



A Computational and Experimental Study of the Electrical and Thermal Properties of Hybrid Nanocomposites based on Carbon Nanotubes and Graphite Nanoplatelets

**M. Safdari & M. Al-Haik**

***Engineering Science and Mechanics Department***

***Virginia Tech***

***Blacksburg VA 24060***

*Workshop on Advances in Computational Mathematics and Engineering*

*In honor of the contributions of M. Y. Hussaini*

*Florida State University, September 28-29, 2012*

- 1 Motivations
- 2 Electrical Properties
  - Background
  - Incorporation of Tunneling
  - Quantifying Electrical Conductivity
- 3 Thermal Properties
  - Background
  - Quantifying Thermal Conductivity
- 4 Experimental Study
  - Background
  - Our Proposal
- 5 Preliminary Results
- 6 Time-Line

Motivations

Electrical  
Properties

Background

Incorporation of  
Tunneling

Quantifying Electrical  
Conductivity

Thermal  
Properties

Background

Quantifying Thermal  
Conductivity

Experimental  
Study

Background

Our Proposal

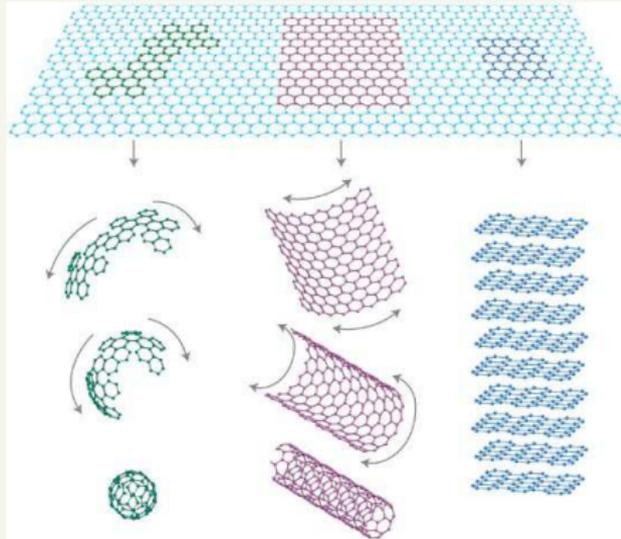
Preliminary  
Results

Time-Line

# What are Carbon Nanotube (CNT) and Graphite Nanoplatelet (GNP)?

## Graphene

One-atom-thick planar sheets of  $sp^2$ -bonded carbon atoms that are densely packed in a honeycomb crystal lattice. (Wikipedia)



Geim et al, 2007

## Motivations

## Electrical Properties

- Background
- Importance of Technology
- Quantifying Electrical Conductivity

## Thermal Properties

- Background
- Quantifying Thermal Conductivity

## Experimental Study

- Background
- Our Proposal

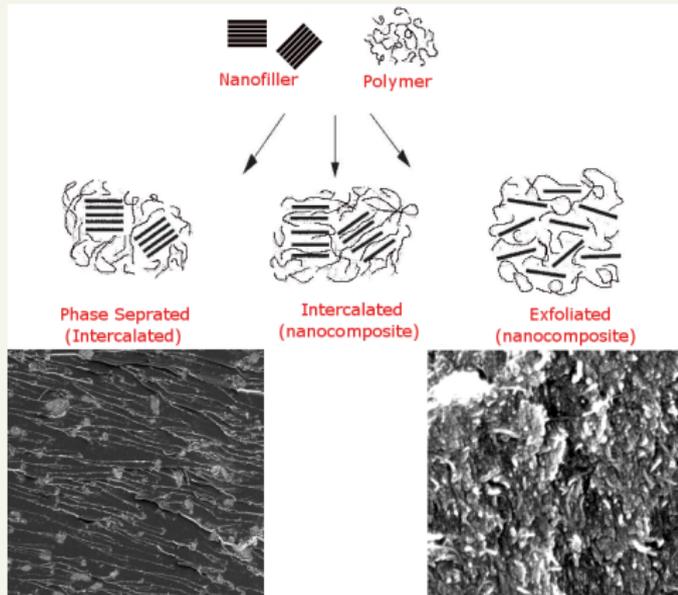
## Preliminary Results

## Time-Line

# What is a polymer nanocomposite?

## Nanocomposites

Composite in which at least one of the phase domains has at least one dimension of the order of nanometers. (IUPAC)



Motivations

Electrical Properties

Background  
 Incorporation of Nanomaterials  
 Quantifying Electrical Conductivity

Thermal Properties

Background  
 Quantifying Thermal Conductivity

Experimental Study

Background  
 Our Proposal

Preliminary Results

Time-Line

# Why polymer nanocomposite (PNC) are important?

## Motivations

### Electrical Properties

- Background
- Importance of Thermal
- Quantifying Electrical Conductivity

### Thermal Properties

- Background
- Quantifying Thermal Conductivity

### Experimental Study

- Background
- Our Proposal

### Preliminary Results

### Time-Line

#### Size effect

- From  $1\mu m^3$  to  $1nm^3$ 
  - Number:  $\times 10^9$
  - Interface surface:  $\times 10^6$
  - particle/particle distance:  $\times 10^{-3}$
- Increased interface volume: e.g. for only 1%vol of nanospheres,  $V_{interface}/V_{polymer} \simeq 63\%vol$  (Winey et al, 2007)
- Quantum confinement effects: Electrical and Optical properties

#### Advantages

- (1–5%) of nanoparticles, equivalent to (15–40%) of traditional fillers
- Easier processing
- Reduced weight/cost
- Unique properties not normally possible with traditional fillers
  - Opacity
  - Reduced permeability
  - Self-passivation
  - **Enhanced thermal/electrical conductivity**
- Overcoming traditionally antagonistic combinations of properties
- Applications: high performance multifunctional materials

# Why nanofillers are important?

## Motivations

### Electrical Properties

- Background
- Importance of Forming
- Quantifying Electrical Conductivity

### Thermal Properties

- Background
- Quantifying Thermal Conductivity

### Experimental Study

- Background
- Our Proposal

### Preliminary Results

### Time-Line

	Approximate Shape	Smallest Dimension (nm)	Aspect Ratio	Elastic Modulus (GPa)	Electrical Conductivity (S/cm)	Thermal Conductivity (W/mK)	Commercial Applications
<b>Traditional Fillers</b>							
Carbon Black	agglomerate of spheres	10-100	1-5	...	10-100	0.1-0.4	tires, hoses, shoes, elastomers
Carbon fiber	rods	5,000-20,000	10-50	300-800	0.1-10	100-1000	aerospace, marine, sporting, medical gaskets, seals
Carbon graphite	plate	250-500	15-50	500-600	1-10	100-500	
E-glass	rod	10,000-20,000	20-30	75	...	...	marine, automotive, filtration
Mineral: $CaCO_3$	sphere platelet	45-70 600-4,000	$\approx 1$ 1-30	35	...	3-5	paper, paint, rubber, plastics
Mineral: silica	agglomerate of spheres	8,000-30,000	5-10	30-200	...	1-10	reinforced plastics, thermal insulator, paint, rubber reinforcing agent
Mineral: talc, china clay	platelet	5,000-20,000	5-10	1-70	...	1-10	paper, consumer goods, construction
<b>Nanoscale Fillers</b>							
Carbon nanofiber	rod	50-100	50-200	500	700-1000	10-20	hoses, aerospace, ESD/EMI shielding, adhesives
SWCNT	rod	0.6-1.8	100-10,000	1500	1000-10,000	Up to 3000	filters, ESD/EMI shielding
Aluminosilicate nanoclay	plate	1-10	50-1000	200-250	...	1-10	automotive, packaging, sporting, tires, aerospace
Nano- $TiO_2$	sphere	10-40	$\approx 1$	230,000	$10^{-11} - 10^{-12}$	12	photocatalysis, gas sensors, paint
Nano- $Al_2O_3$	sphere	300	$\approx 1$	50	10-14	20-30	seal rings, furnace liner tubes, gas laser tubes, wear pads

# Are CNTs and GNPs comparable?

## Motivations

## Electrical Properties

- Background
- Importance of Tuning
- Quantifying Electrical Conductivity

## Thermal Properties

- Background
- Quantifying Thermal Conductivity

## Experimental Study

- Background
- Our Proposal

## Preliminary Results

## Time-Line

Property	Single-walled CNTs	Carbon Nanofibers	GNPs	Copper
<b>Specific Gravity</b> ( $g/cm^3$ )	0.8	$1.8(AG)^a - 2.1(HT)^b$	1.8 – 2.2	8.9
<b>Elastic Modulus</b> (TPa)	$\approx 1$ (axial direction)	$0.4(AG) - 0.6(HT)$	$\approx 1$ (in-plane)	0.117
<b>Strength</b> (GPa)	50 – 500	$2.7(AG) - 7.0(HT)$	$\approx 100 - 400$	220
<b>Resistivity</b> ( $\mu\Omega cm$ )	<b>5 – 50</b>	$55(HT) - 1000(AG)$	<b>50 (in-plane)</b>	<b>1.68</b>
<b>Thermal Conductivity</b> ( $Wm^{-1}K^{-1}$ )	<b>Up to 2,900</b> (estimated)	$20(AG) - 1950(HT)$	<b>5,300 (in-plane)</b> 6 – 30 (c-axis)	<b>401</b>
<b>Magnetic Susceptibility</b> ( $emu/g$ )	$22 \times 10^6$ (radial) $0.5 \times 10^6$ (axial)	N/A	$22 \times 10^6$ ( $\perp$ to plane) $0.5 \times 10^6$ ( $\parallel$ to plane)	$6.4 \times 10^6$
<b>Thermal Expansion</b> ( $K^{-1}$ )	Negligible in the axial direction	$-1 \times 10^{-6}$ (HT: axial)	$-1 \times 10^{-6}$ (in-plane) $29 \times 10^{-6}$ (c-axis)	$3.9 \times 10^{-3}$
<b>Thermal Stability</b> ( $^{\circ}C$ )	> 700 (in air) 2800 (in vacuum)	450 – 650 (in air)	450 – 650 (in air)	165 – 230 (in air) (nanoparticles)
<b>Specific Surface Area</b> ( $m^2/g$ )	Typically 10 – 200 Up to 1,300	10 – 60	Typically 100 – 1,000 up to > 2,600	Typically 40 – 60 (nanoparticles)

<sup>a</sup>AG: as grown

<sup>b</sup>HT: Heat treated (graphitic)

# What are the research objectives of this dissertation?

Motivations

Electrical  
Properties

Background  
Introduction of  
Terminology  
Quantifying Electrical  
Conductivity

Thermal  
Properties

Background  
Quantifying Thermal  
Conductivity

Experimental  
Study

Background  
Our Proposal

Preliminary  
Results

Time-Line

## Research Objectives

For electrical/thermal properties of CNT and GNP-based PNCs:

- Develop quantitative/qualitative models.
- Compare proposed models with literature and discuss discrepancies.
- Extend to hybrid CNTs/GNPs/Polymer nanocomposites.
- Study the advantages of the hybrid nanocomposite.
- Validate proposed models with the in-house experimental results.

