UNIT 3 MODULE 5
PROBABILITIES INVOLVING NEGATIONS, DISJUNCTIONS, and CONDITIONAL PROBABILITY

The following facts follow from our discussions of counting in UNIT 3 MODULE 3 and probability in UNIT 3 MODULE 4.

P(E or F) = P(E) + P(F) – P(E and F)

P(not E) = 1 – P(E)

(Note: these problems can frequently be analyzed with Venn diagrams as well.)

EXAMPLE 3.5.1

According to a recent article from the *New England Journal of Medical Stuff* ,
63% of cowboys suffer from saddle sores,
52% of cowboys suffer from bowed legs,
40% suffer from both saddle sores and bowed legs.

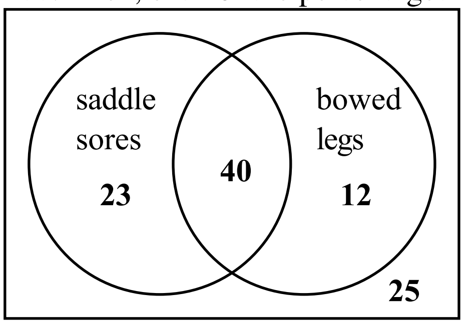
What is the probability that a randomly selected cowboy...

1. ...has saddle sores or bowed legs?
2. ...doesn't have saddle sores?
3. ...has saddle sores but doesn't have bowed legs?
4. ...has saddle sores and bowed legs?
5. ...has neither of these afflictions?

EXAMPLE 3.5.1 SOLUTIONS

As with counting problems, when a probability problem refers to two overlapping categories, we can organize the information with a Venn diagram.

Since the data was given in terms of percentages, we will pretend that the total population is 100. Then, each of the percentages is just a raw number.



1. The number of cowboys who have saddle sores or bowed legs = 23 + 40 + 12 = 75.
So, P(saddle sores or bowed legs) = 75/100 = .75

2. From the diagram, the number of cowboys who don't have saddle sores is
12 + 25 = 37, so
P(doesn't have saddle sores) = 37/100 = .37

We could also get this answer from the complements rule. Since 63% of the cowboys have saddle sores, P(has saddle sores) = .63.

Then, P(don't have saddle sores) = 1 - .63 = .37.

3. The diagram shows us that there are 23 cowboys out of 100 who have saddle sores but don't have bowed legs, so P(has saddle sores but not bowed legs) = 23/100 = .23

4. The diagram shows that 40 cowboys out of 100 have both conditions (this information was also stated directly at the beginning of the problem), so

P(has saddle sores and bowed legs) = 40/100 = .40

5. The diagram shows that there are 25 cowboys out of 100 who have neither affliction, so

P(has neither affliction) = 25/100 = .25

EXAMPLE 3.5.2

A survey of 50 Yugo drivers revealed the following:

30 enjoy waiting for tow trucks
35 enjoy hitchhiking
25 enjoy waiting for tow trucks and hitchhiking

UNIT 3 MODULE 5

What is the probability that a randomly selected Yugo driver...

1. ...enjoys at least one of these activities?

A. .8 B. .65 C. .9 D. 1.8

2. ...doesn't enjoy hitchhiking?

A. .35 B. .15 C. .3 D. .5

3. ...enjoys neither of these activities?

EXAMPLE 3.5.3

The table below shows the distribution of guests on the Jerry Slinger show.
S: screams obscenities P: punches somebody

	S	**S′**	**Totals**
P	14%	8%	22%
P′	52%	26%	78%
Totals	66%	34%	100%

1. What is the probability that a guest screams obscenities or punches somebody?

2. What is the probability that a guest doesn't scream obscenities and doesn't punch anybody?

MUTUALLY EXCLUSIVE EVENTS
Events E and F are *mutually exclusive* if it is not possible for both E and F to occur simultaneously.

This means that P(E and F) = 0.

If events E, F are mutually exclusive, then P(E or F) = P(E) + P(F)

EXAMPLE 3.5.4

In a certain class, 45% of the students are freshmen (F), 30% are sophomores (So)

20% are juniors (J), 5% are seniors (Se)

What is the probability that a randomly chosen student is a junior or senior?

