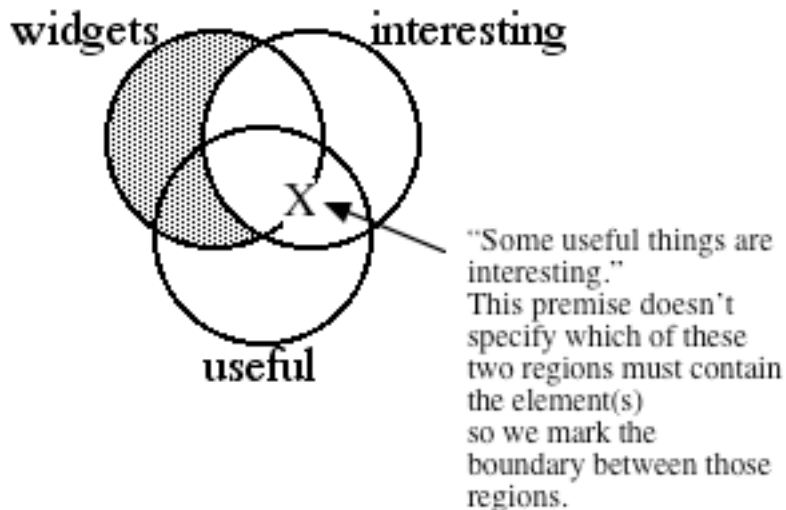
Start by diagramming the universal premise, "All widgets are interesting."

Since "All widgets are interesting," this region, where "widgets" are not "interesting," must contain no elements, so we shade it.

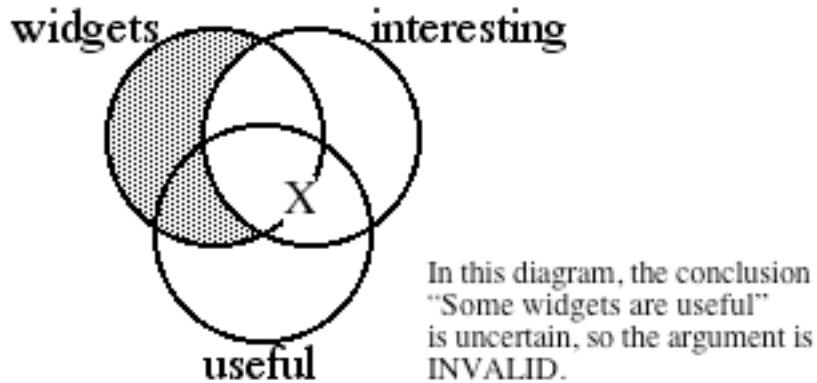


Next, diagram the existential premise, "Some useful things are interesting." This means that there must be at least one element in the overlap of those two circles.

However, that overlap entails two regions, and it is uncertain as which of those two regions contains the element(s), so we place an "X" on their border.



Now that we have diagrammed both premises, we check to see if the marked diagram shows the conclusion, "Some widgets are useful," to be true.



The argument is **invalid**, because the diagram shows that, based on those premises, the conclusion is not certain. That is, the “X” is not in the part of the diagram where “widgets” and “useful” intersect.

**EXAMPLE 2.3.1**

*Test the validity of this argument:*

All elephants are huge creatures.

Some huge creatures have tusks.

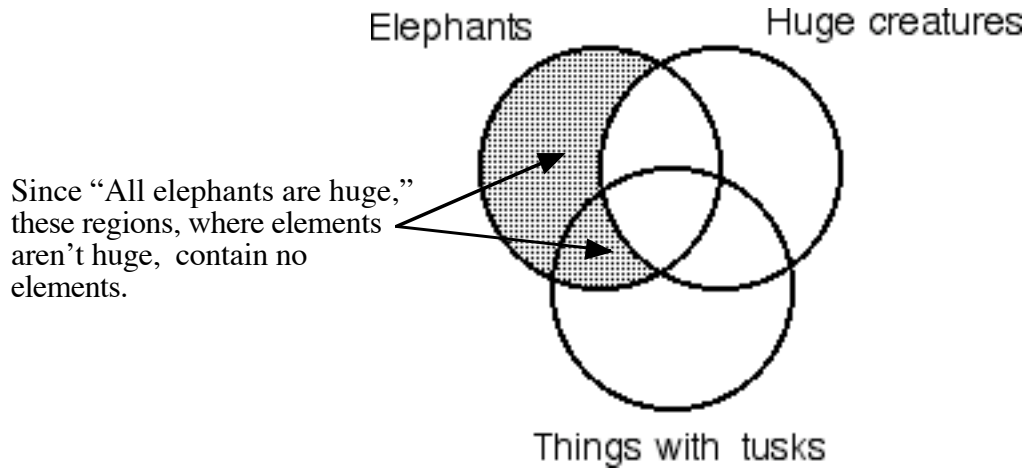
Therefore, some elephants have tusks.