## Motivations

## Electrical Properties

- Background
- Incorporation of Nanotubes
- Quantifying Electrical Conductivity

## Thermal Properties

- Background
- Quantifying Thermal Conductivity

## Experimental Study

- Background
- Our Proposal

## Preliminary Results

## Time-Line

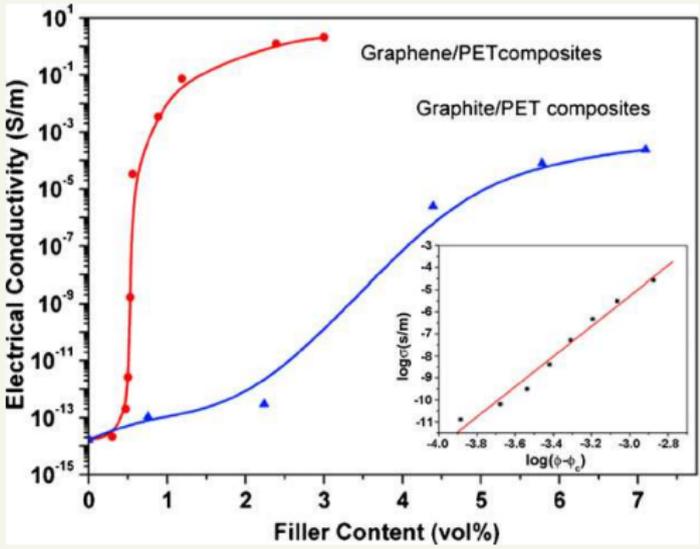
# Electrical properties of CNTs/GNPs based PNCs

What has been done?

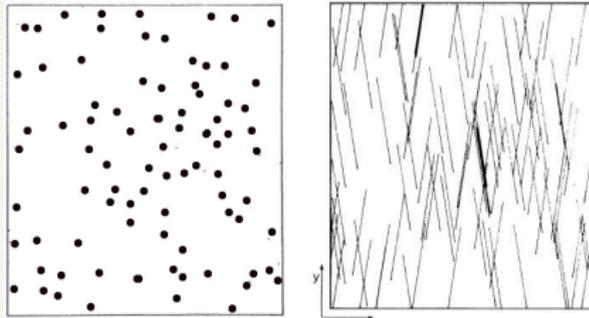
# How does addition of CNTs or GNPs change the electrical conductivity of PNCs?

- Experimental observations
- Sharp increase in the electrical conductivity in a narrow volume fraction
  - S-shaped curve: Classic percolation law
  - Narrow range of volume fractions called percolation threshold ( $\phi_c$ )
  - Very low  $\phi_c$  compared to carbon black, carbon fibers and . . .

- Motivations
- Electrical Properties
  - Background
  - Integration of Technology
  - Quantifying Electrical Conductivity
- Thermal Properties
  - Background
  - Quantifying Thermal Conductivity
- Experimental Study
  - Background
  - Our Proposal
- Preliminary Results
- Time-Line



Kuilia et al, 2010



## Object Percolation

- 1974: Pike et al.  $\Rightarrow$  Statistical Monte Carlo methods
- 1983: Balberg et al. (Sticks)
- Some Key Publications:
  - 1995: Garboczi et al. (Ellipsoids)
  - 2004: Grujicic et al. (CNTs: Low percolation Threshold)
  - 2005: Du et al. (CNTs: Alignment effect)
  - 2007: Li et al. (CNTs: Dispersion state)
  - 2007: Li et al. (GNPs: Low Percolation Threshold)
  - 2007: Berhan et al. (Fibers: Waviness)
  - 2010: Asiaei et al. (CNTs: Geometrical Distribution)

Motivations

Electrical Properties

Background

Integration of Nanomaterials

Quantifying Electrical Conductivity

Thermal Properties

Background

Quantifying Thermal Conductivity

Experimental Study

Background

Our Proposal

Preliminary Results

Time-Line

## Classical Percolation Law

(Stauffer et al,1994): Electrical conductivity can be described by

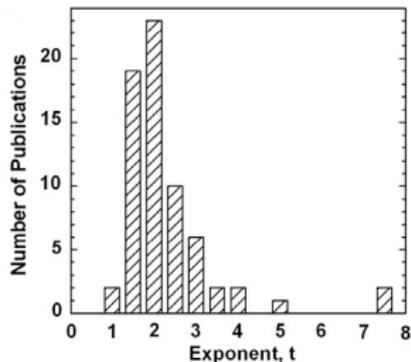
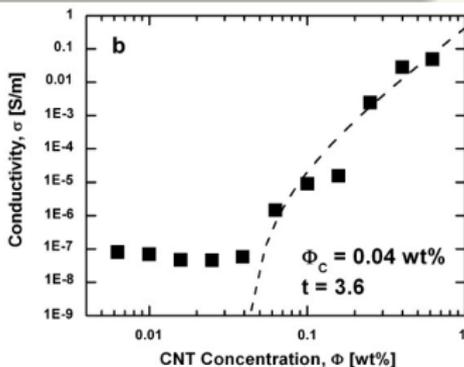
$$\sigma \propto (\phi - \phi_c)^t \quad (1)$$

$\phi$ : filler volume fraction     $\phi_c$ : critical percolation threshold

$t$ : conductivity exponent, for a regular 3D system  $t \approx 1.6 - 2$ , for complicated systems 3.0 or more (Bauhofer et al,2009).

## Limitations

- Narrow applicability range
- Physically interconnected network (No tunneling)
- Only applicable to large particles (Agglomerates)



# Is there any model to quantify tunneling the electrical conductivity from a pair of nanoparticles?

Motivations

Electrical Properties

Background

Introduction of Tunneling

Quantifying Electrical Conductivity

Thermal Properties

Background

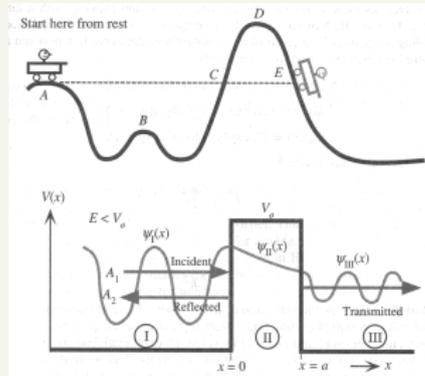
Quantifying Thermal Conductivity

Experimental Study

Background  
Our Proposal

Preliminary Results

Time-Line



Kasap, 2006

Ambrosetti et al 2010

- Switching between percolation/tunneling mechanism governed by parameter  $\epsilon / D$  where  $\epsilon$  is tunneling distance,  $D$  is particle diameter.
- Later in 2010, electrical conductivity a pair of prolate and oblate conductive nanoparticles governed by

$$\sigma_{ij} = \sigma_{pre} \exp\left(-\frac{2\delta_{ij}}{\epsilon}\right) \quad (2)$$

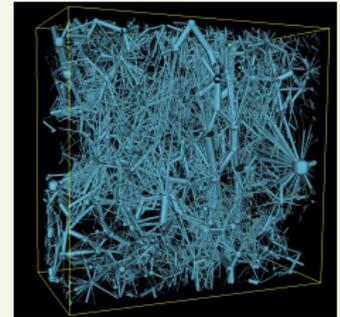
$\delta_{ij}$ : shortest distance between two particle surfaces

$\sigma_{pre}$ : constant exponential prefactor

# How conductive-pair model can be extended to PNCs?



Ambrosetti, 2010



## Global Tunneling Network (GTN)

Using equation 3:

$$\sigma_{ij} = \sigma_{pre} \exp\left(-\frac{2\delta_{ij}}{\epsilon}\right)$$

Constitute a global tunneling network (GTN) and calculate the effective electrical conductivity. GTN method is computationally expensive

Ambrosetti 2010: GTN results can be reproduced by Critical Path (CP) approximation method (Shklovskii,1975)(Pollak,1972)

$$\sigma_c = \sigma_0 \exp\left(-\frac{2\delta_c}{\epsilon}\right) \quad (3)$$

$\delta_c$ : Critical distance found through Monte-Carlo simulation

Motivations

Electrical  
Properties

Background

Incorporation of  
Terminology

Quantifying Electrical  
Conductivity

Thermal  
Properties

Background

Quantifying Thermal  
Conductivity

Experimental  
Study

Background

Our Proposal

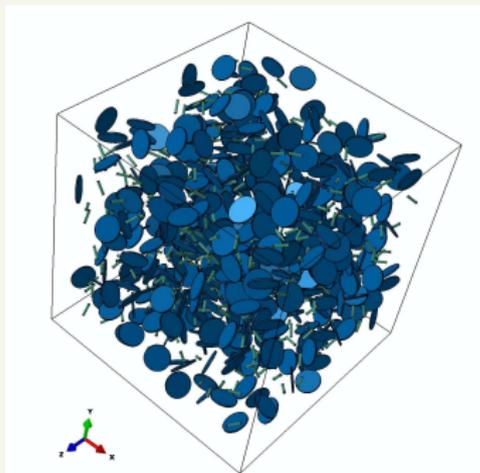
Preliminary  
Results

Time-Line

Electrical properties of CNTs/GNPs based PNCs

What has been done in this dissertation?

# How does tunneling mechanism affect the percolation threshold?



## Motivations

### Electrical Properties

Background

Incorporation of Tunneling

Quantifying Electrical Conductivity

### Thermal Properties

Background

Quantifying Thermal Conductivity

### Experimental Study

Background

Our Proposal

### Preliminary Results

### Time-Line

#### Motivations

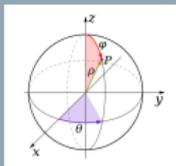
- Computation studies: Ultra low percolation threshold!
- Hardcore/Softcore debate!
- Tunneling was not confirmed for GNPs!

Propose: Monte-Carlo (MC) simulation of GNP-based PNCs

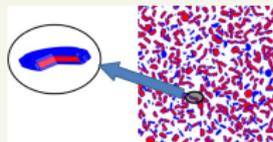
# What are the simulation steps in the proposed MC code?