EXAMPLE 3.5.5

A university awards scholarships on the basis of student performance on a certain placement test. The table below indicates the distribution of scores on that test.

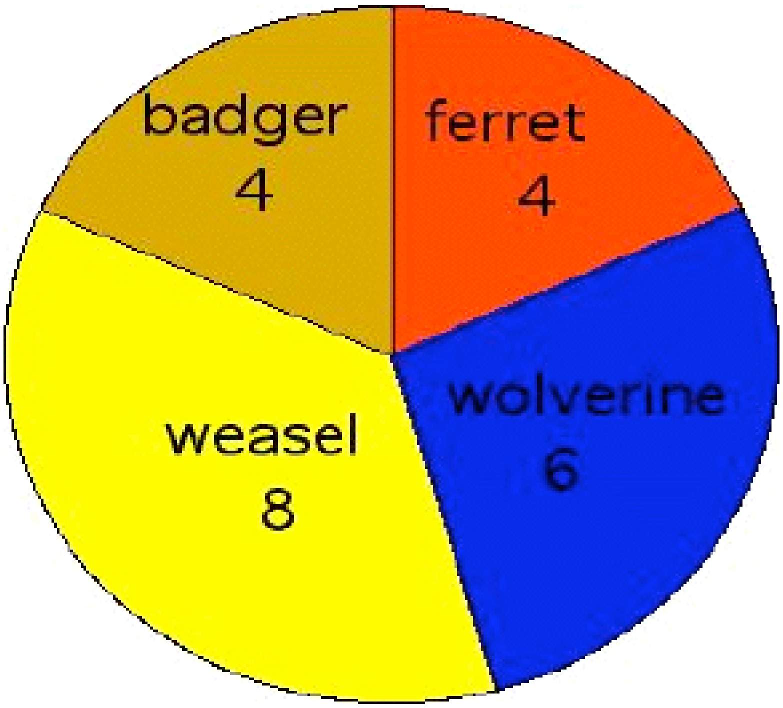
Score	Scholarship	Percentage
0-200	None	13%
201-300	None	23%
301-400	None	26%
401-500	Partial	12%
501-600	Partial	11%
601-700	Partial	9%
701-800	Full	6%

If one student is randomly selected, find the probability that he/she...
1. ...received a partial scholarship.
2. ...didn't have a score in the 201 - 300 range.
3. ...had a score less than 501
4. ...received some kind of scholarship

If 800 students are selected, how many would we expect...
5. ...received no scholarship?
6. ...had scores higher than 600?

EXAMPLE 3.5.6

The pie chart below shows the distribution of animals at Gomer's Petting Zoo.



If one animal randomly goes on a rampage, find the probability that it is...

1. ...a weasel or badger?
2. ...not a badger or ferret?

CONDITIONAL PROBABILITY
Suppose we roll one die.
Let A be the event that the result is the number "2."
Then we know that P(A) = 1/6

However, suppose that before I reveal the result of the die roll, I tell you that an even number has occurred (event E).
Would you still say that P(A) = 1/6?

If we know that and even number has been rolled, then there are only three possible outcomes ([2, 4, 6}), not six, so given this special information it would be reasonable to say that the probability that we rolled a "2" is 1/3.

This is an example of CONDITIONAL probability.
We say that "The probability that the die roll is '2,' **given** that the die roll is 'even,' is 1/3. "

Notation:

P(A, given E) =1/3

or
P(A|E) = 1/3 *The vertical bar separating the names of the events reads "given that."*

General fact:
For any events E, F

$$\mathbf{P(E,\ given\ F) = \frac{P(E\ and\ F)}{P(F)}}$$

which is the same as

$$\mathbf{P(E,\ given\ F) = \frac{n(E\ and\ F)}{n(F)}}$$

If we are referring to population statistics,

$$\mathbf{P(E,\ given\ F) = \frac{portion\ of\ population\ satisfying\ both\ conditions\ E\ and\ F}{portion\ of\ population\ satisfying\ condition\ F}}$$

As a practical matter, conditional probability problems tend to be simpler than these formulas imply. Usually we can solve them simply by thinking in terms of basic probability facts and taking into account the significance of the "given" condition.