**EXAMPLE 2.3.1 SOLUTION**

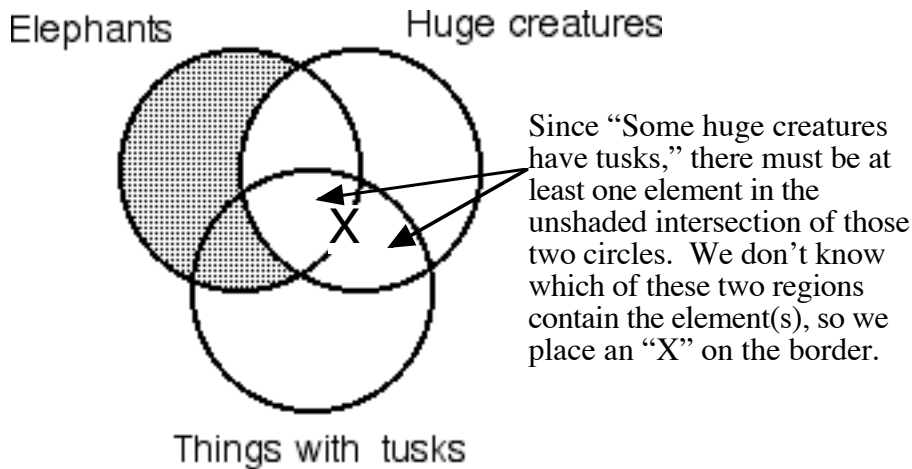
Use shading to diagram the universal premise “All elephants are huge creatures.” Shading indicates that a region has no elements.



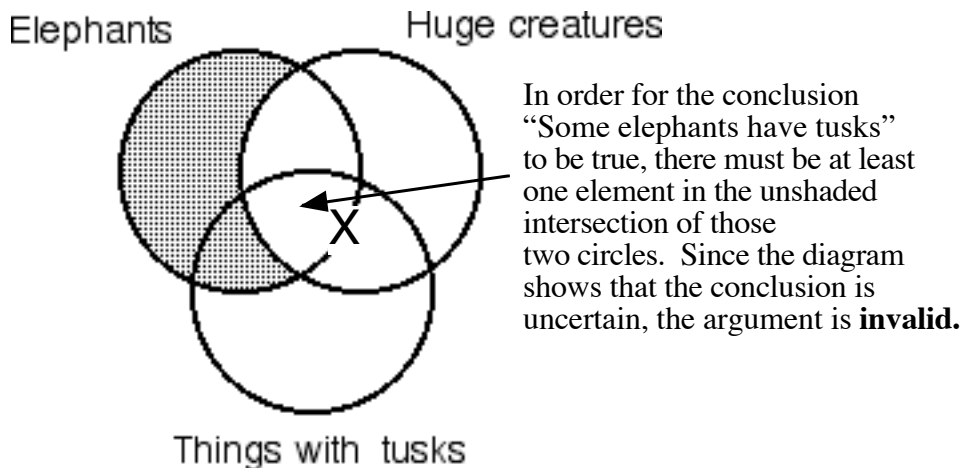
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Next, use an “X” to diagram the existential premise “Some huge creatures have tusks.”



Now that both premises have been diagrammed, if the marked diagram shows that the conclusion is true, then the argument is valid. If the marked diagram shows that the conclusion is false or uncertain, then the argument is invalid.