## Random Spherical Dispersion

X,Y,Z : Randomly selected inside RVE

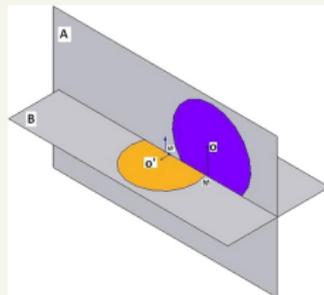
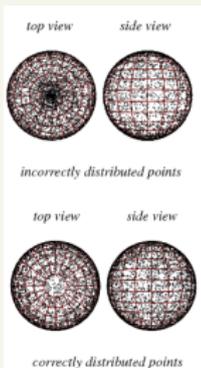


$$\begin{cases} \theta = 2\pi v \\ \varphi = \text{Arccos}(2u - 1) \end{cases} \quad (4)$$



## Fast Algorithm

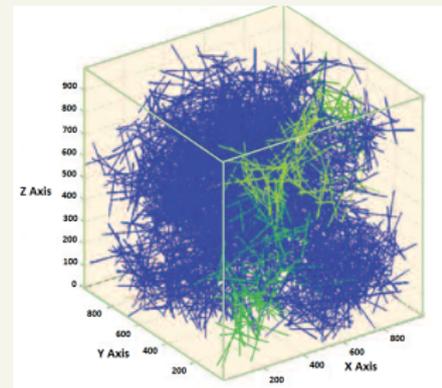
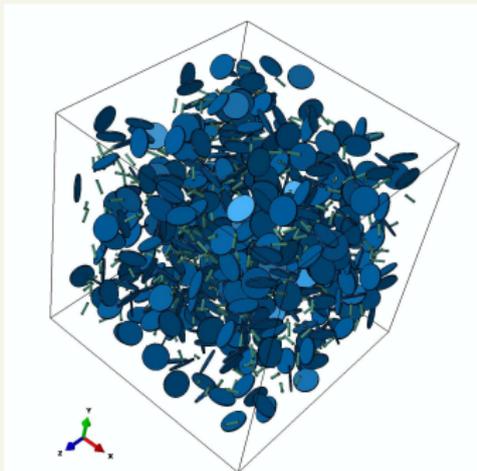
$$\begin{cases} \text{if } D > r + R \Rightarrow NC \\ \text{if } D \leq r + R \Rightarrow C \end{cases} \quad (5)$$



# What are the simulation steps in the proposed MC code?

## Percolation test

- Add a new particle
- Assign a new cluster number to the particle
- Check for tunneling shell interference with the neighbors
- Update the cluster number if interference was detected



Motivations

Electrical  
Properties

Background  
Incorporation of  
Tunneling  
Quantifying Electrical  
Conductivity

Thermal  
Properties

Background  
Quantifying Thermal  
Conductivity

Experimental  
Study

Background  
Our Proposal

Preliminary  
Results

Time-Line

# What is the effect of tunneling distance on the percolation threshold?

Motivations

Electrical Properties

Background  
Incorporation of Tunneling  
Quantifying Electrical Conductivity

Thermal Properties

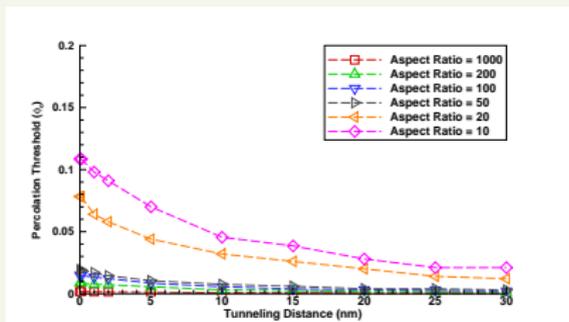
Background  
Quantifying Thermal Conductivity

Experimental Study

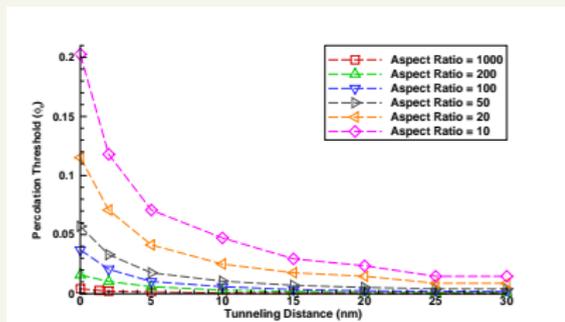
Background  
Our Proposal

Preliminary Results

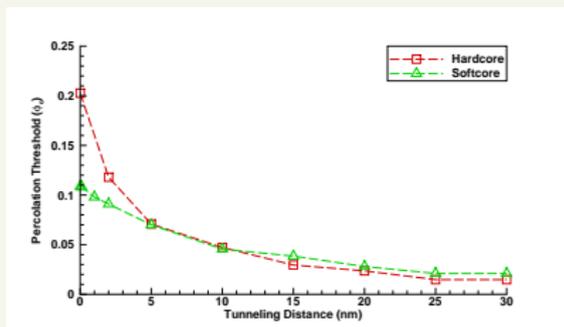
Time-Line



Softcore



Hardcore



Softcore vs. Hardcore  
GNP aspect ratio = 10

# Is there any agreement between the simulation results and experimental observations?

## Validation Study

Study Case	Aspect Ratio	d ( $\mu\text{m}$ )	t (nm)	Predicted Percolation Threshold (%Vol)	Exp. Percolation Threshold (%Vol)
Weng et al. 2004	600	6	10	0.668	4.6
Fukushima and Drzal 2006	1,579	15	9.5	0.294	1.13
Chen et al. 2002	5,000	50	10	0.11	0.67

### Pros/Cons

- Higher percolation threshold but still low!
- Perfect GNPs
- Constant Tunneling Distance
- No agglomeration
- Fast algorithm fails for very high aspect ratio

Motivations

Electrical Properties

Background

Incorporation of Tunneling

Quantifying Electrical Conductivity

Thermal Properties

Background

Quantifying Thermal Conductivity

Experimental Study

Background

Our Proposal

Preliminary Results

Time-Line

# Why MC code Ver 2.0 was needed?

Motivations

Electrical  
Properties

Background

Incorporation of  
Tunneling

Quantifying Electrical  
Conductivity

Thermal  
Properties

Background

Quantifying Thermal  
Conductivity

Experimental  
Study

Background

Our Proposal

Preliminary  
Results

Time-Line

## MC code Ver 2.0

### Features Added:

- Non-Penetration: precise algorithm (separation line method)
- Hybridization: cylinder/disk shaped nanoparticles
- Agglomeration: variable particle geometric factors
- Dispersion: improved particle dispersion technique

# How the proposed MC simulation can predict the electrical conductivity of PNCs?

## Critical Distance

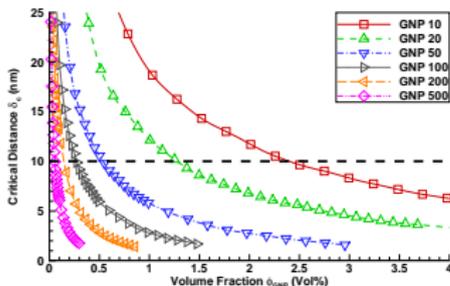
From equation 3:

$$\sigma_c = \sigma_0 \exp\left(-\frac{2\delta_c}{\epsilon}\right)$$

$\delta_c$ : Critical distance found through Monte Carlo simulation

Steps:

- i Generate initial structure
- ii Add penetrable shell to the species
- iii Check for percolation, if no increase shell thickness and go to *ii*, if yes go to *iv*
- iv Repeat *i-iii*, N times and find average shell thickness ( $t_{avg}$ )
- v  $\delta_c = 2 \times t_{avg}$



Critical Distance: GNPs

Motivations

Electrical Properties

Background

Introduction of Nanoparticles

Quantifying Electrical Conductivity

Thermal Properties

Background

Quantifying Thermal Conductivity

Experimental Study

Background

Our Proposal

Preliminary Results

Time-Line

# What are MC predictions for the electrical conductivity of PNCs loaded with CNTs and GNPs?

Motivations

Electrical Properties

Background  
Implementation of  
Forming  
Quantifying Electrical  
Conductivity

Thermal Properties

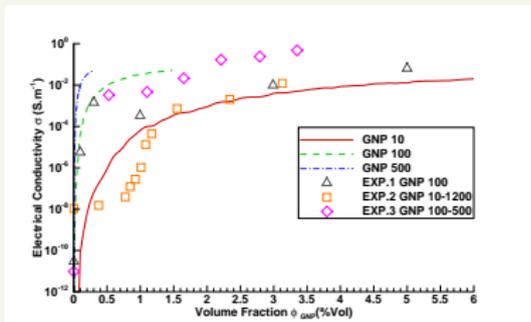
Background  
Quantifying Thermal  
Conductivity

Experimental Study

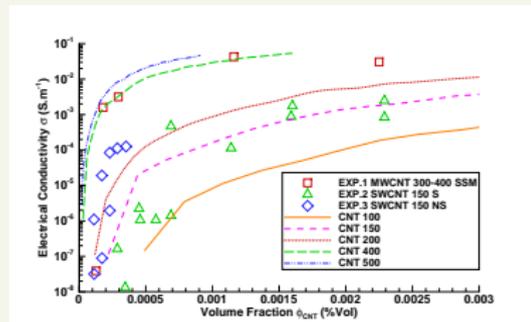
Background  
Our Proposal

Preliminary Results

Time-Line



Validation study: GNPs



Validation study: CNTs

## Future Work: Modeling Uncertainties

Work on assumptions:

- Perfect Geometry
- No Fragmentation, Folding, Waviness
- No Agglomeration
- Uniform Distribution

Motivations

Electrical  
Properties

Background  
Integration of  
Sensing  
Quantifying Electrical  
Conductivity

Thermal  
Properties

Background  
Quantifying Thermal  
Conductivity

Experimental  
Study

Background  
Our Proposal

Preliminary  
Results

Time-Line

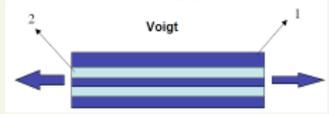
## Thermal properties of CNTs/GNPs based PNCs

What has been done?