EXAMPLE 3.5.7
In a box we have a bunch of puppies:
4 brown bulldogs
2 gray bulldogs
5 brown poodles
3 gray poodles

If one puppy is selected, what is the probability that the puppy is...

1. ...brown?
2. ...a poodle?
3. ...gray or a bulldog?
4. ...brown and a bulldog?
5. ...a bulldog, **given** that it is gray?
6. ...brown, **given** that it is a poodle?

SOLUTION TO EXAMPLE 3.5.7 #5 and #6
5. We want to find the probability that a puppy is a bulldog, given that it is gray. This means that we have already selected the puppy, and we know that it is one of the five gray puppies. Among the five gray puppies, two of them are bulldogs, so
P(bulldog|gray) = 2/5

6. We want to find the probability that a puppy is brown, given that it is a poodle. This means that we have already selected the puppy and we know that it is one of the eight poodles. Among the eight poodles, five of them are brown, so
P(brown|poodle) = 5/8

EXAMPLE 3.5.8
A survey of Gators indicates that 7% are charming, 4% are modest, and 3% are both charming and modest. Find the probability that a Gator is modest, given that he/she is charming.
A. .75 B. .03 C. .43 D. .25

EXAMPLE 3.5.9
Refer to the data on scholarships in the table presented earlier (see **EXAMPLE 3.6.9).**

Score	Scholarship	Percentage
0-200	None	13%
201-300	None	23%
301-400	None	26%
401-500	Partial	12%
501-600	Partial	11%
601-700	Partial	9%
701-800	Full	6%

1. What is the probability that a randomly-chosen student received a full scholarship, given that he/she received some sort of scholarship?

2. What is the probability that a randomly chosen student received a scholarship, given that he/she had a score less than 501?

EXAMPLE 3.5.10
The table below shows the distribution according to cumulative GPA of juniors at Normal University.

GPA	% of Juniors
0.00 – 1.99	16%
2.00 – 2.49	24%
2.50 – 2.99	28%
3.00 – 3.49	22%
3.50 – 4.00	10%

1. Find the probability that a randomly selected junior's GPA is greater than 1.99, given that it is less than 3.50.
A. .22 B. .88 C. .82 D. .74

2. Referring to the data above, find the probability that a randomly selected junior's GPA is in the 2.50 - 3.49 range, given that it is greater than 1.99.

EXAMPLE 3.5.11
Recently, Gomer took his Yugo to Honest Al's Yugo Repair Shop for a brake job.
Later, while driving home, a wheel fell off of the car. When Gomer returned to Honest Al's to complain that the wheel must have fallen off because of the brake job was done incorrectly, Honest Al produced a ream of statistics from NHTSA that showed that for this type of brake job, the probabilty that the wheel will fall off, even if the work is done incorrectly, is only about 0.008.
Based on that data, Honest Al graciously offered to cover 1% of the cost of repairing the damage to Gomer's car.
What question should Gomer have asked?

C: work done correctly F: wheel fell off

EXAMPLE 3.5.11 SOLUTION
The statistic that Honest Al cited would be useful if we were tried to predict whether Gomer's wheel would fall off. Since the wheel has definitely fallen off, that statistic is meaningless. This illustrates a common and fundamental error in the use of statistics: treating events as random and uncertain even though those events have already occurred. Gomer should have asked a question that takes into account the fact that the wheel has already fallen off, such as "Given that the wheel has fallen off, what's the probability that the work was done incorrectly?"

If he had looked at Honest Al's data, here's what he would have seen:

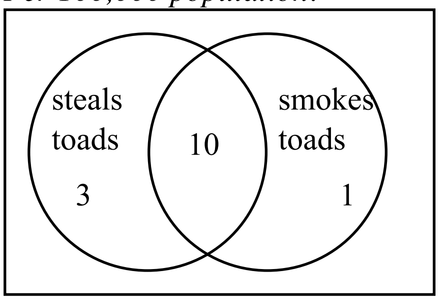
	F	F′	Totals
C	1	860	861
C′	5	650	655
Totals	6	1510	1516

C: work done correctly F: wheel fell off

The data shows that the probability that the work was done incorrectly, given that the wheel has fallen of, is 5/6 or roughly .833. Rather than paying 1% if the cost of replacing Gomer's vehicle, it would be more reasonable for Honest Al to pay 83% of the cost.

