Consider a two-dimensional, compressible mixing layer flow formed by a splitter plate with a blunt trailing edge. The lower stream has free stream Mach number $M_1 = 0.6$ with a boundary layer momentum thickness θ_1^* at $x^* = x_1^*$, while the upper stream has free stream Mach number $M_2 = 0.1$ with momentum thickness $\theta_2^* = \theta_1^*$ at $x^* = x_1^*$. The splitter plate has width $w^* = 2\theta_1^*$ with a shape consisting of a flat plate section capped by a super-ellipse at the trailing edge. With all lengths nondimensionalized by θ_1^*, the definition of the splitter plate surface is:

$$\left(\frac{2y}{w}\right)^2 + \left(1 + \frac{x}{\mathrm{AR}}\right)^m = 1 \quad , \qquad -AR \leq x \leq 0 \tag{1}$$

$$y = \pm\frac{w}{2} \quad , \qquad x < -AR \tag{2}$$

with aspect ratio AR = 2.5 and order $m = 6$. The Reynolds number based on the lower free stream properties is $Re_{\theta_1^*} = \frac{\rho_1^* u_1^* \theta_1^*}{\mu_1^*} = 250$, and the Prandtl number is $Pr = 0.7$. The governing equations are the two-dimensional Navier-Stokes equations:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \frac{\partial \mathbf{V}_x}{\partial x} + \frac{\partial \mathbf{V}_y}{\partial y} \tag{3a}$$

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ u(p + \rho E) \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ v(p + \rho E) \end{bmatrix} \tag{3b}$$

$$\mathbf{V}_x = \frac{M_1}{Re_{\theta_1^*}} \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ u\tau_{xx} + v\tau_{xy} - \frac{1}{Pr} q_x \end{bmatrix} \quad \mathbf{V}_y = \frac{M_1}{Re_{\theta_1^*}} \begin{bmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ u\tau_{xy} + v\tau_{yy} - \frac{1}{Pr} q_y \end{bmatrix} \tag{3c}$$

$$E = \frac{1}{\gamma} T + \frac{1}{2}\left(u^2 + v^2\right) \tag{3d}$$

where ρ is the density, u, v are the velocity components, p is the pressure, T is the temperature, E is the total energy per unit mass, τ is the viscous stress tensor, q is the heat flux vector, and $\gamma = \frac{C_p}{C_v} = 1.4$ is the ratio of specific heats. All variables are non-dimensionalized using reference length θ_1^* and the following lower free stream dimensional quantities: speed of sound $c_1^* = \sqrt{(\gamma - 1) C_v T^*}$, time θ_1^*/c_1^*, density ρ_1^*, pressure $\rho_1^* {c_1^*}^2$, temperature $(\gamma - 1) T_1^*$, and viscosity μ_1^*. The following constitutive relations close the system

$$p = \frac{\gamma - 1}{\gamma} \rho T \tag{4a}$$

$$q_x = -k\frac{\partial T}{\partial x} \qquad q_y = -k\frac{\partial T}{\partial y} \tag{4b}$$

$$\tau_{xx} = \mu\left(\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right) \quad \tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \quad \tau_{yy} = \frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial u}{\partial x} \tag{4c}$$

$$\mu = k = ([\gamma - 1]T)^{0.7} \tag{4d}$$

Steady State Solution

First find a steady, laminar solution to (3) for the given geometry and flow conditions. The rectangular solution domain is given by $-50 \leq x \leq 100$, $-100 \leq y \leq 100$, with the trailing edge of the splitter plate at the origin. The flow state at the inflow boundary at $x_1 = -50$ is approximated above and below the plate by solutions to the compressible boundary layer equations with zero pressure gradient. Free stream conditions above the plate are $T_2 = T_1$ and $\rho_2 = \rho_1$. Boundary conditions on the plate surface are the no slip condition $u = v = 0$ and the isothermal condition $T_{wall} = T_1$. The steady solution is denoted $\bar{\mathbf{U}} = [\bar{\rho} \;\; \bar{\rho}\bar{u} \;\; \bar{\rho}\bar{v} \;\; \bar{\rho}\bar{E}]^T$. Plot streamwise velocity profiles of $\bar{u}(y)$ at $x = 5$, $x = 40$, and $x = 75$, so that mean mixing layer solutions may be compared.