# What are the upper bound and the lower bound for the properties?

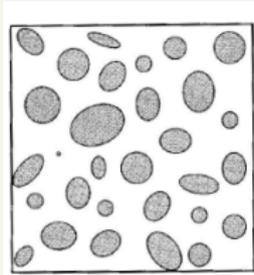
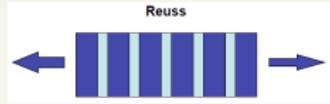
Upper Bound: Arithmetic Average or Voigt Model

$$\sigma_e = \sum_{i=1}^N \phi_i \sigma_i \quad (6)$$

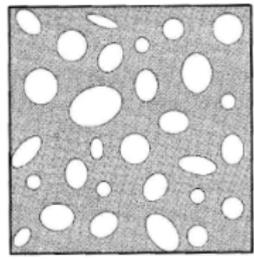


Lower Bound: Harmonic Average or Reuss Model

$$\frac{1}{\sigma_e} = \sum_{i=1}^N \frac{\phi_i}{\sigma_i} \quad (7)$$

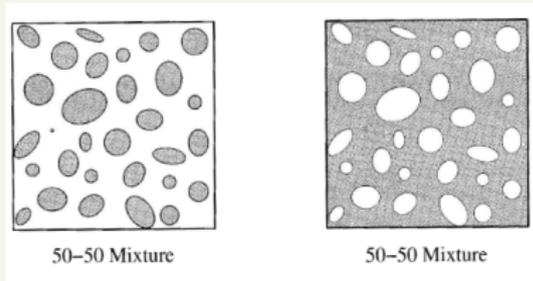


50-50 Mixture



50-50 Mixture

# What are the microstructural details and how to represent them through statistical descriptor functions?



Dr. Adams Lecture Notes

## Microstructural details

- Volume fractions
- Orientations/Sizes/Shapes
- Spatial distribution
- Connectivity
- Clustering
- Surface areas of interface

Motivations

Electrical Properties

Background  
Importance of  
Topology  
Quantifying Electrical  
Conductivity

Thermal Properties

Background  
Quantifying Thermal  
Conductivity

Experimental Study

Background  
Our Proposal

Preliminary Results

Time-Line

# Which classical methods are utilized to predict the thermal conductivity of PNCs loaded with CNTs and GNPs?

Motivations

Electrical Properties

Background  
Incorporation of Nanomaterials  
Quantifying Electrical Conductivity

Thermal Properties

Background  
Quantifying Thermal Conductivity

Experimental Study

Background  
Our Proposal

Preliminary Results

Time-Line

## Classical approaches

### Other Methods:

- Empirical/Semiempirical methods (afdl,1976)
- Mean Field methods (Chen, 1992)
- Variational energy-based methods (Hori,1999)
- Asymptotic methods (Guinovart, 2005)
- Finite elements method (Shenogina, 2005)

Statistical continuum methods (Torquato,2005)

## Others vs Statistical

- Only for periodic microstructure (Mean Field) → Random Microstructure
- Only providing bounds (Variational Methods) → Bounds/Exact Expressions
- Only for weak contrast between the properties (Almost all) → Weak/Strong Contrast
- Slow but detailed (FEM) → Fast and Detailed

# How does statistical continuum methods work?

Motivations

Electrical Properties

Background  
 Interpretation of Tomography  
 Quantifying Electrical Conductivity

Thermal Properties

Background  
 Quantifying Thermal Conductivity

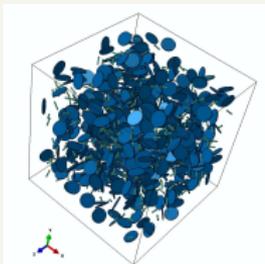
Experimental Study

Background  
 Our Proposal

Preliminary Results

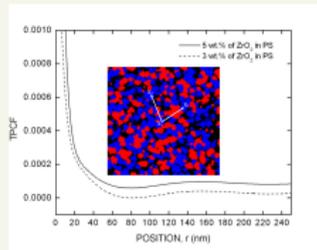
Time-Line

Start with microstructure



**Experimental/Computational Reconstruction**

Connect microstructural details to the effective properties



Baniassadi et al 2011

**Statistical Descriptor Functions**  
**N-point Correlation Functions**  
**Lineal-Path Functions**  
**Cluster Functions etc.**

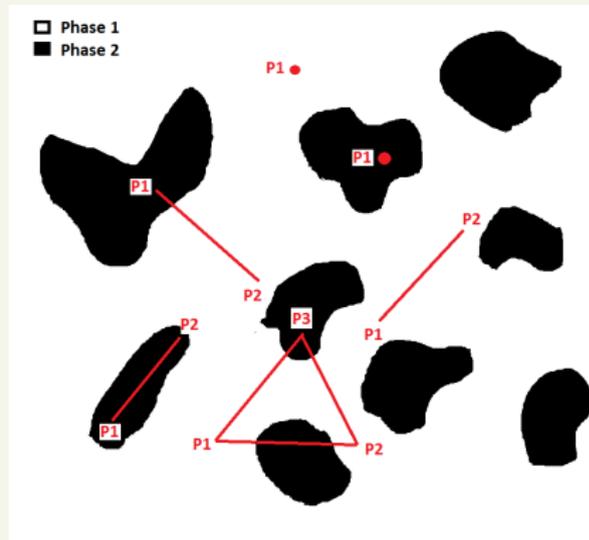


Utilize one of the available solutions:  
**Cluster Expansions**  
**Exact Contrast Expansions etc.**

# What are the N-point correlation functions?

## N-point Correlation Functions

- $N = 1 \rightarrow$  information about microstructure volume fraction
- $N = 2 \rightarrow$  information about microstructure geometry
- $N = 3 \rightarrow$  more information about microstructure
- ...
- $N = \infty \rightarrow$  Full reconstruction



Motivations

Electrical Properties

Background  
 Interpretation of Tomography  
 Quantifying Electrical Conductivity

Thermal Properties

Background  
 Quantifying Thermal Conductivity

Experimental Study

Background  
 Our Proposal

Preliminary Results

Time-Line

Motivations

Electrical  
Properties

Background

Integration of  
Terminology

Quantifying Electrical  
Conductivity

Thermal  
Properties

Background

Quantifying Thermal  
Conductivity

Experimental  
Study

Background

Our Proposal

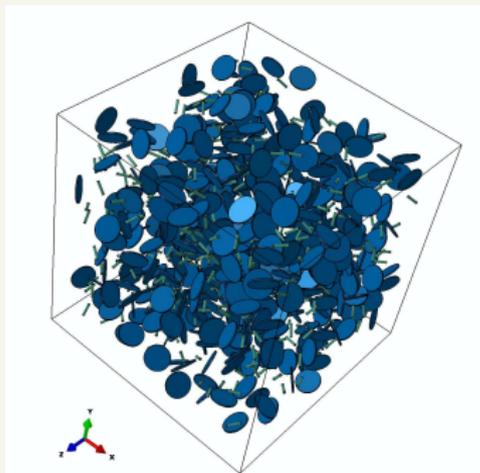
Preliminary  
Results

Time-Line

Thermal properties of CNT/GNP based PNCs

What has been done in this dissertation?

# What is the approach for predicting the thermal conductivity of PNCs in this dissertation?



Motivations

Electrical Properties

Background  
Importance of  
Therming  
Quantifying Electrical  
Conductivity

Thermal Properties

Background  
Quantifying Thermal  
Conductivity

Experimental Study

Background  
Our Proposal

Preliminary Results

Time-Line

## Proposed Study

- Microstructure Generation: MC code
- Descriptor Functions: TPCFs + Approximate 3-point Correlation Functions
- Expansion: Modified Multiphase Strong-Contrast Expansion

In summary: 6 Steps

# Step-1&2: What are the differences between two-point correlation functions (TPCFs) of isotropic and anisotropic microstructures?

Motivations

Electrical Properties

Background  
 Independence of  
 Tortuosity  
 Quantifying Electrical  
 Conductivity

Thermal Properties

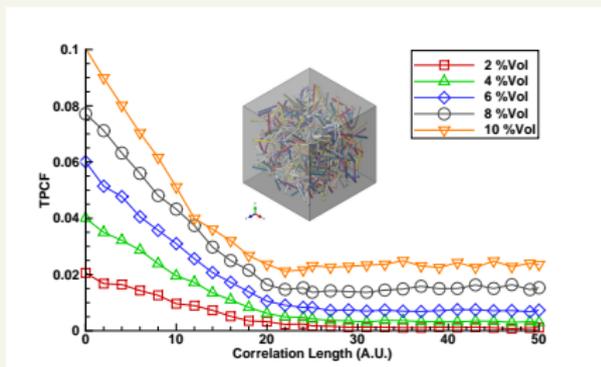
Background  
 Quantifying Thermal  
 Conductivity

Experimental Study

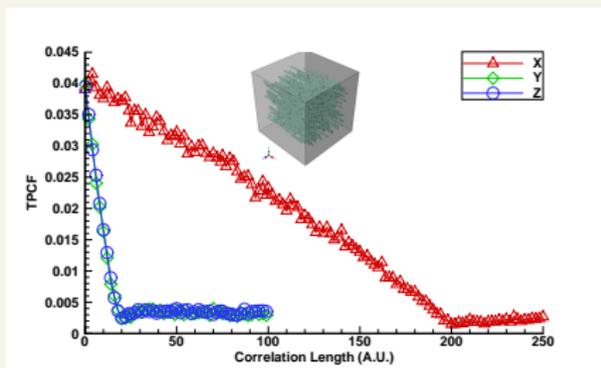
Background  
 Our Proposal

Preliminary Results

Time-Line



Isotropic Tubes



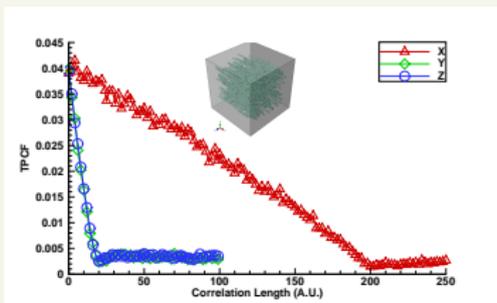
Anisotropic Tubes (Perfectly aligned in x-direction)

# Step-1&2: How TPCFs describe an anisotropic microstructure?

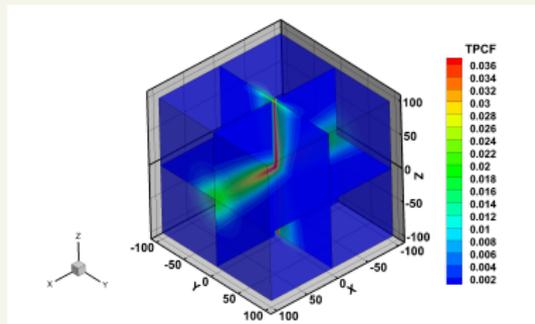
Modified Corson approximation <sup>1</sup>

$$S_2^{(ij)}(r, \theta, \varphi) = a_{ij} + b_{ij} \exp(c_{ij}(\theta, \varphi)r) \quad (8)$$

$c_{ij}(\theta, \varphi)$ : Found by curve fitting( $\theta, \varphi$  are spherical Angles)



Anisotropic



3D TPCF cut section

<sup>1</sup>P.B. Corson "Correlation functions for predicting properties of heterogeneous materials. II. Empirical construction of spatial correlation functions for two-phase solids", Journal of Applied Physics, 1974

