Radiation and Refraction of Sound Waves Through a Two-Dimensional Shear Layer

The propagation of sound in a turbulent shear flow can be described by a solution of the linearized Euler equations. The base flow is assumed to be the long-time average of the turbulent flow field. Viscous and nonlinear effects enter the problem through their influence in determining the base flow. However, the linearized Euler equations not only provide a solution for sound propagation, they also admit unstable solutions and instability waves can be triggered. In the complete physical problem, these instabilities contribute to the turbulence and are limited and modified by non-linear and viscous effects. In that sense they are non-physical solutions, if the actual problem to be solved is for sound propagation in a turbulent sheared flow. It should be remembered that the inhomogeneous linearized Euler equations only represent a mathematical model of part of the complete physical problem.

The purpose of this benchmark problem is to find ways to suppress the non-physical instabilities but to retain that part of the solution associated with the sound propagation.

The problem to be solved is very similar to the Category 5 problem at the Third Computational Aeroacoustics Workshop on Benchmark Problems: "Generation and Radiation of Acoustic Waves From a 2D Shear Layer." The operating conditions have been changed and, in the present problem, **the required solution should only consist of the acoustic part of the solution: not the instability wave that is generated**. The problem consists of an energy source inside a two-dimensional jet that generates an acoustic wave that is refracted as it moves through the jet shear layer. The benchmark problem requires the solution of the two-dimensional linearized Euler equations to calculate the fluctuations associated with only the sound radiation and refraction through the jet. The locations of the required output data and its format are given below. The numerical solutions will be evaluated by comparison with an exact analytical solution.

The linearized Euler equation for a parallel jet can be written as

$$\mathsf{L}\left(\frac{\partial}{\partial \boldsymbol{x}}, \frac{\partial}{\partial t}, \boldsymbol{x}\right) \boldsymbol{U}(\boldsymbol{x}, t) = \boldsymbol{S}(\boldsymbol{x}) \cos(\omega_o t), \tag{1}$$

where

$$\boldsymbol{L} = \begin{bmatrix} \boxdot & \bar{\rho}(y)\frac{\partial}{\partial x} & \frac{\partial\bar{\rho}(y)}{\partial y} + \bar{\rho}(y)\frac{\partial}{\partial y} & 0 \\ 0 & \boxdot & \frac{\partial\bar{u}(y)}{\partial y} & \frac{1}{\bar{\rho}(y)}\frac{\partial}{\partial x} \\ 0 & 0 & \boxdot & \frac{1}{\bar{\rho}(y)}\frac{\partial}{\partial y} \\ 0 & \gamma\bar{p}\frac{\partial}{\partial x} & \gamma\bar{p}\frac{\partial}{\partial y} & \boxdot \end{bmatrix}, \quad \boldsymbol{U} = \begin{Bmatrix} \rho \\ u \\ v \\ p \end{Bmatrix}, \quad \boldsymbol{S} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \Lambda(\boldsymbol{x}) \end{Bmatrix}, \qquad (2)$$

$$\boxdot = \frac{\partial}{\partial t} + \bar{u}(y)\frac{\partial}{\partial x} \quad , \quad \Lambda(\boldsymbol{x}) = A\exp[-(B_x x^2 + B_y y^2)]$$

The mean flow variables are denoted by an overbar and are given by

$$\bar{u}(y) = \begin{cases} u_j \exp[-(\ln 2)(y/b - h/b)^2] & y \geq h \\ u_j & 0 \leq y \leq h \end{cases} \qquad (3)$$

$$\frac{1}{\bar{\rho}(y)} = -\frac{1}{2}\frac{\gamma - 1}{\gamma\bar{p}}(\bar{u}(y) - u_j)\bar{u}(y) + \frac{1}{\rho_j}\frac{\bar{u}(y)}{u_j} + \frac{1}{\rho_\infty}\frac{u_j - \bar{u}(y)}{u_j} \qquad (4)$$

$$\bar{p} = \text{constant} = 103330 \text{ m}^{-1} \text{ kg s}^{-2} \qquad (5)$$

The parameters for the problem are given in the Table. $M_j = u_j/a_j$, $a_j = (\gamma R T_j)^{1/2}$.

M_j	T_j K	T_∞ K	R $\text{m}^2\text{s}^{-2}\text{K}^{-1}$	γ	h m	b m	A $\text{kgm}^{-1}\text{s}^{-3}$	B_x m^{-2}	B_y m^{-2}
0.756	600	300	287.0	1.4	0.0	1.3	0.001	0.04 ln(2)	0.32 ln(2)

The source frequency $\omega_o = 76$ rad/s. Note that this source frequency generates an instability wave that can overwhelm the acoustic solution. Therefore, the numerical scheme must filter out the instability wave.

The physical domain Ω is a rectangle with dimensions $\Omega = [-50, 150] \times [0, 50]$. A symmetry boundary condition should be used along the x-axis.

Calculate p at the start of a cycle and $\bar{p^2}$ along:

1. $y = 15, \quad -50 \leq x \leq 150$
2. $y = 50, \quad -50 \leq x \leq 150$
3. $x = 100, \quad 5 \leq y \leq 50$

Output data in ASCII text format with three columns: x, y, and p or $\bar{p^2}$. Specify the grid layout, memory used, CPU time, and the computational scheme (including the boundary conditions) used for the numerical simulation.

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