Single Airfoil Gust Response Problem

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The purpose of this problem is to test the ability of a CFD/CAA code to accurately predict the unsteady aerodynamic and aeroacoustic response of a single airfoil to a two-dimensional, periodic vortical gust.

Consider the airfoil configuration shown in Figure 1. The airfoil has chord length c and angle of attack α. The upstream velocity is

$$\vec{U} \;=\; U_\infty \,\vec{\mathrm{i}} \;+\; \vec{a}\,\cos[\vec{k}\cdot(\vec{x}-\vec{\mathrm{i}}\,U_\infty t)] \tag{1}$$

where $\vec{x} = (x_1, x_2)$ denotes the spatial coordinates, $\vec{a} = (a_1, a_2)$ is the gust amplitude vector with $a_1 = -\,\epsilon\, U_\infty\, k_2/|\vec{k}|$, $\;a_2 = \epsilon\, U_\infty\, k_1/|\vec{k}|$, $\vec{k}$ is the wave number vector, and ϵ is a small parameter satisfying $\epsilon \ll 1$.

The governing equations are the 2-D Euler equations

$$\frac{\partial \rho}{\partial t} \;+\; \frac{\partial}{\partial x}(\rho\, u) \;+\; \frac{\partial}{\partial y}(\rho\, v) \;=\; 0 \tag{2}$$

$$\frac{\partial}{\partial t}(\rho u) \;+\; \frac{\partial}{\partial x}(\rho\, u^2 + p) \;+\; \frac{\partial}{\partial y}(\rho\, u\, v) \;=\; 0 \tag{3}$$

$$\frac{\partial}{\partial t}(\rho v) \;+\; \frac{\partial}{\partial x}(\rho\, u\, v) \;+\; \frac{\partial}{\partial y}(\rho\, v^2 + p) \;=\; 0 \tag{4}$$

$$\frac{\partial E_t}{\partial t} \;+\; \frac{\partial}{\partial x}[(E_t + p)\, u] \;+\; \frac{\partial}{\partial y}[(E_t + p)\, v] \;=\; 0 \tag{5}$$

where ρ, u, v, p and E_t denote the fluid density, velocity, pressure, and internal energy per unit volume.

Since the gust amplitude $\vec{a}$ satisfies $|\vec{a}| \ll U_\infty$, one can alternatively solve the linearized unsteady Euler equations

$$\frac{D_0 \rho'}{Dt} + \rho' \vec{\nabla}\cdot\vec{U}_0 + \vec{\nabla}\cdot(\rho_0 \vec{u}) = 0 \tag{6}$$

$$\rho_0\Big(\frac{D_0 \vec{u}}{Dt} + \vec{u}\cdot\vec{\nabla}\vec{U}_0\Big) + \rho' \vec{U}_0\cdot\vec{\nabla}\vec{U}_0 = -\vec{\nabla} p' \tag{7}$$

$$\frac{D_0 s'}{Dt} = 0, \tag{8}$$

where $\frac{D_0}{Dt} = \frac{\partial}{\partial t} + \vec{U}_0\cdot\vec{\nabla}$ is the material derivative associated with the mean flow, $\vec{u} = (u', v')$, primed quantities are the unknown perturbation variables, and "0" subscripts denote steady mean flow quantities which must be independently solved for and are assumed to be known.

Nondimensionalize the Euler equations as follows:

x_1, x_2	by	$\frac{c}{2}$
$\vec{U} = (u, v)$	by	U_∞
c_0 (sound speed)	by	U_∞
ρ	by	ρ_∞
p	by	$\rho_\infty U_\infty^2$
T	by	T_∞
t	by	$\frac{c}{2U_\infty}$
$\omega = k_1 U_\infty$	by	$\frac{2U_\infty}{c}$
k_1, k_2	by	$\frac{2}{c}$

If solving the linearized unsteady Euler equations, nondimensionalize the mean flow variables as above, and the perturbation variables as follows:

$\vec{u} = (u', v')$	by	U_∞
ρ'	by	ρ_∞
p'	by	$\rho_\infty U_\infty^2$
T'	by	T_∞
$\vec{a}$	by	U_∞

For the following two cases, solve the gust response problem for a Joukowski airfoil in a two-dimensional gust with $k_2 = k_1$ for reduced frequencies $k_1 = 0.1$, 1.0, and 2.0. The nondimensional upstream velocity is $\vec{U} = \vec{\mathrm{i}} + \epsilon\,\vec{a}\,\cos(\vec{k}\cdot\vec{x} - k_1\,t)$, where $\vec{a} = (a_1, a_2) = (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$. Take $\epsilon = .02$.

For Case 1, the airfoil has a 12% thickness ratio, free stream Mach number $M_\infty = 0.5$, angle of attack $\alpha = 0^o$, and a camber ratio of zero.

