Cascade-Gust Interaction Problem

Geometry

The two-dimensional geometry, shown in Fig. 1, is the unrolled section of a realistic three-dimensional fan outlet guide vane stator. The cascade has a gap-to-chord ratio of $d/c = 2/3$ with the inflow and outflow planes located at $x_{\mp} = \mp 3/2c$. The airfoil definition is given in the accompanying ASCII file and reproduced at the end of this note.

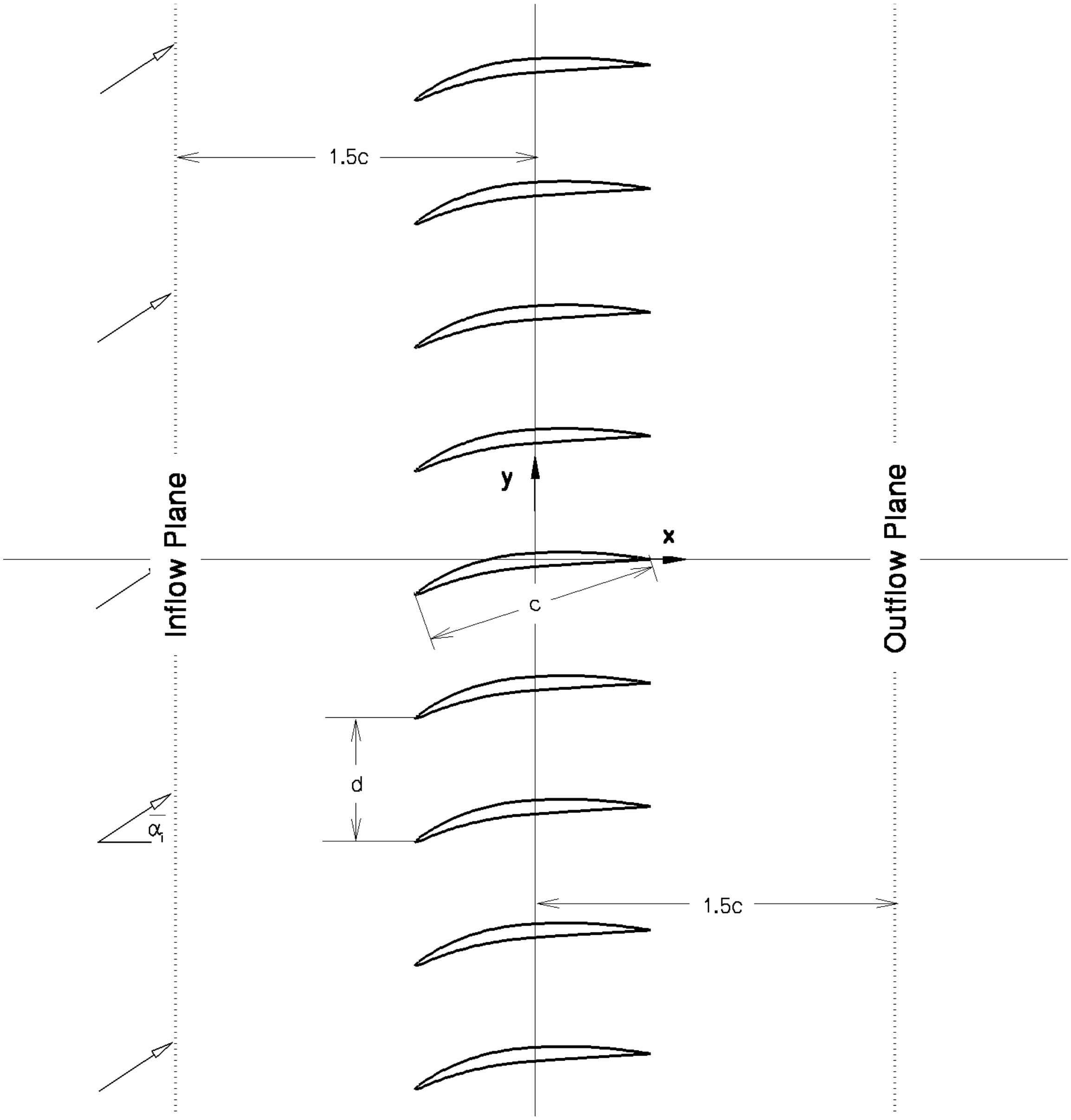

Fig. 1 - Stator Cascade Geometry

Inflow/Outflow Conditions and Gust Input

The mean (i.e., time-averaged) inflow/outflow conditions are:

$$\text{inflow conditions: } \begin{cases} \bar{P}_i = 1 \\ \bar{T}_i = 1 \\ \bar{\alpha}_i = 36^\circ \end{cases}, \qquad \text{outflow condition: } \quad \bar{p}_o / \bar{P}_i = 0.92$$

where $\bar{P}_i$ and $\bar{T}_i$ are the normalized inflow plane mean stagnation pressure and mean stagnation temperature. $\bar{\alpha}_i$ is the mean flow angle and $\bar{p}_o$ the normalized outflow plane mean static pressure. Assume the flow to be inviscid and isentropic throughout the domain.

The inflow gust (produced, say, by the wake of an upstream blade row) is given, <u>at the inflow plane</u>, by

$$\vec{u}_g'(y,t) = \left\{ a_1 \cos\left(k_y y - \omega t\right) + a_2 \cos\left(2\left(k_y y - \omega t\right)\right) + a_3 \cos\left(3\left(k_y y - \omega t\right)\right) \right\} \hat{e}_\beta$$

$$\rho_g'(y,t) = 0, \qquad p_g'(y,t) = 0$$

$$\hat{e}_\beta = \cos(\beta)\,\hat{e}_x - \sin(\beta)\,\hat{e}_y, \qquad \beta = 50^\circ$$

$$\omega = 3\pi/4, \qquad k_y = 11\pi/9, \qquad \begin{cases} a_1 = 5\times10^{-3} \\ a_2 = 3\times10^{-3} \\ a_3 = 7\times10^{-4} \end{cases}$$

