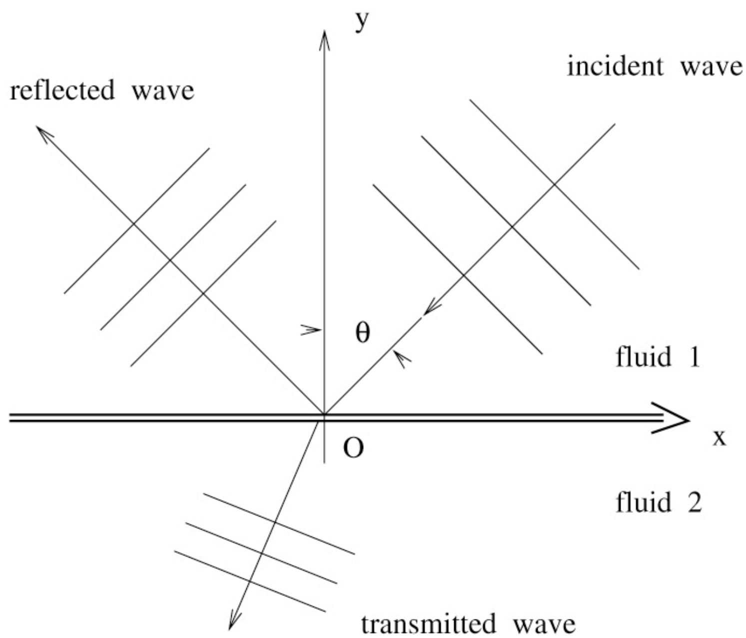


This problem is intended to test the discrete formulation and computation of sound transmitted through a surface of discontinuity. Here the surface of discontinuity is formed by the interface of two fluids with different densities and sound speeds as shown in the figure. An incident acoustic wave at an angle of incidence $\theta$ impinges on the interface. Part of the wave is transmitted and part of it is reflected. For computation purpose, we will use the following length, velocity, time, pressure and density scales. Subscripts 1 and 2 indicate fluid 1 and 2.

length scale = $L$

velocity scale = $a_1$ (sound speed in region 1)

time scale = $\frac{L}{a_1}$

density scale = $\bar{\rho}_1$ (density of fluid in region 1)

pressure scale = $\bar{\rho}_1 a_1^2$

$$\square = \frac{\square_2}{\square_1}, \qquad \square = \frac{\square_2}{\square_1}$$

The governing equations for small amplitude disturbances in fluid 1 are

$$\frac{\partial \rho_1}{\partial t} + \left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y}\right) = 0$$

$$\frac{\partial u_1}{\partial t} = -\frac{\partial p_1}{\partial x}$$

$$\frac{\partial v_1}{\partial t} = -\frac{\partial p_1}{\partial y}$$

$$\frac{\partial p_1}{\partial t} + \left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y}\right) = 0$$

The governing equations for small amplitude disturbances in fluid 2 are,

$$\frac{\partial \rho_2}{\partial t} + \square\left(\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y}\right) = 0$$

$$\square\frac{\partial u_2}{\partial t} = -\frac{\partial p_2}{\partial x}$$

$$\square\frac{\partial v_2}{\partial t} = -\frac{\partial p_2}{\partial y}$$

$$\frac{\partial p_2}{\partial t} + \square\left(\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y}\right) = 0$$

The dynamic and kinematic boundary conditions at the fluid interface are,

$$p_1 = p_2, \qquad v_1 = v_2$$

Now consider a plane wave at an incident angle $\theta$ and frequency $\omega$ given below

$$\begin{bmatrix} p_1 \\ u_1 \\ v_1 \\ \rho_1 \end{bmatrix}_{\text{incidence wave}} = \mathrm{Re}\begin{bmatrix} 1 \\ \sin\theta \\ \cos\theta \\ 1 \end{bmatrix} e^{-i\omega(\sin\theta x + \cos\theta y + t)}$$

($\mathrm{Re} =$ real part of.) Determine the intensity and direction of the transmitted and reflected waves for the two cases with $\theta = 20^\circ$ and $65^\circ$. The frequency and other parameters are $\square = 0.7$, $\square = 0.694$ and $\square = 1$. Plot contours of $p$ at intervals of $0.25$ at the beginning of a cycle.