Motivations

Electrical Properties

Background

Integration of Technology

Quantifying Electrical Conductivity

Thermal Properties

Background

Quantifying Thermal Conductivity

Experimental Study

Background

Our Proposal

Preliminary Results

Time-Line

# Step-3: How was the strong-contrast expansion extended?

## Extensions

- Original 2-phase → Multi-phase
- Usually second order expansion → third-order expansion
- Three-point correlation functions → approximated from TPCFs
- Sensitive to reference phase → A novel method to reduce sensitivity

$$\left(\frac{1}{\sigma_0} \sigma_e - I\right)^{-1} = \frac{1}{3} \left( \frac{1}{\sum_{\alpha=1}^3 \phi_{\alpha} b_{\alpha 0}} - 1 \right) I - \mathbf{A}_2 - \mathbf{A}_3 \quad (9)$$

$$\mathbf{A}_2 = \frac{\sigma_0}{\left(\sum_{\alpha=1}^3 \phi_{\alpha} b_{\alpha 0}\right)^2} \int \sum_{\alpha=1}^3 \sum_{\beta=1}^3 \left( b_{0\alpha} b_{0\beta} [S_2^{(\alpha\beta)}(1, 2) - S_1^{(\alpha)}(1) S_1^{(\beta)}(2)] \right) H^{(0)}(1, 2) d2 \quad (10)$$

$$\mathbf{A}_3 = \frac{3\sigma_0}{\left(\sum_{\alpha=1}^3 \phi_{\alpha} b_{\alpha 0}\right)^2} \int \int \left( \sum_{\alpha=1}^3 \sum_{\beta=1}^3 \sum_{\gamma=1}^3 b_{0\alpha} b_{0\beta} b_{0\gamma} S_3^{(\alpha\beta\gamma)}(1, 2, 3) - \frac{1}{\sum_{\alpha=1}^3 \phi_{\alpha} b_{\alpha 0}} \sum_{\alpha=1}^3 \sum_{\beta=1}^3 \sum_{\gamma=1}^3 \sum_{\delta=1}^3 b_{0\alpha} b_{0\beta} b_{0\gamma} b_{0\delta} S_2^{(\alpha\beta)}(1, 2) S_2^{(\gamma\delta)}(2, 3) \right) H^{(0)}(1, 2) H^{(0)}(2, 3) d2 d3 \quad (11)$$

Motivations

Electrical Properties

Background

Importance of Tomography

Quantifying Electrical Conductivity

Thermal Properties

Background

Quantifying Thermal Conductivity

Experimental Study

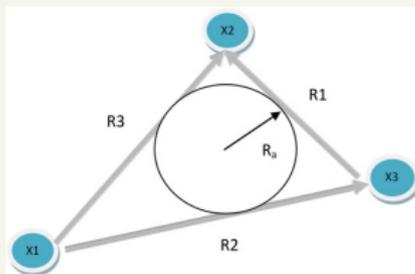
Background

Our Proposal

Preliminary Results

Time-Line

# Step-4: How was three-point correlation functions from approximated from TPCFs?



Baniassadi et al, 2012

$$S_3^{(\alpha\beta\gamma)}(1, 2, 3) \approx W_1^3 \frac{S_2^{(\alpha\beta)}(1, 2)S_2^{(\alpha\gamma)}(1, 3)}{S_1^{(\alpha)}(1)} + W_2^3 \frac{S_2^{(\alpha\beta)}(1, 2)S_2^{(\beta\gamma)}(2, 3)}{S_1^{(\beta)}(2)} + W_3^3 \frac{S_2^{(\beta\gamma)}(2, 3)S_2^{(\alpha\gamma)}(1, 3)}{S_1^{(\gamma)}(3)} \quad (12)$$

$$W_i^3 = \frac{R_i}{\sum_{j=1}^3 R_j}, \quad i = 1 \dots 3 \quad (13)$$

Motivations

Electrical Properties

Background

Importation of Tomography

Quantifying Electrical Conductivity

Thermal Properties

Background

Quantifying Thermal Conductivity

Experimental Study

Background

Our Proposal

Preliminary Results

Time-Line

# Step-5: How did the sensitivity of multiphase third order Strong-Contrast expansion reduce?

## sensitivity analysis

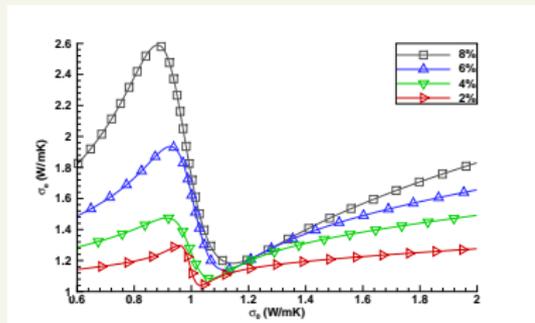
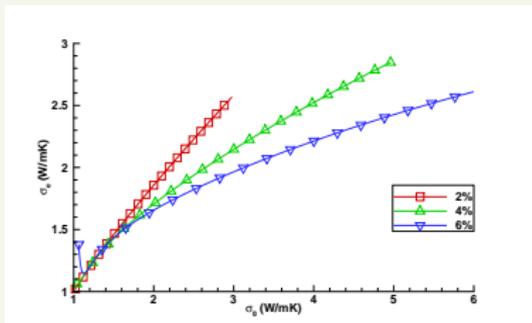
Equation 9 can be solved for  $\sigma_e$ :

$$\sigma_e = \sigma_0 \left( I + \left( \frac{1}{\sum_{\alpha=1}^3 \phi_{\alpha} b_{\alpha 0}} - 1 \right) I - A_2 - A_3 \right)^{-1} = f(\sigma_0, \text{microstructure}) \quad (14)$$

Thus

$$e(\sigma_0) = (f(\sigma_0) - \sigma_{\text{true}})^2 \quad (15)$$

looking for  $\partial e / \partial \sigma_0 = 0 \rightarrow$  Candidate points



Sensitivity diagram for  $\sigma_{\text{tube}}/\sigma_{\text{matrix}} = 100$

Motivations

Electrical Properties

Background

Importation of Technology

Quantifying Electrical Conductivity

Thermal Properties

Background

Quantifying Thermal Conductivity

Experimental Study

Background

Our Proposal

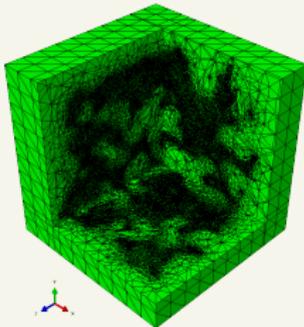
Preliminary Results

Time-Line

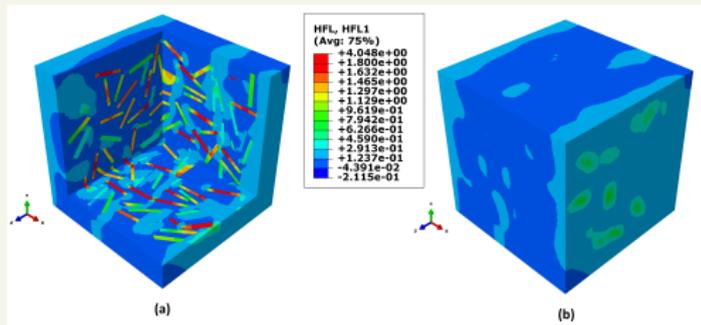
# Step-6: How was the validity of the proposed method verified?

## Finite Element Study

- Package: Abaqus 6.10-EF
- Pre/post-processing: Python
- Assumption: Perfect thermal bounding
- Element: DC3D4
- BC's: Constant temperature left/right sides of RVE
- Analysis: Steady-state Thermal Analysis (1 - 96 hours)
- Result: Volume averaged heat-flux vector
- Convergence: Local and global mesh refinement



Internal view of a Sample RVE



10%Vol disk-shaped inclusions (Aspect ratio= 10 and  $\sigma_{disk} = 100\sigma_{matrix}$ )

Motivations

Electrical Properties

Background  
Importation of Forming  
Quantifying Electrical Conductivity

Thermal Properties

Background  
Quantifying Thermal Conductivity

Experimental Study

Background  
Our Proposal

Preliminary Results

Time-Line

# How was agreement between Modified SC and FEA?

Motivations

Electrical Properties

Background  
Independence of  
Terminology  
Quantifying Electrical  
Conductivity

Thermal Properties

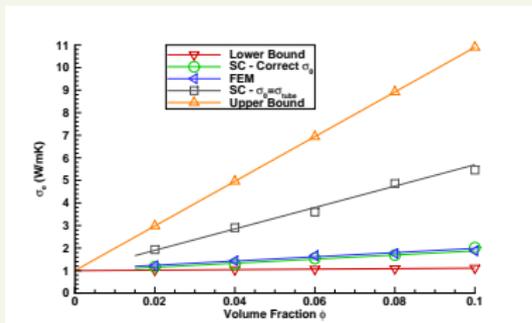
Background  
Quantifying Thermal  
Conductivity

Experimental Study

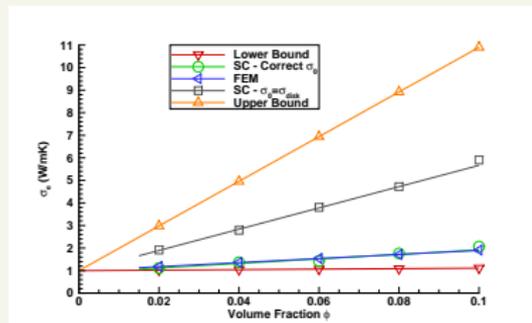
Background  
Our Proposal

Preliminary Results

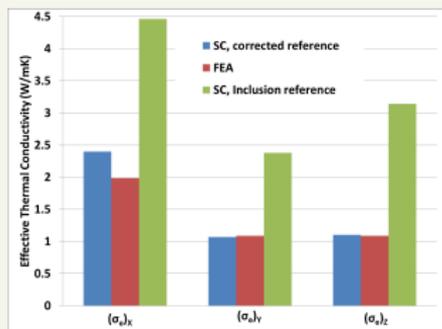
Time-Line



Tube



Disk



Tube- Aligned in x-direction

## Motivations

## Electrical Properties

- Background
- Integration of Terahertz
- Quantifying Electrical Conductivity

## Thermal Properties

- Background
- Quantifying Thermal Conductivity

## Experimental Study

- Background
- Our Proposal

## Preliminary Results

## Time-Line

### Electrical/Thermal Properties

Enhancement for PNCs based on:

- CNTs . . . ✓
- GNPs . . . ✓
- CNTs + GNPs . . . ?