where ω is the fundamental reduced frequency[1], k_y is the transverse wavenumber[2], and a_i's are the gust harmonic amplitudes[3].

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[1] Frequency is normalized by the chord divided by the ambient speed of sound.

[2] Wavenumber is normalized by the vane chord.

[3] Gust harmonic amplitudes are normalized by the ambient speed of sound.

Requirements

Solve the time-dependent <u>inviscid</u> flow equations for this geometry subject to the specified inflow/outflow mean conditions and the fluctuating inflow velocity distortion.

(1) Compute the unsteady solution until periodicity in pressure is achieved by showing that at least two successive periods are identical[4]. Periodicity must be achieved on both the airfoil surface and the inflow/outflow boundaries.

(2) Once periodicity is achieved, compute the pressure frequency spectra on the reference airfoil on both the upper and lower surfaces at $x = (-0.25c,\ 0.00,\ +0.25c)$, on the inflow boundary at $(x, y) = \{(-1.5c,\ \text{-}0.3c), (-1.5c,\ 0.0), (-1.5c,\ 0.3c)\}$, and on the outflow boundary at $(x, y) = \{(1.5c,\ \text{-}0.3c), (1.5c,\ 0.0), (1.5c,\ 0.3c)\}$. Express the spectral results in dB using the standard definition $20\ \log(p_{\text{r.m.s.}} / p_{\text{ref.}})$, where $p_{\text{ref.}} = 20\ \mu\text{Pa}$.

(3) Extract the harmonic pressure distributions on the inflow and outflow boundaries (i.e., on $x = \mp 1.5c$ lines) at the fundamental frequency ω and apply a Fourier transform in y direction to identify the spatial (i.e., mode order) structure of the pressure perturbations. Express the result in dB for each mode order. Repeat the process for the frequencies 2ω and 3ω.