EXAMPLE 3.5.12
Suppose that the data below comes from the FBI Uniform Crime Statistics.
It conveys information about the number of Americans (per 100,000 population) involved in the crimes of toad theft and toad smoking.

Per 100,000 population:



A rare, exotic toad has been stolen from the Tallahassee Museum. Police are searching for Gomer, who is the only known toad-smoker in town. Meanwhile, Gomer's lawyers have spoken out publicly. Referring to the data shown above, they state that, since only 10 out 100,000 people are both toad-stealers and toad-smokers, it is extremely unlikely that Gomer is the guilty party, and so the police should focus their investigation elsewhere.
What do you think about this claim?

EXAMPLE 3.5.14

The conventional test for tuberculosis (TB) is only about 50% accurate. Does this mean that if you test positive for TB, then the probability that you actually have TB is about .5? Suppose that the table below summarizes the results of the TB screening for a sample of 500 people. In this table, TB means "A person has tuberculosis," and P means "A person tests positive for TB."

	TB	TB′	Totals
P	9	250	259
P′	1	240	241
Totals	10	490	500

Use this information to find the probability that a person who tests positive for TB actually has the disease.

PRACTICE EXERCISES

Table A below shows the distribution of undergraduate students at Normal University according to the number of credit hours for which they are registered this semester. Table B below shows the distribution of students at Normal University according to cumulative G.P.A.

TABLE A

# of credit hours	% of students
11 or fewer	12%
12	31%
13	6%
14	8%
15	21%
16	9%
17	2%
18 or more	11%

TABLE B

cumulative G.P.A.	% of students
0.00 - 0.80	14%
0.81 - 1.60	16%
1.61 - 2.40	38%
2.41 - 3.20	17%
3.21 - 4.00	15%

1 - 4: Refer to the appropriate table to determine the probability that a randomly selected student:

1. has a G.P.A. greater than 0.80.
A. .16 B. .86 C. .81 D. .14

2. is registered for 12 or 13 credit hours.
A. .516 B. .186 C. .37 D. .91

3. is registered for more than 16 credit hours.
A. .13 B. .22 C. .31 D. .09

4. has a G.P.A. that is not in the 0.81 - 3.20 range.
A. .14 B. .15 C. ..71 D. .29

5 - 6: Statistics for a certain carnival game reveal that the contestants win a large teddy bear 1% of the time, win a small teddy bear 4% of the time, win a feather attached to an alligator clip 35% of the time, and lose the rest of the time. What is the probability that a randomly selected player…

5. …wins a teddy bear.
A. .4 B. .05 C. .5 D. .005

6. …doesn't lose.
A. .65 B. .35 C. .4 D. .04

7. A survey of 50 informed voters revealed the following:
32 believe that Earth has been visited by space aliens
28 believe that Elvis is still alive
20 believe that Earth has been visited by space aliens and Elvis is still alive.
According to this data, what is the probability that a randomly selected informed voter believes that Earth has been visited by space aliens or Elvis is still alive?

A. .40 B. .60 C. .80 D. 1.20

8. Referring to the data in #7 above, what is the probability that a randomly selected voter doesn't believe that Earth has been visited by space aliens and doesn't believe that Elvis is still alive?
A. .20 B. .1584 C. .40 D. .80

9. A group of Harley-Davidson enthusiasts were recently asked "How many tattoos do you have?" The responses are summarized in the following table:

# of tattoos	% of respondents
0	2%
1	4%
2	3%
3	5%
4 or more	86%

What is the probability that a randomly chosen respondent has at least one tattoo?
A. .02 B. .04 C. .80 D. .98

10. Referring to the data in the table for #9, what is the probability that a respondent has 2 or 3 tattoos?
A. .8 B. .08 C. .15 D. .0015

11 : The table below shows the distribution according to salary of the employees of a large corporation.

annual salary	% of employees
\$0 - 9,999	4%
10,000 - 29,999	38%
30,000 - 59,999	32%
60,000 - 99,999	17%
100,000 or more	9%

