

HACKING MATHEMATICS

UNIT 3 MODULE 3 MISCELLANEOUS COUNTING PROBLEMS

EXAMPLE 3.3.1

Refer to the Shakesperean Insult-o-Matic from MODULE 1 (EXAMPLE 3.1.17).

Suppose we are going to form a three-part Shakespearean insult by choosing two different adjectives from column A and one noun phrase from column B.

Assume that rearranging the order of the adjectives does not change the meaning of the insult (for example, "You poisonous, dreadful toad" is **the same as** "You dreadful, poisonous toad"). How many different three-part insults are possible?

SOLUTION

We can use the Fundamental Counting Principle to solve this problem. However, we must take into account that the order in which the two adjectives are chosen or listed does not matter.

There are 12 adjectives from which to choose, and we want to choose two of them. Since order doesn't matter, the number of ways in which this can be done is $C(12, 2) = 66$.

There are 16 possible noun phrases with which they may be combined. The total number of ways to combine the two adjectives with a noun phrase is $(66)(16) = 1056$.

EXAMPLE 3.3.2

Suppose there are 8 Representatives and 6 Senators on a congressional conference committee. Three of each will be chosen to receive a birthday present, even though today isn't their birthday. How many 6-person groups are possible? (Note: they will each receive the same gift.)

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EXAMPLE 3.3.3

There are 12 female aviators and 8 male aviators at Billy Bob's Blimp-o-Drome. Billy Bob is going to choose a pilot and co-pilot for a special women-only flight, and another pilot and co-pilot for a special men-only flight. How many combinations are possible?

EXAMPLE 3.3.4

Suppose that by show of hands, we find that 24 people in this room are 18 years old, and 20 people are 19 years old. If we ask the question, "How many are 18 or 19 years old?" will the answer be $(24)(20) = 480$?

FACTS

In counting problems or probability problems, "AND" corresponds to "MULTIPLY" while "OR" corresponds to "ADD"

$$n(A \text{ and } B) = n(A) \times n(B)$$

$$n(\text{either } A \text{ or } B) = n(A) + n(B)$$

The second formula above assumes that A and B are mutually exclusive conditions.

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EXAMPLE 3.3.5

A couple is expecting the birth of a baby. If the child is a girl, they will choose her first name and middle name from this list of their favorite girls' names: Betty, Beverly, Bernice, Bonita, Barbie.

If the child is a boy, they will choose his first name and middle name from this list of their favorite boys' names: Biff, Buzz, Barney, Bart, Buddy, Bert.

In either case, the child's first name will be different from the middle name. How many two-part names are possible?

- A. 50 B. 600 C. 61 D. 900

SOLUTION

In this case, the child's name will be **either** a girl's name **or** a boy's name, so the number of possible names will be the number of possible girl's names plus the number of possible boy's names.

Number of possible girl's names:

We form a two-part girl's name by making two dependent decisions.

1. Choose first name: *5 options*.
 2. Choose middle name: *4 options*.
- $(5)(4) = 20$ different two-part girl's names.

Number of possible boy's names:

We form a two-part boy's name by making two dependent decisions.

1. Choose first name: *6 options*.
 2. Choose middle name: *5 options*.
- $(6)(5) = 30$ different two-part boy's names.

Total number of two-part names
= number of girl's names + number of boy's names
= $20 + 30 = 50$

The correct choice is A.

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EXAMPLE 3.3.6

The mathematics department is going to hire a new instructor. They want to hire somebody who possesses **at least four** of the following traits:

1. Honest;
2. Trustworthy;
3. Loyal;
4. Gets along well with others;
5. Well-groomed;
6. Good handwriting

In how many ways is it possible to combine at least four of these traits?

EXAMPLE 3.3.7

Six Democrats and five Republicans are in a jail cell. Either three Democrats or three Republicans will be chosen for a police line-up. How many different line-ups are possible? (We assume that a line-up is determined not only by the people who are chosen, but also by the order in which they are "lined-up.")

- A. 990 B. 7200 C. 33 D. 180

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EXAMPLE 3.3.8

Six Democrats and 5 Republicans are in a jail cell. Either 3 Democrats or 3 Republicans will be chosen to pick up trash alongside the highway. How many different 3-person groups are possible?

- A. 30 B. 200 C. 7200 D. 180

EXAMPLE 3.3.9

Gomer's Auto Body Shoppe is still trying to unload some surplus paint by offering the following special deals:

For \$99 you get a two-tone paint job, using one color for the car's top and a different color for the body.

For \$129 you get a three-tone paint job, using one color for the car's top, a different color for the body, and a third color for the hub caps.

The available colors are:

1. hot pink
2. puke green
3. Gator orange
4. flaming chartreuse

How many different color schemes are possible?