Problem 1

Once the steady solution is obtained, solve the initial value problem of a pressure pulse superimposed on the steady flow, with initial condition

$$\mathbf{U}(x,y,0) = \bar{\mathbf{U}}(x,y) + 0.05 \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{f(x,y)}{\gamma-1} \end{bmatrix}, \qquad f(x,y) = e^{-\ln 2\left(\left[\frac{x+20}{4}\right]^2 + \left[\frac{y+20}{4}\right]^2\right)} \tag{5}$$

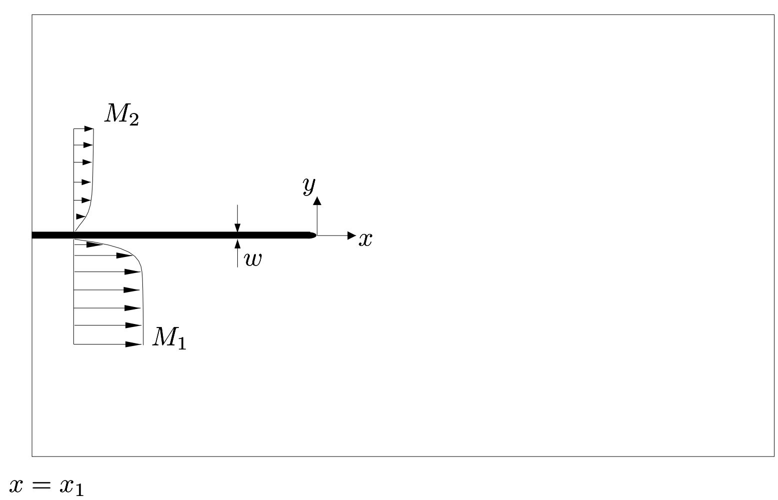
where $\bar{\mathbf{U}}$ is the steady solution. The unsteady calculation is performed on the same domain as the steady calculation over the time period $0 \le t \le 1200$. Give the disturbance pressure $p' = p - \bar{p}$ along the line $y = -3$ at times $t = 200, 400, \ldots, 1200$. Give the data at 401 points equally spaced along the interval $-50 \le x \le 150$. Also give the disturbance pressure time history for $0 \le t \le 1200$ at $(x,y) = (-30,1)$ and at $(x,y) = (50,50)$. Give data in time increments of $\Delta t = 0.5$. Each data set should contain $t,\ x,\ y,\ p'$ and be saved in `FORMAT(4(1X,E15.6))`.

Problem 2

Solve the initial value problem where a "vortex" is initiated upstream of the trailing edge below the splitter plate. The initial condition is given by:

$$\begin{bmatrix} \rho \\ u \\ v \\ p \end{bmatrix}(x,y,0) = \begin{bmatrix} \bar{\rho} \\ \bar{u} \\ \bar{v} \\ \bar{p} \end{bmatrix}(x,y) + \begin{bmatrix} \left(1 - \frac{\gamma-1}{2} M_v^2 \exp\left(1 - \left[\frac{r}{\sigma}\right]^2\right)\right)^{\frac{1}{\gamma-1}} - 1 \\ -M_v(y-y_0)\exp\left(\frac{1-\left[\frac{r}{\sigma}\right]^2}{2}\right) \\ M_v(x-x_0)\exp\left(\frac{1-\left[\frac{r}{\sigma}\right]^2}{2}\right) \\ \frac{1}{\gamma}\left(\left[1 - \frac{\gamma-1}{2} M_v^2 \exp\left(1 - \left[\frac{r}{\sigma}\right]^2\right)\right]^{\frac{\gamma}{\gamma-1}} - 1\right) \end{bmatrix} \tag{6}$$

where $(x_0, y_0) = (-35, -8)$, $M_v = 0.1$, $\sigma = 1$, and $r = \sqrt{(x-x_0)^2 + (y-y_0)^2}$. Find the solution for $0 \le t \le 1200$, and give the same data as for Problem 1.



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