For Case 2, change α to 2^o and the camber ratio to .02.

The airfoil geometries can be generated as follows. Set

$$\zeta_1 = r_0\, e^{i\,\theta} \;+\; \zeta_{0'} \tag{9}$$

where

$$\zeta_{0'} \;=\; -\,\epsilon_1 \;+\; i\,\epsilon_2 \tag{10}$$

is a complex constant. Letting $z = x + i\,y$ denote the airfoil coordinates in the complex z plane, the transformation

$$z = \left(\zeta_1 \;+\; \frac{d^2}{\zeta_1}\right) e^{-\,i\alpha} \tag{11}$$

transforms the ζ_1 circle defined by equation (9) into the desired airfoil shape.

For Case 1, use $r_0 = 0.54632753$, $\epsilon_1 = 0.05062004$, $\epsilon_2 = 0$, $d^2 = 0.24572591$, $\alpha = 0$. Discretize the ζ_1 circle in θ, starting from 0 and going to $2\,\pi$, and then apply equation (11) to get the airfoil coordinates. The values $\theta = 0$ and $\theta = 2\,\pi$ map into the trailing edge point.

For Case 2, use $r_0 = 0.54676443$, $\epsilon_1 = 0.05062004$, $\epsilon_2 = 0.02185310$, $d^2 = 0.24572591$, $\alpha = 0.034906585$. Discretize the ζ_1 circle in θ, starting from $\theta = -\beta$ and going to $\theta = 2\pi - \beta$, where $\beta = 0.039978687$, and then apply equation (11) to get the airfoil coordinates. The values $\theta = -\beta$ and $\theta = 2\pi - \beta$, map into the trailing edge point.

The above procedure for generating the airfoil geometries will generate a Joukowski airfoil of chord length 2, situated very nearly between $x = -1$ and $x = 1$. The airfoil geometries are shown in Figure 2.

For both Case 1 and Case 2, march the discrete equations in time until the solution becomes periodic. On the airfoil surface, calculate the RMS pressure $\sqrt{\overline{(p')^2}}$. In the far field, calculate the intensity $\overline{(p')^2}$ at the following three locations: (i) on a circle of radius $R = 2$ (one chord length) centered about the airfoil center; (ii) on a circle of radius $R = 4$ (two chord lengths); (iii) on a circle of radius $R = 8$ (four chord lengths).

State whether the solution is from the Euler equations or linearized Euler equations. Also state the grid dimensions for each calculation, the number of complete periods computed, the CPU time per period, and the type of machine the calculations were run on.

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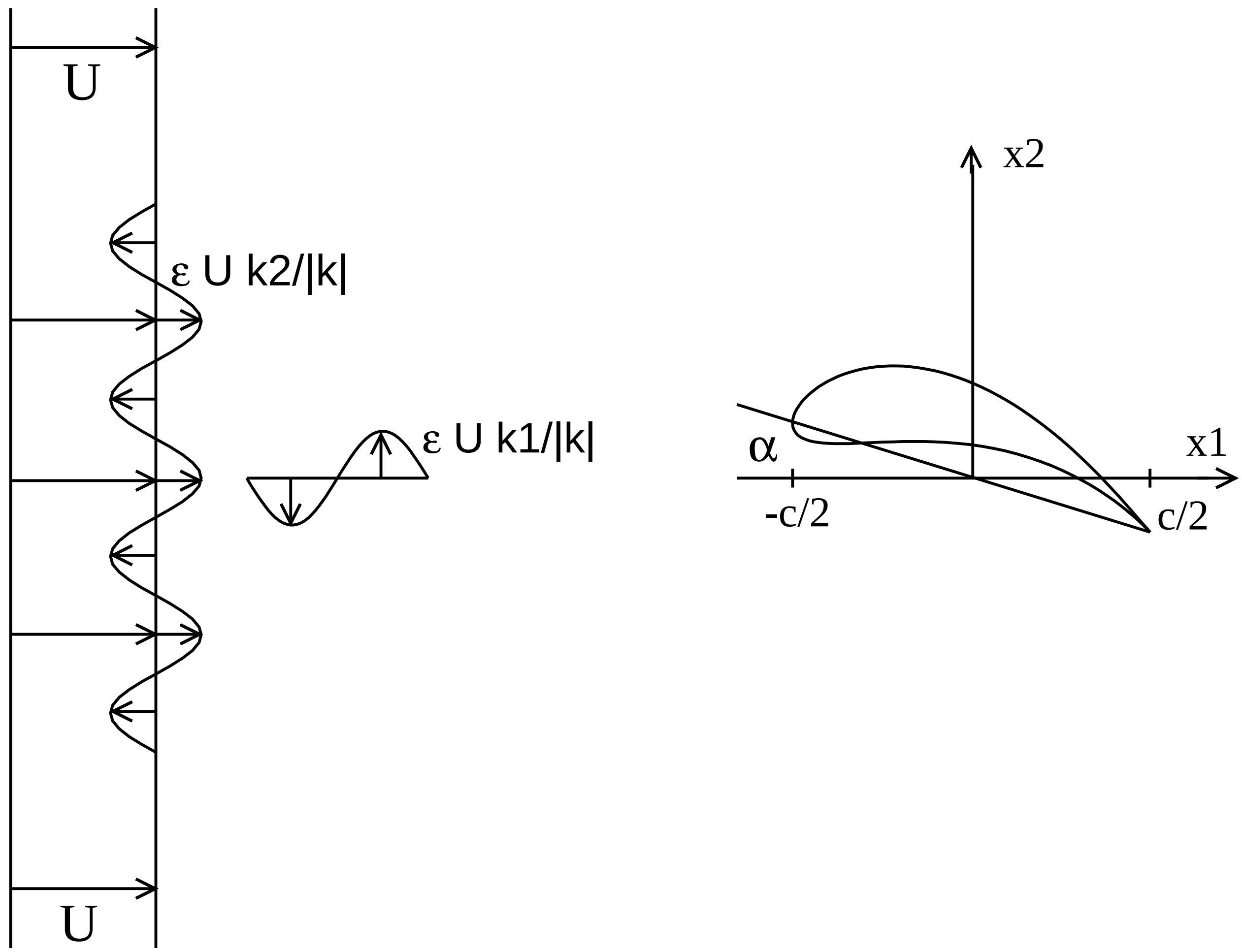