- A. 24 B. 36 C. 27 D. none of these

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EXAMPLE 3.3.10

Gomer is going to order a frozen tofu cone. The following toppings are available:

granola crumbles

seaweed sprinkles

carob chips

frosted alfalfa sprouts

roasted soybeans

1. How many topping combinations are possible if he will choose either three or four toppings?

A. 50 B. 10 C. 15 D. 21

2. How many combinations are possible if he may choose all, some or none of the items?

EXAMPLE 3.3.11

At the animal shelter there are seven new puppies and six new kittens.

Either three puppies or three kittens will be randomly chosen and donated to a local nursing home. How many different three-animal collections are possible?

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EXAMPLE 3.3.12

A survey of glamorous celebrities revealed that
30 wear dark glasses
32 smoke cigars
28 wear dark glasses and smoke cigars
1 does neither of these things

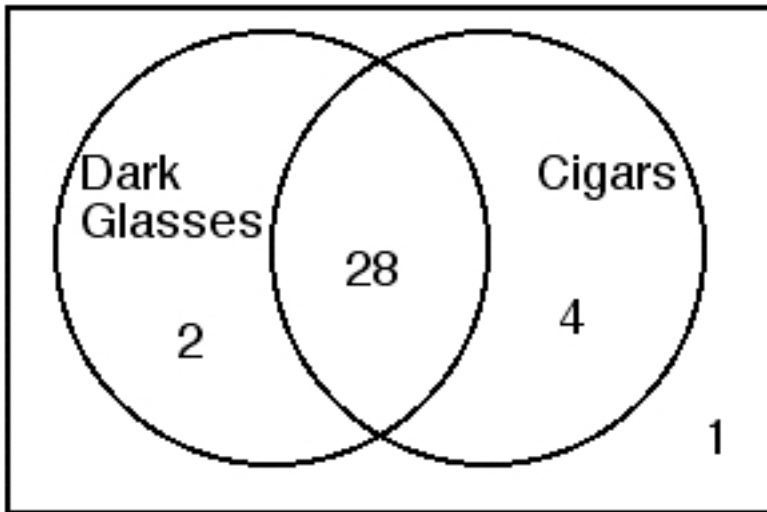
1. How many wear dark glasses and don't smoke cigars?
A. 540 B. 2 C. 35 D. 3
2. How many wear dark glasses or smoke cigars?

FACT

When a counting problem (or probability problem) refers to two categories that overlap (that is, categories that *aren't* mutually exclusive), the information can be organized with a Venn diagram.

EXAMPLE 3.3.12 SOLUTION

The information refers to two overlapping categories: "wears dark glasses" and "smokes cigars." We can organize the information in the following two-circle Venn diagram:



1. To answer the question "How many wear dark glasses and don't smoke cigars?" we recognize that the diagram shows 2 people who are within the set "wear dark glasses" yet are outside of the set "smoke cigars," so the answer is 2.
2. To answer the question "How many wear dark glasses or smoke cigars" we add all the numbers in those two circles: $2 + 28 + 4 = 34$

There is another fact that can be used to answer question #2 above (but not question #1) without making the Venn diagram:

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FACT

For categories or conditions A, B: $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$

Applying this fact to the question "How many wear dark glasses or smoke cigars?" we have:

$$\begin{aligned} &n(\text{wear dark glasses or smoke cigars}) \\ &= n(\text{wear dark glasses}) + n(\text{smoke cigars}) - n(\text{wear dark glasses and smoke cigars}) \\ &= 30 + 32 - 28 = 34. \end{aligned}$$

EXAMPLE 3.3.13

Among a group of Gators, 26 are modest, 28 are charming, 24 are modest and charming, 864 are not modest and not charming.

How many are...

1. ...charming but not modest?
2. ...modest or charming?

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EXAMPLE 3.3.14

The editorial board of the *National Requirer* has decided to launch two new series of investigative reports. One series will involve the theme

"**Elvis** ____ **my** ____!"

For this series, the writers will generate a story line by filling in the first blank with an item from this list:

"*loved*"; "*sang to*"; "*got sick on*"; "*smoked*" and filling in the second blank with an item from this list:

"*fig tree*"; "*living room carpet*"; "*cat*".

The other series will involve the theme "**Aliens** ____ **my** ____!"

The writers will generate a story line by filling in the first blank with an item from this list:

"*attacked*"; "*rescued*"; "*probed*";

and filling in the second blank with an item from this list:

"*home town*"; "*trailer park*"; "*granny's arrest record*"

How many story lines are possible?

EXAMPLE 3.3.15

Pizza Shack is offering the following special deal this month: for one low price (\$6.99) you can get a large pizza with up to 4 different toppings. There are 8 toppings from which to choose. How many different topping combinations are possible?