Note: The benchmark solution to this problem will be computed using a frequency-domain linearized Euler code called LINFLUX which has been extensively tested at United Technology Research Center and NASA Glenn Research Center.

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[4] The maximum difference between the spectra of two successive periods must be less than 1% at any of the three input frequencies.

Airfoil Section Data[5]

Suction Side		Pressure Side	
x	*y*	*x*	*y*
-0.5000E+00	-0.1901E+00	-0.5000E+00	-0.1901E+00
-0.5003E+00	-0.1894E+00	-0.4994E+00	-0.1906E+00
-0.5003E+00	-0.1885E+00	-0.4984E+00	-0.1907E+00
-0.4999E+00	-0.1874E+00	-0.4973E+00	-0.1907E+00
-0.4994E+00	-0.1862E+00	-0.4961E+00	-0.1904E+00
-0.4985E+00	-0.1845E+00	-0.4942E+00	-0.1899E+00
-0.4962E+00	-0.1813E+00	-0.4905E+00	-0.1885E+00
-0.4916E+00	-0.1757E+00	-0.4841E+00	-0.1855E+00
-0.4837E+00	-0.1674E+00	-0.4741E+00	-0.1802E+00
-0.4716E+00	-0.1561E+00	-0.4600E+00	-0.1723E+00
-0.4550E+00	-0.1415E+00	-0.4411E+00	-0.1620E+00
-0.4443E+00	-0.1325E+00	-0.4291E+00	-0.1557E+00
-0.4334E+00	-0.1237E+00	-0.4170E+00	-0.1496E+00
-0.4201E+00	-0.1134E+00	-0.4024E+00	-0.1425E+00
-0.4066E+00	-0.1035E+00	-0.3878E+00	-0.1357E+00
-0.3960E+00	-0.9598E-01	-0.3765E+00	-0.1306E+00
-0.3852E+00	-0.8869E-01	-0.3652E+00	-0.1257E+00
-0.3744E+00	-0.8159E-01	-0.3538E+00	-0.1209E+00
-0.3621E+00	-0.7390E-01	-0.3411E+00	-0.1157E+00
-0.3497E+00	-0.6645E-01	-0.3283E+00	-0.1107E+00
-0.3372E+00	-0.5925E-01	-0.3155E+00	-0.1059E+00
-0.3235E+00	-0.5176E-01	-0.3018E+00	-0.1009E+00
-0.3096E+00	-0.4456E-01	-0.2879E+00	-0.9611E-01
-0.2956E+00	-0.3767E-01	-0.2740E+00	-0.9151E-01
-0.2845E+00	-0.3246E-01	-0.2631E+00	-0.8806E-01
-0.2733E+00	-0.2744E-01	-0.2522E+00	-0.8472E-01
-0.2621E+00	-0.2261E-01	-0.2412E+00	-0.8150E-01
-0.2507E+00	-0.1798E-01	-0.2303E+00	-0.7839E-01
-0.2391E+00	-0.1346E-01	-0.2191E+00	-0.7537E-01
-0.2274E+00	-0.9145E-02	-0.2079E+00	-0.7247E-01
-0.2156E+00	-0.5032E-02	-0.1967E+00	-0.6969E-01
-0.2038E+00	-0.1124E-02	-0.1855E+00	-0.6702E-01
-0.1919E+00	0.2586E-02	-0.1743E+00	-0.6449E-01
-0.1799E+00	0.6091E-02	-0.1630E+00	-0.6207E-01
-0.1678E+00	0.9390E-02	-0.1518E+00	-0.5978E-01
-0.1557E+00	0.1248E-01	-0.1405E+00	-0.5759E-01
-0.1436E+00	0.1538E-01	-0.1292E+00	-0.5553E-01
-0.1313E+00	0.1806E-01	-0.1179E+00	-0.5358E-01
-0.1191E+00	0.2052E-01	-0.1065E+00	-0.5176E-01
-0.1068E+00	0.2275E-01	-0.9516E-01	-0.5007E-01
-0.9448E-01	0.2475E-01	-0.8376E-01	-0.4851E-01
-0.8214E-01	0.2655E-01	-0.7233E-01	-0.4706E-01
-0.6978E-01	0.2816E-01	-0.6090E-01	-0.4571E-01
-0.5739E-01	0.2962E-01	-0.4946E-01	-0.4442E-01
-0.4370E-01	0.3108E-01	-0.3614E-01	-0.4299E-01
-0.2999E-01	0.3240E-01	-0.2282E-01	-0.4162E-01
-0.1627E-01	0.3358E-01	-0.9490E-02	-0.4028E-01
-0.2543E-02	0.3466E-01	0.3839E-02	-0.3897E-01
0.1119E-01	0.3564E-01	0.1718E-01	-0.3767E-01
0.2494E-01	0.3651E-01	0.3051E-01	-0.3640E-01