# Is there any advantage in hybridization?

Motivations

Electrical Properties

Background  
 Interpretation of Terminology  
 Quantifying Electrical Conductivity

Thermal Properties

Background  
 Quantifying Thermal Conductivity

Experimental Study

Background  
 Our Proposal

Preliminary Results

Time-Line

Study	Electrical Conductivity	Thermal Conductivity	Description
Yu et al 2008	▼	▲	CNTs/GNPs/polymer
Lie et al 2008	▲	N/A	CNTs/GNPs/polymer
Tung et al 2009	▲	N/A	CNTs/graphene/polymer
Kim et al 2009	▲	N/A	CNTs/graphene/polymer
Yan et al 2010	▲	N/A	CNTs/graphene/polymer
Hong et al 2010	▲	N/A	CNTs/graphene/polymer
Kumar et al 2010	▲	▲	CNTs/GNPs/polymer

## Proposal

Study: hybrid CNTs/GNPs PNCs both computational + experimental

Motivations:

- Confirm synergistic effects
- Shed more light on physics
- Further validation

# What is proposed for experimental study in this dissertation?

## Experimental Study

- Samples: CNT/epoxy; GNP/epoxy; CNT/GNP/epoxy
- Measurement: Electrical/Thermal Conductivity
- Microscopy/FIB/Reconstruction



Motivations

Electrical Properties

Background  
Incorporation of Nanomaterials  
Quantifying Electrical Conductivity

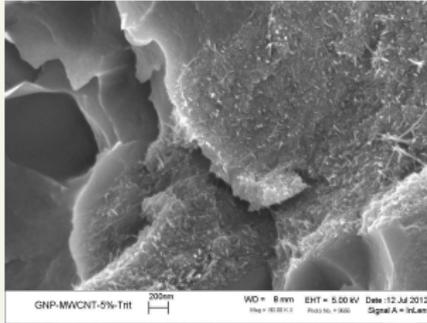
Thermal Properties  
Background  
Quantifying Thermal Conductivity

Experimental Study  
Background  
Our Proposal

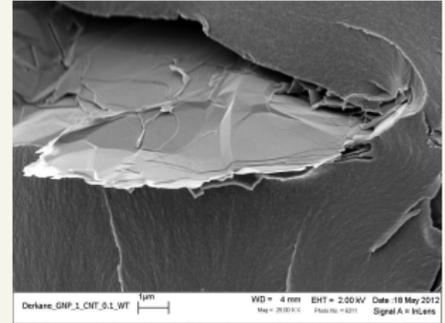
Preliminary Results

Time-Line

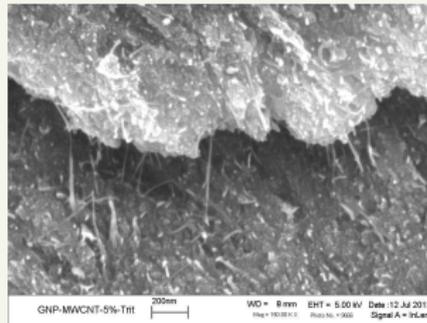
# What was the preliminary observation?



CNT/epoxy



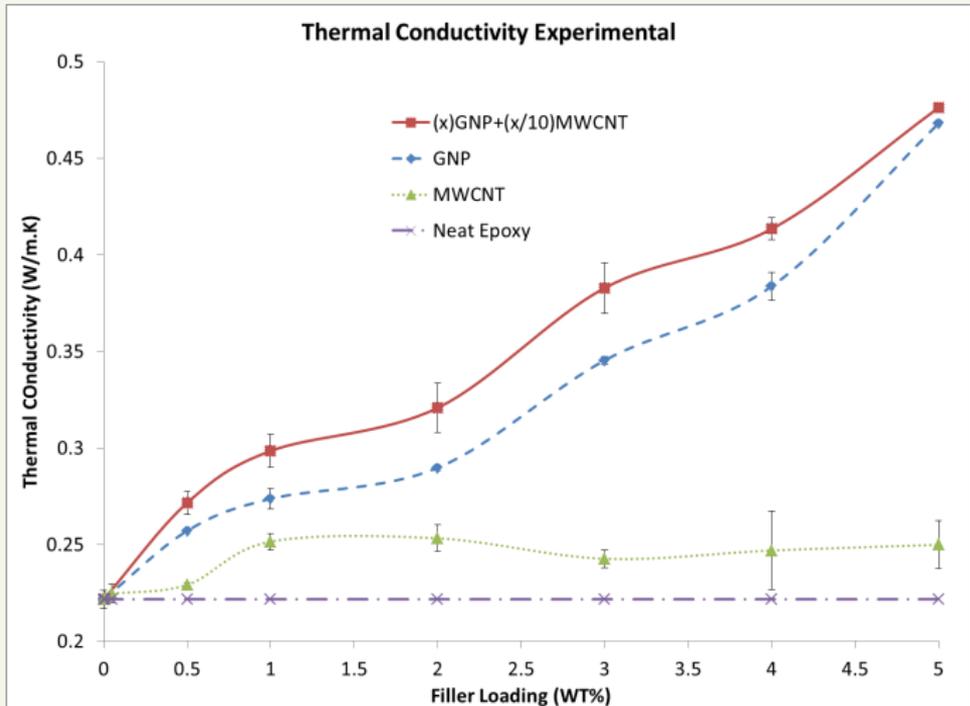
GNP/epoxy



CNT/GNP/epoxy

- Motivations
- Electrical Properties
  - Background
  - Integration of Tinning
  - Quantifying Electrical Conductivity
- Thermal Properties
  - Background
  - Quantifying Thermal Conductivity
- Experimental Study
  - Background
  - Our Proposal
- Preliminary Results
- Time-Line

# What are thermal conductivity measurements results?



Motivations

Electrical Properties

Background  
Integration of  
Terminology  
Quantifying Electrical  
Conductivity

Thermal Properties

Background  
Quantifying Thermal  
Conductivity

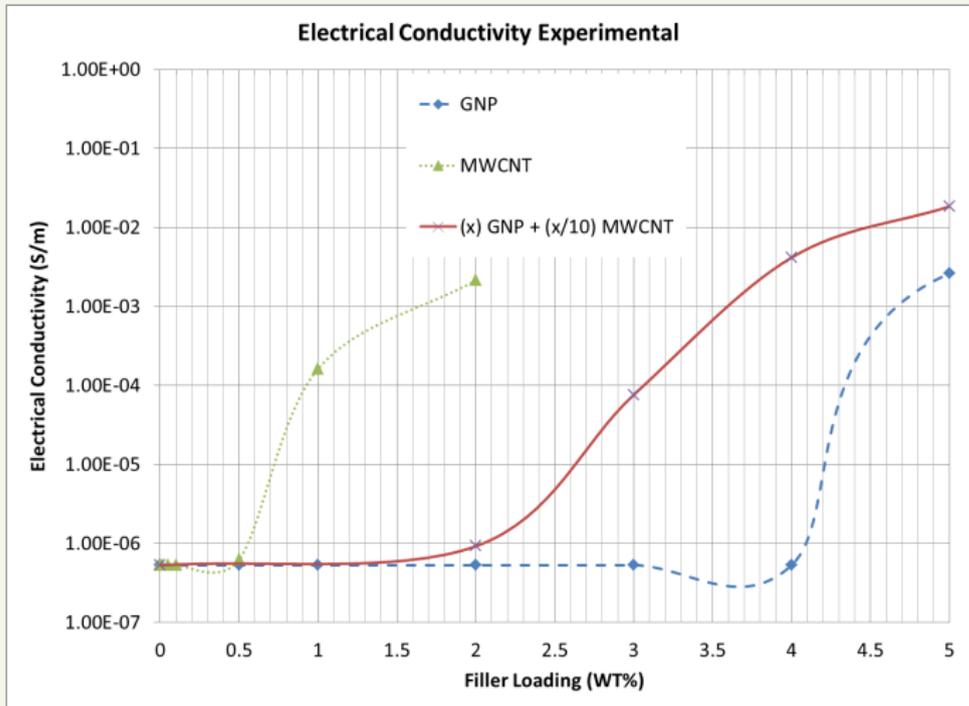
Experimental Study

Background  
Our Proposal

Preliminary Results

Time-Line

# What are electrical conductivity measurements results?



- Motivations
- Electrical Properties
  - Background
  - Importance of Tinning
  - Quantifying Electrical Conductivity
- Thermal Properties
  - Background
  - Quantifying Thermal Conductivity
- Experimental Study
  - Background
  - Our Proposal
- Preliminary Results
- Time-Line

# What are the next steps and their time-line?

Motivations

Electrical Properties

Background  
Importance of Terminology  
Quantifying Electrical Conductivity

Thermal Properties

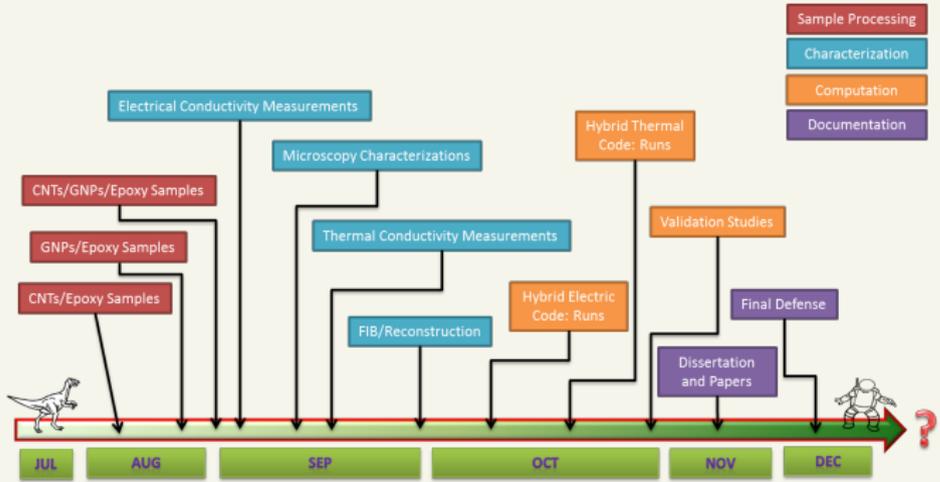
Background  
Quantifying Thermal Conductivity

Experimental Study

Background  
Our Proposal

Preliminary Results

Time-Line



Motivations

Electrical Properties

Background  
Incorporation of Termination  
Quantifying Electrical Conductivity

Thermal Properties

Background  
Quantifying Thermal Conductivity

Experimental Study

Background  
Our Proposal

Preliminary Results

Time-Line

## Publications:

- Safdari, M., Baniassadi, M., Garmestani, H. and Al-Haik, M. S. "A Modified Strong-Contrast Expansion for Estimating the Effective Thermal Conductivity of Multiphase Heterogeneous Materials", *Journal of Applied Physics*, Under review
- Safdari, M. and Al-Haik, M. S. "Electrical Conductivity of Synergistically Hybridized Nanocomposites based on Graphite Nanoplatelets (GNPs) and Carbon Nanotubes (CNTs)", *Nanotechnology*, Accepted
- Baniassadi, M., Safdari, M., Ghazavizadeh, A., Garmestani, H., Ahzi, S., Gracio, J., and Ruch, D. "Incorporation of electron tunnelling phenomenon into 3d monte carlo simulation of electrical percolation in graphite nanoplatelet composites.", *Journal of Physics D: Applied Physics*, 44(45):455306–455313, 2011.
- Ghazavizadeh, A., Baniassadi, M., Safdari, M., Atai, A., Ahzi, S., Patlazhan, S., Gracio, J., and Ruch, D. "Evaluating the Effect of Mechanical Loading on the Electrical Percolation Threshold of Carbon Nanotube Reinforced Polymers: A 3D Monte-Carlo Study." *Journal of Computational and Theoretical Nanoscience*, 8(10):2087–2099, 2011.
- Asiaei, S., Khatibi, A. A., Baniassadi, M., and Safdari, M. "Effects of carbon nanotubes geometrical distribution on electrical percolation of nanocomposites: A comprehensive approach.", *Journal of Reinforced Plastics and Composites*, 29: 818-829, 2009.