A. 70 B. 1680 C. 163 D. 32

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EXAMPLE 3.3.16

Homer pays big bucks to the Gomermatic Investment Corporation (GIC) to manage his retirement account. Homer hired GIC on the basis of the following event:

Last January, Homer received an unsolicited mailing from GIC. The mailing listed seven prominent stocks from the New York Stock Exchange. For each of the seven stocks, there was a prediction as to whether or not that stock's value would increase by at least 5% as of May 1. When May 1 rolled around, Homer found that all seven predictions were correct. He received a follow-up mailing from GIC, boasting of their spectacular ability to predict stock market events. Based on this phenomenal performance at picking winners and losers, Homer instantly signed up as a client of GIC.

What questions might have occurred to a less gullible investor?

WORLD WIDE WEB NOTE

For more practice problems visit the companion website and try THE FUNDAMENTAIZER PART 2.

PRACTICE EXERCISES

1. In a pet shop, there are 8 puppies and 6 kittens. 3 puppies and 2 kittens will be neutered. In how many ways can the 5 animals be selected?

A. 2002 B. 71 C. 840 D. 2400

2. 5 women and 4 men are trying out for the cheerleading team. If 2 women and 2 men will be selected, how many different 4-person groups are possible?

A. 16 B. 60 C. 18 D. 80

3. A couple is expecting a baby. They don't yet know whether the child will be a boy, or a girl. They will name the child by choosing a first name and middle name from either of these two lists, depending upon the sex of the baby:

girls' names: Sally, Sue, Sara, Stephanie

boys' names: Josh, Jacob, Jonah, Jeremiah, John.

The child's first name will be different from the middle name. How many possible names are there?

A. 240 B. 400 C. 41 D. 32

4. Carmen's Carry-Out offers two different menus to her customers: A customer may choose a 3-course meal from either the Spanish Menu or the Oriental Menu, but the two menus may not be mixed. The Spanish menu consists of 3 different appetizers, 4 different entrees, and 2 different desserts. The Oriental menu consists of 2 different appetizers, 3 different entrees, and 3 different desserts. How many different 3-course meals are possible?

A. 432 B. 42 C. 864 D. 84

5. Referring to #5, how many different 3-course meals are possible if the two menus may be combined?

A. 175 B. 1296 C. 72 D. 17

6. A survey of 100 Leon County voters, following the November, 2000 elections, revealed the following data: 38 voted for Bush for President, 28 voted for McCollum for U.S. Senator, and

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25 voted for both Bush for President and McColluumfor Senator. How many of those surveyed voted for at least one of the two candidates mentioned above?

- A. 66 B. 63 C. 41 D. 91

7. Piggo's Pizza is offering a special deal: for \$5 you get a large pizza with up to 4 toppings. The toppings available are: sardines, Spam, Hershey's Kisses, corn, peanuts, pickles. No topping may be repeated on a single pizza. How many different topping combinations are possible?

- A. 10 B. 64 C. 57 D. 36

8. In #6 above, how many didn't vote for Bush?

- A. 10 B. 72 C. 75 D. 62

9. A pet shop has 8 puppies and 6 kittens. Either 2 puppies or 3 kittens will be donated to the Mary Kay Cosmetics Product Testing Labs. In how many ways can the selection be made?

- A. 112 B. 8064 C. 48 D. 560

10. In a jail cell, there are 8 Democrats and 6 Republicans. Either 4 Democrats or 4 Republicans will be selected to go out and pick up trash along the highway. How many different 4-person groups are possible?

- A. 1050 B. 85 C. 102 D. 1001

11. How many subsets are in a 12-element set? (Note: this is a question that you could have answered prior to Test 1.)

- A. 24 B. 144 C. 4096 D. 66

12. A basketball team has a home uniform and an away uniform. For the home uniform, there is a choice of 3 different jerseys, 4 styles of trunks, 2 styles of shoes, and 2 styles of stockings. For the away uniform, there is a choice of 2 styles of jersey, 2 styles of trunks, 1 style of shoe, and 2 styles of stockings. All items in the home uniform are different from those in the away uniform, and items from the two uniforms may not be mixed. How many different uniform configurations are possible?

- A. 384 B. 288 C. 54 D. 56

13. On a certain day, 20 customers purchased Burley Cigarettes from Moe's QuikShop. Of the 20, 14 were bikers, 16 had tattoos, and 12 were bikers with tattoos. How many were neither bikers, nor tattooed?

- A. 18 B. 10 C. 2 D. 4

14. Refer to the situation in #7 above. How many different 3-topping combinations are possible?

- A. 24 B. 20 C. 12 D. 60

15. A new health-food-trash-food emporium, I Definitely Believe It's Tofu, has opened next to Publix. Their specialty is the frozen tofu cone, with toppings. There are five toppings from which to choose: carob chips, granola, prunes, sunflower seeds, or (of course) seaweed sprinkles. A customer may order a cone with any combination of toppings (or no toppings at all: *au naturel*). How many different possibilities are there?

