Multi-Geometry Scattering Problem

The problem considered here is the scattering of sound generated by a spatially distributed, axisymmetric, acoustic source from multiple rigid circular cylinders. This case provides a stringent test of the ability of high-order CAA codes to handle increasingly complex geometries. It also serves to examine the performance of well-established low-order CFD codes, developed for complex geometries, when applied to the simulation of aeroacoustic phenomena. In addition, this problem provides a demonstration of numerical robustness, long-time stability and suitability of far-field radiation treatments in the presence of multiple scattering bodies.

The acoustic scattering problem is governed by the linearized Euler equations, which may be written in two spatial dimensions as

$$\frac{\partial \rho}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0 \tag{2}$$

$$\frac{\partial v}{\partial t} + \frac{\partial p}{\partial y} = 0 \tag{3}$$

$$\frac{\partial p}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = S \tag{4}$$

The flow variables in the above expressions are fluctuating quantities and have been non-dimensionalized by the following scales:

length scale	=	diameter of largest circular cylinder, D_{max}
velocity scale	=	ambient speed of sound, c_∞
time scale	=	$\frac{D_{max}}{c_\infty}$
density scale	=	ambient gas density, ρ_∞
pressure scale	=	$\rho_\infty c_\infty^2$

The time-dependent acoustic source term on the right-hand side of the energy equation is assumed axisymmetric and is written in the source-centered coordinate system as

$$S = \exp\left[-\ln 2 * \left\{ \frac{x_S^2 + y_S^2}{b^2} \right\} \right] \sin(\omega t) \tag{5}$$

where $\omega = 8\pi$ and $b = 0.2$

Case 1

The first case, shown in Fig. 1, consists of two cylinders of unequal diameters ($D_1 = 1.0, D_2 = 0.5$), with a co-linearly located source equidistant from the center of each cylinder. In the (x_S, y_S)-coordinate system centered on the source, the location of the cylinders are given by $L_1 = (-4, 0), L_2 = (4, 0)$.

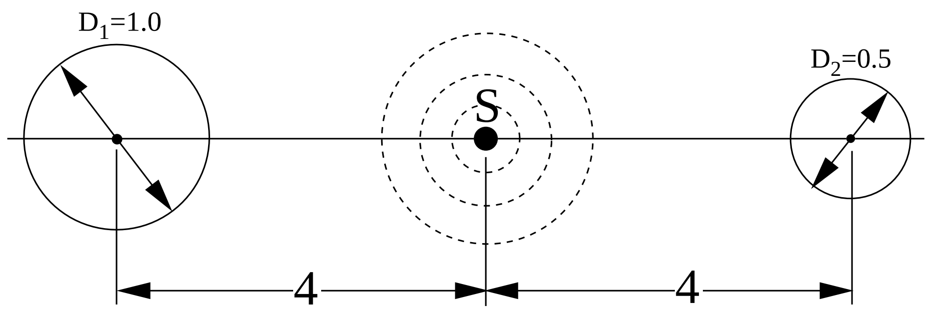

Figure 1: Geometry for acoustic scattering Case 1

Compute the time-averaged pressure $< p^2 >$ along the surface of both cylinders. Compute $< p^2 >$ along the centerline between the two cylinders, to the left of the leftmost cylinder and to the right of the rightmost cylinder, for $-9 \leq x_S \leq 9$. Provide details on both the spatial and temporal discretizations employed. This includes: (a) the total number of grid points or elements used, (b) the typical mesh spacing in the near field in terms of points per wave, (c) the farfield boundary location, (d) the number of time steps per period of the source, and (e) the total number of time steps used to achieved a fully time-asymptotic solution.

Case 2

The second case, shown in Fig. 2, consists of three cylinders with diameters $D_1 = 1.0$ and $D_2 = D_3 = 0.75$. The locations of the cylinders with respect to the source are given by $L_1 = (-3, 0), L_2 = (3, 4), L_3 = (3, -4)$.

Compute the time-averaged pressure $< p^2 >$ along the surface of cylinders 1 and 2. Compute $< p^2 >$ along the centerline, to the right and left of cylinder 1, for $-8 \leq x_S \leq 8$. Provide details on both the spatial and temporal discretizations employed as specified above for Case 1.

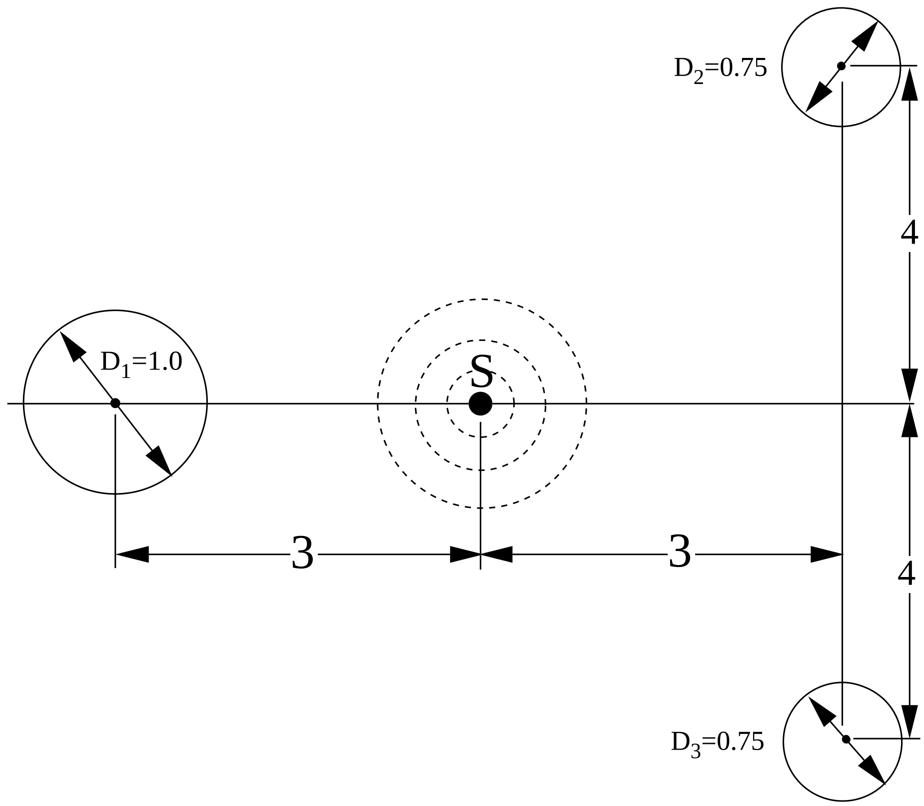

Figure 2: Geometry for acoustic scattering Case 2

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