### EXAMPLE 2.3.2

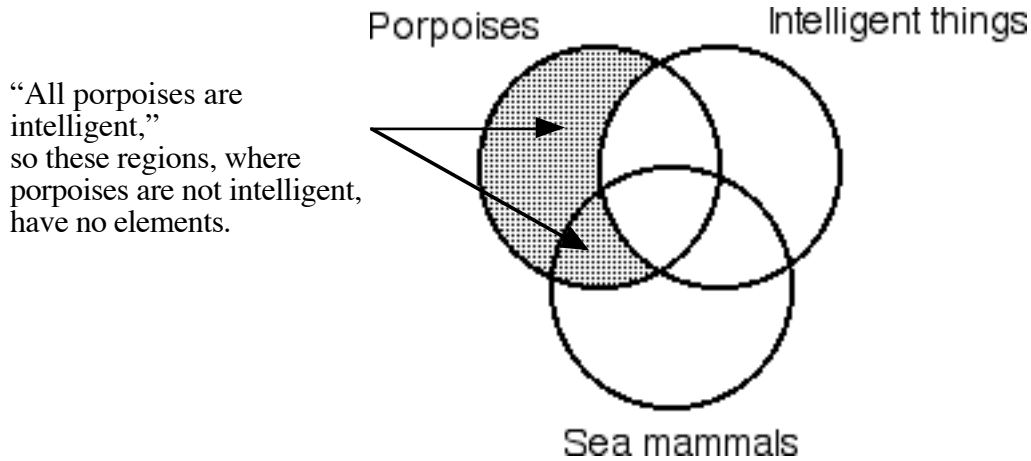
Test the validity of the argument:

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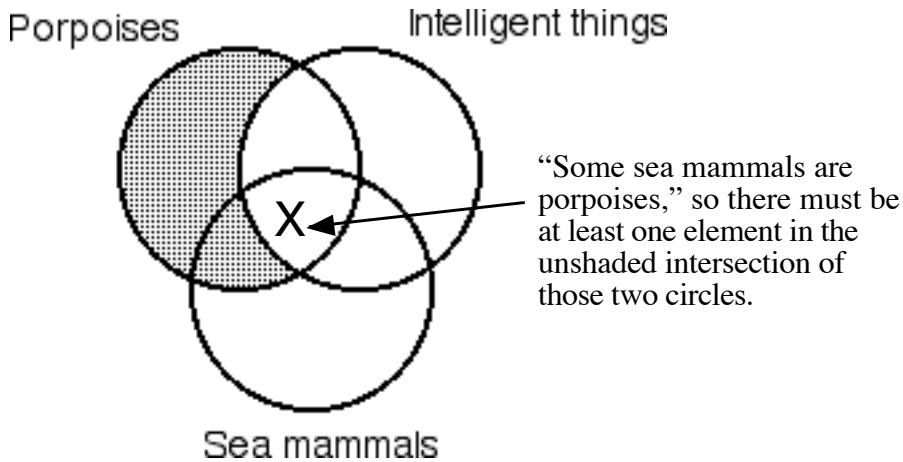
All porpoises are intelligent. Some sea mammals are porpoises. Therefore, some sea mammals are intelligent.

**EXAMPLE 2.3.2 SOLUTION**

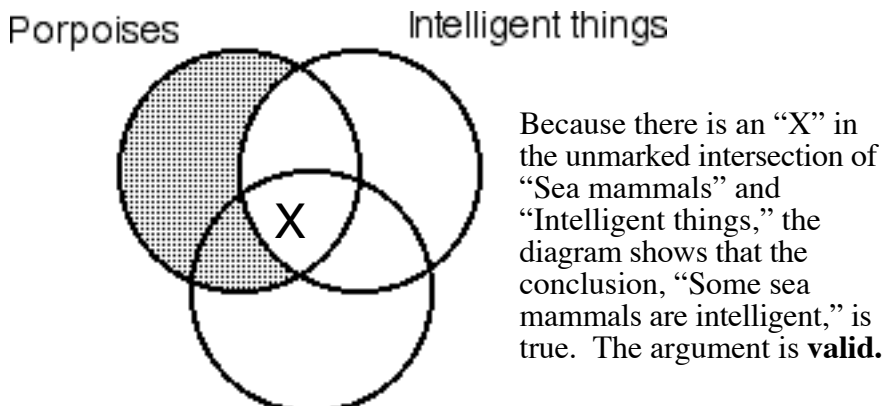
First use shading to diagram the universal premise.



Next use an “X” to diagram the existential premise.



If the marked diagram shows that the conclusion is true, then the argument is valid. If the marked diagram shows that the conclusion is uncertain or false, then the argument is invalid.



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### **EXAMPLE 2.3.3**

Test the validity of the argument:

All cows like to chew. Some dairy animals don't like to chew.

Therefore, some dairy animals aren't cows.

### **EXAMPLE 2.3.4**

Test the validity of the argument:

Some lawyers are judges. Some judges are politicians. Therefore, some lawyers are politicians.