- A. 10 B. 120 C. 32 D. 25

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16. Refer to the situation in #15 above. How many different topping combinations are possible if at least 2 toppings will be chosen?

- A. 6 B. 18 C. 26 D. 14

17. A wrestling promoter needs to select 2 wrestlers to fight in the main event for Wrestlepalooza III. The main event will involve one of these two themes: Masked Mayhem (featuring two masked wrestlers), or Whiskered Warfare (featuring two bearded wrestlers). The promoter has eight masked wrestlers and seven bearded wrestlers from whom to choose. In how many ways may he form a match for the main event?

- A. 98 B. 49 C. 14 D. 588

18. Ships of the navy of the Democratic People's Republic of North Tyrania communicate at sea, using flags to transmit coded messages according to the following scheme: each ship has a set of 6 flags (the same six flags are on each ship). A code message is formed by choosing some combination of the flags, and hanging them from the mainmast. A code message is determined entirely by the flags chosen, and not by the order in which they are arranged. Thus, a code message could involve no flags at all ("we are unable to come to the mast at this time; please leave your message at the beep") or it could involve 1, 2, 3, 4 or 5 flags, or it could involve all 6 flags ("if it's my wife, I ain't here"). How many different messages are possible?

- A. 720 B. 120 C. 64 D. 36

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19. In #10 above, 4 Democrats and 4 Republicans will be selected. How many different 8-person groups are possible?

- A. 1050 B. 85 C. 102 D. 1001

20. For his birthday, Gomer is going to invite some friends to a party at Chuck E. Cheese's. He has compiled a list of 10 of his closest friends, but he can only afford to invite 6 of them. If he randomly chooses 6 friends from the list of 10, how many different 6-person collections are possible?

- A. 60 B. 5040 C. 210 D. 1,000,000

21. The board of directors of a lobbying group consists of 6 men and 5 women. Two men and two women will be chosen to attend a conference in Sopchoppy. How many different 4-person groups are possible?

- A. 330 B. 150 C. 25 D. 600

22. Gomer is shopping for a cat. He wants a cat that possesses **at least three** of the following traits: I. Has aloof personality; II. Has rodent breath; III. Has stripes; IV. Purrs when happy; V. Twitches tail when angry. Assuming that a combination of traits includes at least three of the traits, how many combinations of traits are possible?

- A. 16 B. 10 C. 20 D. 32

23. There are 6 Democrats and 6 Republicans on the local zoning commission. They are going to randomly choose a chairperson and treasurer from among themselves, subject to the following agreements: both officeholders will belong to the same party, and no person will hold more than one position. How many different outcomes are possible?

- A. 132 B. 900 C. 30 D. 60

24. Select the answer that correctly completes the sentence.

$P(14, 10) =$

- A. 24,024
B. the number of ways to choose 10 people from a group of 14 and assign them into 10 empty seats.
C. the number of 10-element subsets in a 14-element set.
D. the number of ten-letter passwords that can be formed using letters chosen from the set $S = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n\}$, if repeated letters are allowed.

25. In how many different ways is it possible to choose a Chairperson, Associate Chairperson, and Parliamentarian from a list of 7 candidates, assuming that no candidate can hold more than one position?

- A. 210 B. 343 C. 16 D. 35

26. The Egotists' Club consists of 6 men and 7 women. They will send either 2 men or 2 women to the national Egotists' convention. How many different 2-person groups are possible?

- A. 42 B. 315 C. 72 D. 36

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ANSWERS TO LINKED EXAMPLES

EXAMPLE 3.3.2 1120

EXAMPLE 3.3.3 7392

EXAMPLE 3.3.4 Since 24 are 18 years old and 20 are 19 years old, the number who are either 18 or 19 years old is $24 + 20 = 48$.

EXAMPLE 3.3.6 22

EXAMPLE 3.3.7 D

EXAMPLE 3.3.8 A

EXAMPLE 3.3.9 B

EXAMPLE 3.3.10 1. C 2. 32

EXAMPLE 3.3.11 55

EXAMPLE 3.3.13 1. 4 2. 30

EXAMPLE 3.3.14 21

EXAMPLE 3.3.15 C

EXAMPLE 3.3.16 What are the names and addresses of other people who were sent this solicitation?

ANSWERS TO PRACTICE EXERCISES

- | | | | | | |
|--------------|--------------|--------------|--------------|--------------|--------------|
| 1. C | 2. B | 3. D | 4. B | 5. A | 6. C |
| 7. C | 8. D | 9. C | 10. B | 11. C | 12. D |
| 13. C | 14. B | 15. C | 16. C | 17. B | 18. C |
| 19. A | 20. C | 21. B | 22. A | 23. D | 24. B |
| 25. A | 26. D | | | | |