11. Find the probability that a randomly chosen employee's salary is in the \$0,000 - \$9,999 range or in the \$60,000 - \$99,999 range.
A. .2032 B. .0068 C. .57 D. .21

12. The table below summarizes the distribution of a number a dogs. If one of these dogs is randomly selected, find the probability that it doesn't have fleas or is a bulldog.

	beagle	**poodle**	**bulldog**	**totals**
fleas	21	17	9	47
no fleas	9	13	5	27
totals	30	30	14	74

A. 0.49 B. 0.36 C. 0.41 D. 0.55

13. A survey of bulldogs reveals that 28% of them agree with the statement "cats are yummy." Among a group of 900 bulldogs how many would we expect to agree with the statement "cats are yummy?"
A. 572 B. 25 C. 648 D. 252

14. At the Wee Folks Gathering there are 45 jolly hobbits, 27 grumpy hobbits, 5 jolly leprechauns and 27 grumpy leprechauns.
If one person is randomly selected, find the probability that he/she is a leprechaun or jolly.
A. .77 B. .74 C. .048 D. .05

15. A survey of 50 informed voters revealed the following:
32 believe that Earth has been visited by space aliens
28 believe that Elvis is still alive
20 believe that Earth has been visited by space aliens and Elvis is still alive.
According to this data, what is the probability that a randomly chosen voter doesn't believe that Elvis is still alive, given that he/she believes that Earth has been visited by space aliens?
A. .375 B. .6 C. .24 D. .25

16. A group of Harley-Davidson enthusiasts were recently asked "How many tattoos do you have?" The responses are summarized in the following table:

# of tattoos	% of respondents
0	2%
1	4%
2	3%
3	5%
4 or more	86%

What is the probability that a randomly chosen Harley-Davidson enthusiast has more than one tattoo, given that he/she has fewer than 4 tattoos?
A. .08 B. .04 C. .57 D. .29

17. The table below shows the distribution according to salary of the employees of a large corporation.

annual salary	% of employees
$0 - 9,999	4%
10,000 - 29,999	38%
30,000 - 59,999	32%
60,000 - 99,999	17%
100,000 or more	9%

Find the probability that a randomly chosen employee's salary is more than $9,999, given that it is less than $60,000.
A. 1.297 B. .946 C. .543 D. .70

18. In a basket, there are 10 ripe peaches, 8 unripe peaches, 12 ripe apples, and 4 unripe apples. If one fruit is randomly chosen, find the probability that it is a peach, given that it is unripe.

A. .50 B. 44 C. .67 D. .33

ANSWERS LINKED EXAMPLES

EXAMPLE 3.5.2	1. A (40/50 = .8)	2. C (15/50 = .3)	3. 10/50 = .2	
EXAMPLE 3.5.5	1. .12 + .11 + .09 = .32			
	2. 1 - .23 = .77			
	3. .13 + .23 + .26 + .12 = .74			
	4. .12 + .11 + .09 + .06 = .38			
	5. (.62)(800) = 496			
	6. (.15)(800) = 120			
EXAMPLE 3.5.6	1. 12/22 = .545			
	2. 14/22 = .636			
EXAMPLE 3.5.7	1. 9/14	2. 8/14	3. 9/14	4. 4/14
	5. 2/5	6. 5/8		
EXAMPLE 3.5.8	C			
EXAMPLE 3.5.9	1. .158	2. .162		
EXAMPLE 3.5.10	1. C	2. .676		
EXAMPLE 3.5.12	The statistical claim doesn’t take into account the fact that Gomer is a toad smoker. The data shows that the probability that a person steals toads, given that he or she smokes toads, is 10/11.			
EXAMPLE 3.5.13	The statistic would be useful if we were trying to predict whether Nicole would be murdered. Since she was already murdered, the statistic is meaningless. A meaningful question would be, “Given that a woman has been murdered, what is the probability that her murderer was her abusive husband?”			
EXAMPLE 3.5.14	.0347			

ANSWERS TO PRACTICE EXERCISES

1. B	**2.** C	**3.** A	**4.** D	**5.** B	**6.** C
7. C	**8.** A	**9.** D	**10.** B	**11.** D	**12.** A
13. D	**14.** B	**15.** A	**16.** C	**17.** B	**18.** C