### **WORLD WIDE WEB NOTE**

For practice on arguments involving categorical syllogisms and diagramming, visit the companion website and try The CATEGORIZER.

### **EXAMPLE 2.3.6**

*Test the validity of each argument.*

A. Some fish are tasty. All fish can swim. Therefore, some tasty things can swim.

B. Some doctors are dentists. Some dentists are surgeons. Therefore, some doctors are surgeons.

C. All hogs are smelly. Some swine aren't hogs. Therefore, some swine aren't smelly.

D. All burglars are criminals. Some thieves are criminals. Therefore, some burglars are thieves.

E. Some food preparers aren't cooks. All chefs are cooks. Some food preparers aren't chefs.

F. No thieves are saintly. Some congressmen are thieves. Therefore, some congressmen aren't saintly.

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### EXAMPLE 2.3.7

Consider these premises:

All poodles are dogs. All dogs bark.

We should easily recognize that a valid conclusion is "All poodles bark."

Question: since "**All** poodles bark" is a valid conclusion, wouldn't "**Some** poodles bark" also be a valid conclusion? After all, "some" sounds like a softer condition than "all," so common sense suggests that "some A are B" should be a valid conclusion whenever "all A are B" is a valid conclusion.

However, the answer to the question above is "no." In fact, "all" does not imply "some," due to this peculiarity of the word "all:" A statement like "All poodles bite" is true even if there were no poodles. In other words, if all of the poodles went extinct, the statement "All poodles bite" would still be (vacuously) true, but the statement "Some poodles bite" would be false (because "some poodles bites" means that there must be at least one poodle).

This example gives rise to the following observation, which holds for all arguments.

**If every premise in an argument has the form "All are...," a valid conclusion will not have the form "Some are..."** (unless a premise specifies that all of the sets in the argument are non-empty).

### PRACTICE EXERCISES

**1 - 18:** Test the validity of each argument.

1. All horses have hooves. Some horses eat oats. Therefore, some oat-eaters have hooves.
2. Some mammals are bats. All bats can fly. Thus, some mammals can fly.
3. No dogs can talk. Some searchers are dogs. Thus, some searchers can't talk.
4. Some squirrels fly. Some squirrels gather acorns. Therefore, some acorn-gatherers fly.
5. Some scavengers eat road-kill. All crows eat road-kill. Therefore, all crows are scavengers.
6. All successful politicians are persuasive. Some lawyers are persuasive. Thus, some lawyers are successful politicians.
7. Some skaters drink Surge. Some skaters drink Citra. Therefore, some Surge drinkers are Citra drinkers.
8. All tattooers are body-artists. Some tattooers drive Harleys. Therefore, some body-artists drive Harleys.
9. Some multiple-choice questions are tricky. No easy questions are tricky. Therefore, some multiple-choice questions are not easy.
10. Some snails live under rocks. All snails are slimy. Therefore, some things that live under

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11. All primates are curious. Some primates are carnivores. Thus, some carnivores are curious.
12. All astronauts are bold. Some pilots are not bold. Therefore, some astronauts are not pilots.
13. Some ants are aggressive. All ants are insects. Therefore, some insects are aggressive.
14. All plumbers use monkey wrenches. Some mechanics don't use monkey wrenches. Hence, some mechanics aren't plumbers.
15. All spies are secretive. Some agents aren't spies. Therefore, some agents aren't secretive.
16. Some poodles yap too much. Some dogs are poodles. Therefore, some dogs yap too much.
17. Some poodles are dogs. All poodles yap too much. Thus, some dogs yap too much.
18. All senators are politicians. Some corrupt people aren't politicians. Therefore, some corrupt people aren't senators.

### ANSWERS TO LINKED EXAMPLES

**EXAMPLE 2.3.3** Valid

**EXAMPLE 2.3.4** Invalid

**EXAMPLE 2.3.6**    A. Valid                      B. Invalid                      C. Invalid  
                            D. Invalid                      E. Valid                      F. Valid

### ANSWERS TO PRACTICE EXERCISES

**1.** Valid    **2.** Valid    **3.** Valid    **4.** Invalid    **5.** Invalid    **6.** Invalid  
**7.** Invalid    **8.** Valid    **9.** Valid    **10.** Valid    **11.** Valid    **12.** Invalid  
**13.** Valid    **14.** Valid    **15.** Invalid    **16.** Invalid    **17.** Valid    **18.** Valid