## Other Publications:

- Tehrani, M., Safdari, M., Yarri, A., Razavi, Z., Case, S. W., Dahmen, K., Garmestani, H., and Al-Haik, M. S. "Hybrid carbon fiber / carbon nanotubes composites for structural damping applications.", *Carbon*, Under review
- Safdari, M. and Al-Haik, M. S. "Optimization of stress wave propagation in a multilayered elastic/viscoelastic hybrid composite based on carbon fibers/carbon nanotubes." *Polymer Composites*, 33(2):196–206, 2012.
- Tehrani, M., Safdari, M., and Al-Haik, M. "Nanocharacterization of creep behavior of multiwall carbon nanotubes/epoxy nanocomposite." *International Journal of Plasticity*, 27(6):887 – 901, 2011.

Thanks for your attention!

Questions??



Motivations

Electrical  
Properties

Background  
Introduction of  
Terminology  
Quantifying Electrical  
Conductivity

Thermal  
Properties

Background  
Quantifying Thermal  
Conductivity

Experimental  
Study

Background  
Our Proposal

Preliminary  
Results

Time-Line



## Carbon Nanotubes for Targeted Drug Delivery to Ovarian Cancer

A. Alipour<sup>1</sup>, R. Zeineldin<sup>2</sup> and M. Al-Haik<sup>1</sup>

*Department of Engineering Science & Mechanics  
Virginia Tech  
Blacksburg, VA 24060*

*Department of Pharmaceutical Sciences  
Massachusetts College of Pharmacy & Health Sciences  
Worcester, MA 01608*

*Workshop on Advances in Computational Mathematics and Engineering  
In honor of the contributions of M. Y. Hussaini  
Florida State University Sept 28-30, 2012*

10/2/12

1

## Therapy in ovarian cancer (OVCA)

- Targeting OVCA through tumor markers or over-expressed receptors
- Expression of folate receptor alpha (FR $\alpha$ ) or folate receptor beta (FR beta), both are 38 KDa GPI-anchored membrane glycoproteins is detected at high frequency in cancers.

10/2/12

2

## Advantages of functionalized CNTs for drug delivery

- CNTs  $\leq 1 \mu\text{m}$  deliver to cells: protein, nucleic acids, drugs
- Non-immunogenic (short-term studies)
- Little toxicity (short-term studies)
- Cleared rapidly from body
- High thermal conductivity
- Hollow cylinders can introduce molecules into internal space
- Easy uptake

10/2/12

3

## Dispersion by Functionalizing CNTs



500 mg

Dispersion



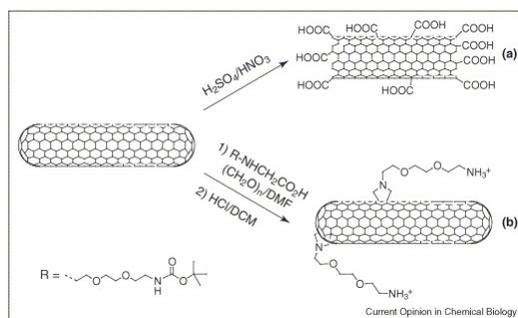
2 mg in 500 mL

1. Chemical conjugation
2. Adsorption (involves ultra sonication) e.g. phospholipid-polyethylene glycol (PL-PEG)  
ultrasonication – 1 to 2 hr

10/2/12

4

## Chemical Modifications / Conjugations



Organic functionalisation of carbon nanotubes. Pristine single- or multi-walled carbon nanotubes can be **(a)** treated with acids to purify them and generate carboxylic groups at the terminal parts, or **(b)** reacted with amino acid derivatives and aldehydes to add solubilizing moieties around the external surface.

Bianco & coworkers 2005. Current Opinion in Chemical Biology 9:674-679

10/2/12

5

## PEGylation

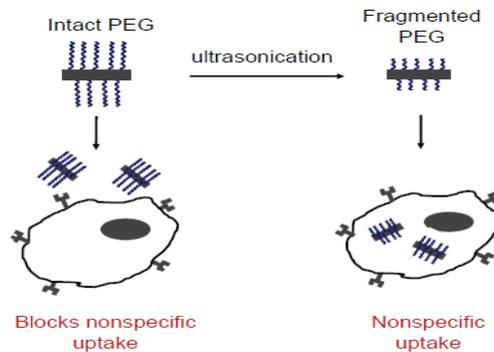
- Increases half-life in circulation
- Reduce non-specific uptake by cells
- Blocks non-specific binding to proteins
  
- Evaluation of cellular uptake of SWNTs functionalized by adsorbing PL-PEG led to unexpected findings:
  - PL-PEG2000 (i.e., the MW of PEG is ~2000) to SWNTs did not reduce uptake of SWNTs
  - PL-PEG5000 gave contradictory results

10/2/12

6

## Hypothesis

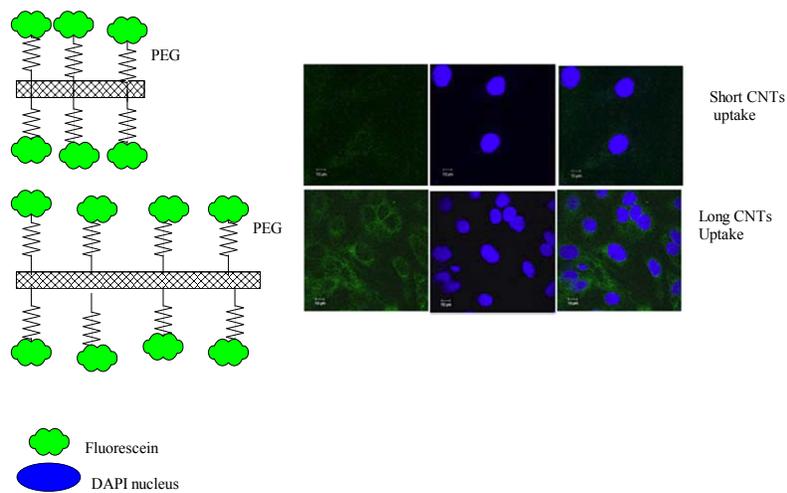
- Integrity of PEG is important to prevent nonspecific uptake of SWNTs



10/2/12

7

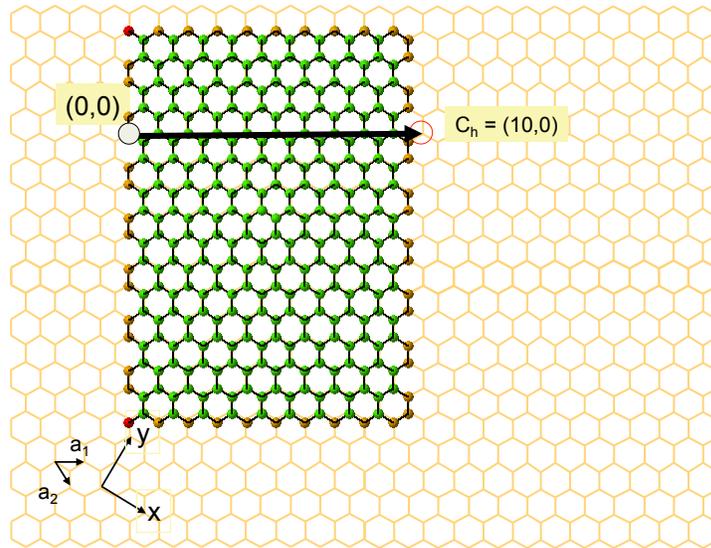
## Evaluation of the effect of PEG/CNTs length on the uptake of SWNTs by cells



10/2/12

8

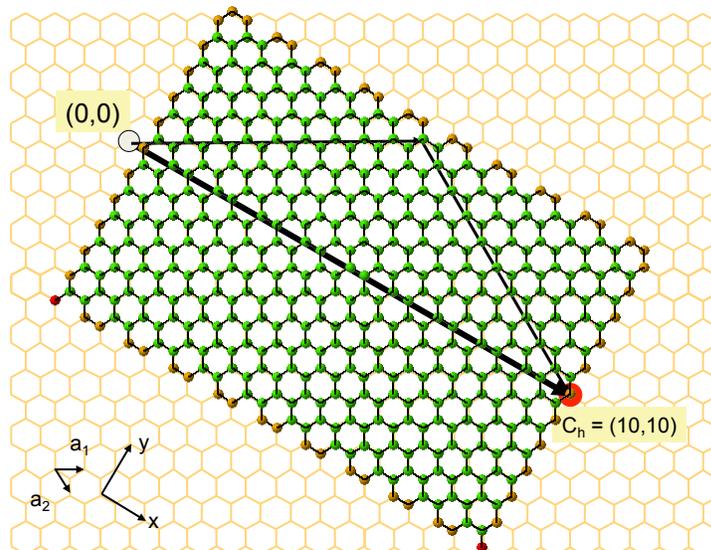
### Wrapping (10,0) SWNT (zigzag)



10/2/12

9

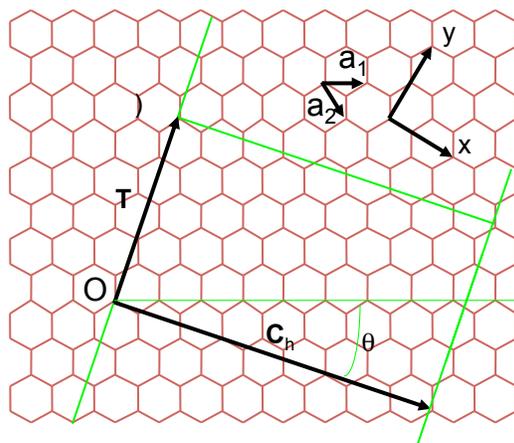
### Wrapping (10,10) SWNT (armchair)