Figure 1   Airfoil in a two-dimensional, periodic gust.

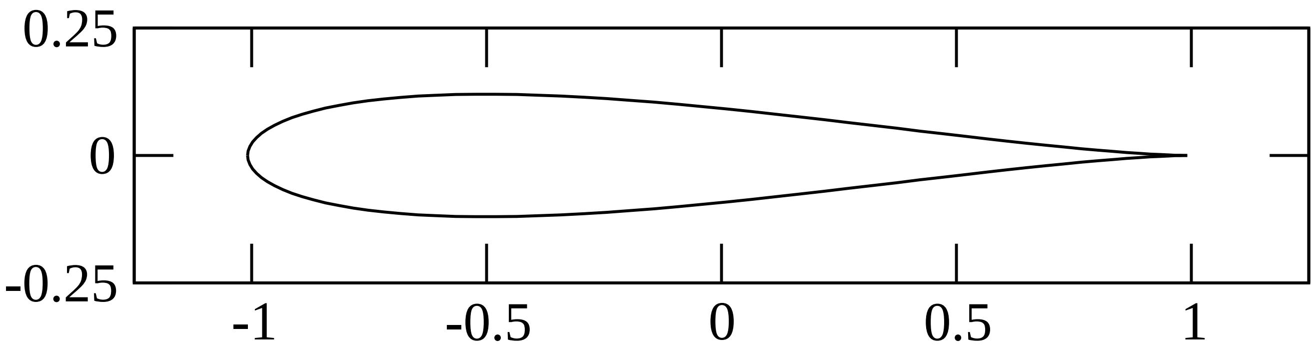

Figure 2a   Airfoil geometry for Case 1.

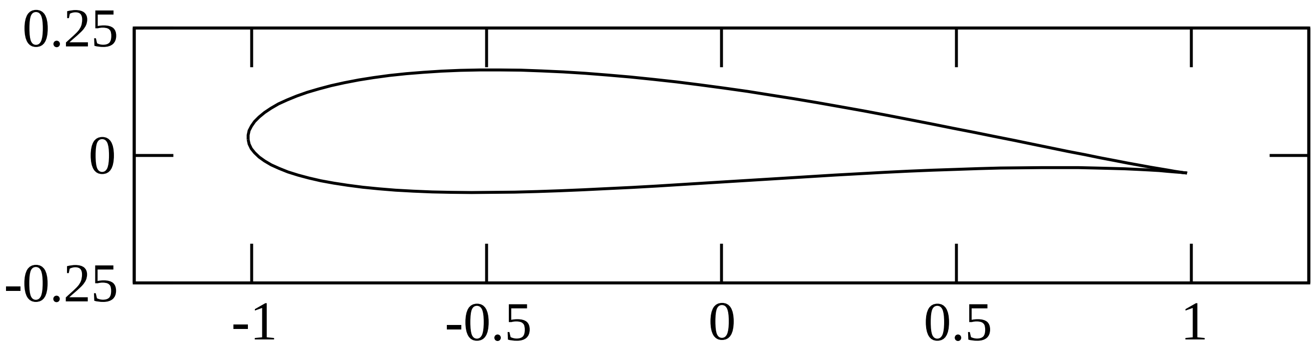

Figure 2b Airfoil geometry for Case 2.