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[5] These coordinates are normalized by the vane chord.

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0.3868E-01    0.3729E-01    0.4386E-01   -0.3516E-01
0.5244E-01    0.3796E-01    0.5720E-01   -0.3395E-01
0.6619E-01    0.3851E-01    0.7056E-01   -0.3278E-01
0.7995E-01    0.3896E-01    0.8392E-01   -0.3163E-01
0.9372E-01    0.3929E-01    0.9728E-01   -0.3051E-01
0.1075E+00    0.3950E-01    0.1106E+00   -0.2939E-01
0.1212E+00    0.3959E-01    0.1240E+00   -0.2827E-01
0.1350E+00    0.3957E-01    0.1374E+00   -0.2714E-01
0.1488E+00    0.3943E-01    0.1508E+00   -0.2602E-01
0.1625E+00    0.3917E-01    0.1642E+00   -0.2490E-01
0.1760E+00    0.3880E-01    0.1773E+00   -0.2381E-01
0.1894E+00    0.3832E-01    0.1904E+00   -0.2272E-01
0.2028E+00    0.3773E-01    0.2035E+00   -0.2165E-01
0.2163E+00    0.3703E-01    0.2166E+00   -0.2057E-01
0.2288E+00    0.3627E-01    0.2288E+00   -0.1958E-01
0.2412E+00    0.3542E-01    0.2411E+00   -0.1858E-01
0.2537E+00    0.3448E-01    0.2533E+00   -0.1759E-01
0.2662E+00    0.3343E-01    0.2655E+00   -0.1660E-01
0.2809E+00    0.3208E-01    0.2800E+00   -0.1544E-01
0.2956E+00    0.3060E-01    0.2945E+00   -0.1428E-01
0.3103E+00    0.2898E-01    0.3089E+00   -0.1313E-01
0.3226E+00    0.2752E-01    0.3211E+00   -0.1217E-01
0.3348E+00    0.2597E-01    0.3332E+00   -0.1121E-01
0.3471E+00    0.2434E-01    0.3453E+00   -0.1026E-01
0.3614E+00    0.2229E-01    0.3596E+00   -0.9145E-02
0.3758E+00    0.2012E-01    0.3739E+00   -0.8034E-02
0.3966E+00    0.1676E-01    0.3946E+00   -0.6431E-02
0.4102E+00    0.1442E-01    0.4082E+00   -0.5386E-02
0.4181E+00    0.1300E-01    0.4161E+00   -0.4778E-02
0.4254E+00    0.1166E-01    0.4235E+00   -0.4215E-02
0.4306E+00    0.1067E-01    0.4291E+00   -0.3789E-02
0.4434E+00    0.8175E-02    0.4423E+00   -0.2792E-02
0.4562E+00    0.5572E-02    0.4554E+00   -0.1799E-02
0.4690E+00    0.2895E-02    0.4686E+00   -0.8073E-03
0.4817E+00    0.1831E-03    0.4817E+00    0.1831E-03
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