10/2/12

10

## Hexagonal Lattice (Definition of Chiral Vectors)



Chiral vector:

$$\mathbf{C}_h = n\mathbf{a}_1 + m\mathbf{a}_2$$

$$\mathbf{a}_1 = \left( \frac{3}{2}a_{cc}, \frac{\sqrt{3}}{2}a_{cc} \right)$$

$$\mathbf{a}_2 = \left( \frac{3}{2}a_{cc}, -\frac{\sqrt{3}}{2}a_{cc} \right)$$

$a_{cc}$  is the C-C bond length (0.142 nm).

$$\theta = \arccos\left( \frac{\sqrt{3}(n+m)}{2\sqrt{n^2+m^2+nm}} \right)$$

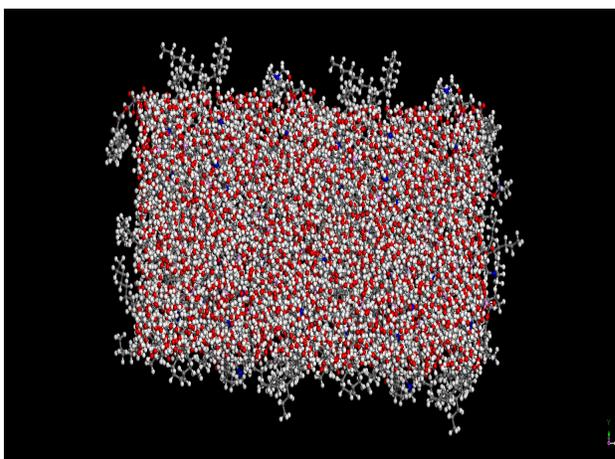
$$D_n = \frac{\sqrt{3}}{\pi} a_{cc} \sqrt{(n^2+m^2+nm)}$$

10/2/12

11

## The Membrane

72 dimyristoyl-sn-glycerophosphatidylcholine (DMPC) lipids, -a common membrane and 2716 water molecules making an overall 17,796 atoms structure.



10/2/12

12

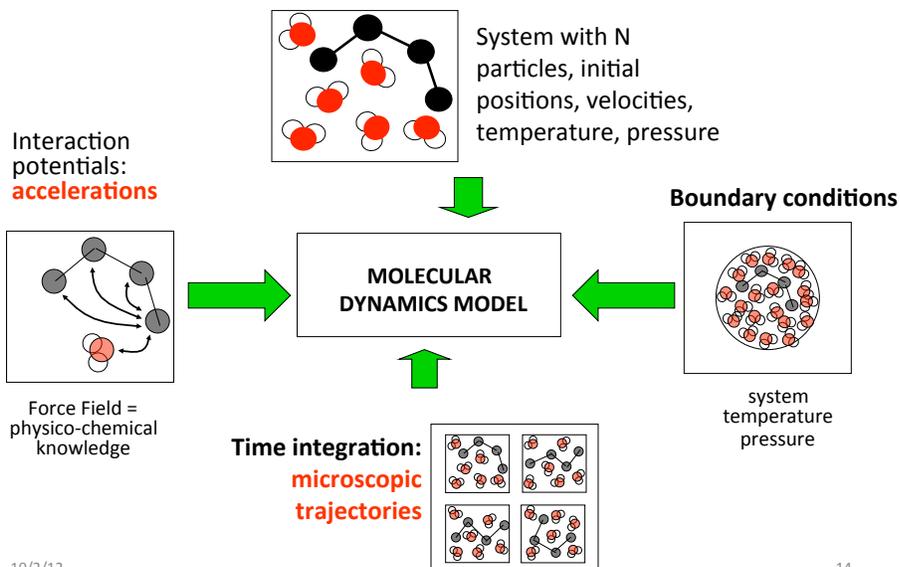
## The CNTs

		H	C	Diameter (nm)	Length (nm)	Chiral Angle $\theta^\circ$
(10,0) NT-	short	40	600	0.783	6.5541	0.00
	long		1280		13.472	
(10,5) NT	short	50	600	1.036	5.2281	19.11
	long		1300		10.972	
(10,7) NT	short	54	600	1.158	4.8061	24.18
	long		1308		10.212	
(10,10) NT-	short	60	600	1.356	4.1278	30.00
	long		1320		8.932	

10/2/12

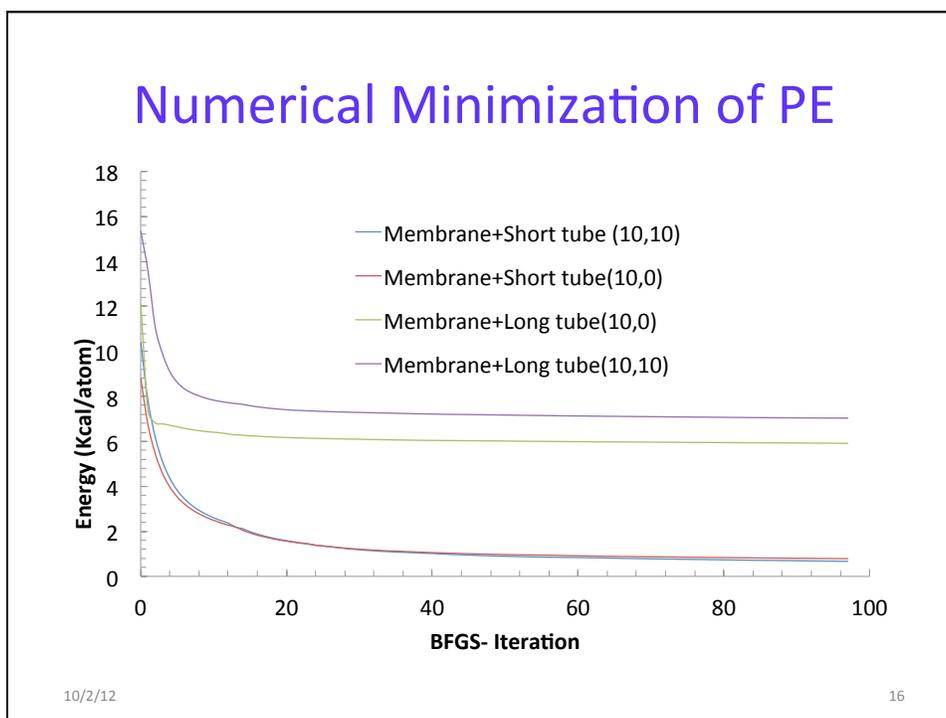
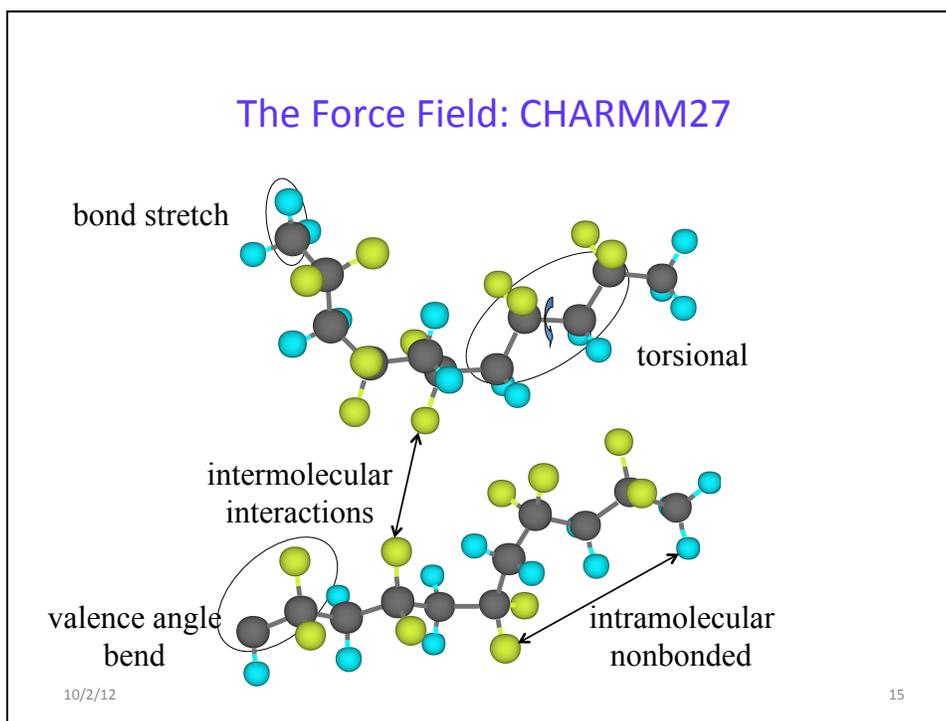
13

## Molecular Dynamics Simulation Basics



10/2/12

14



## Generating Trajectory

Newton Eqn:

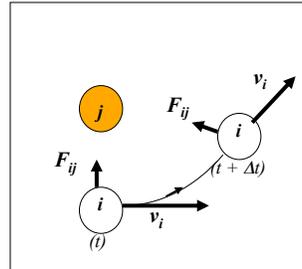
$$\vec{F}_{ij} = -\frac{\partial \phi_{ij}}{\partial \vec{r}_{ij}} \hat{r}_{ij} \quad \sum_j \vec{F}_{ij} = m_i \frac{d\vec{v}_i}{dt}$$

Updated Position:

$$\vec{r}_i(t + \Delta t) = \vec{r}_i(t) + (\Delta t)\vec{v}_i(t) + \frac{1}{2}(\Delta t)^2 \frac{\vec{F}_i(t)}{m_i}$$

Updated Velocity:

$$\vec{v}_i\left(t + \frac{\Delta t}{2}\right) = \vec{v}_i(t) + \frac{1}{2}(\Delta t) \frac{\vec{F}_i(t)}{m_i} \quad \vec{v}_i(t + \Delta t) = \vec{v}_i\left(t + \frac{\Delta t}{2}\right) + \frac{1}{2}(\Delta t) \frac{\vec{F}_i(t + \Delta t)}{m_i}$$



10/2/12

17

## Controlling Thermodynamic Variables T and P

- Statistical ensembles connect microscopic to macroscopic/thermodynamic
- NVE (microcanonical )
- NVT (Canonical )
- NPT (Isothermal-isobaric )
- Thermostats, barostats, etc., allow one to choose appropriate ensembles.

$$V = 1/M * \sum_{i=1}^M V_i$$

- **Nose-Hoover** thermostat is used for the temperature control

$$T_i = \frac{1}{3Nk_B} \sum_{i=1}^N m_i v_i^2$$

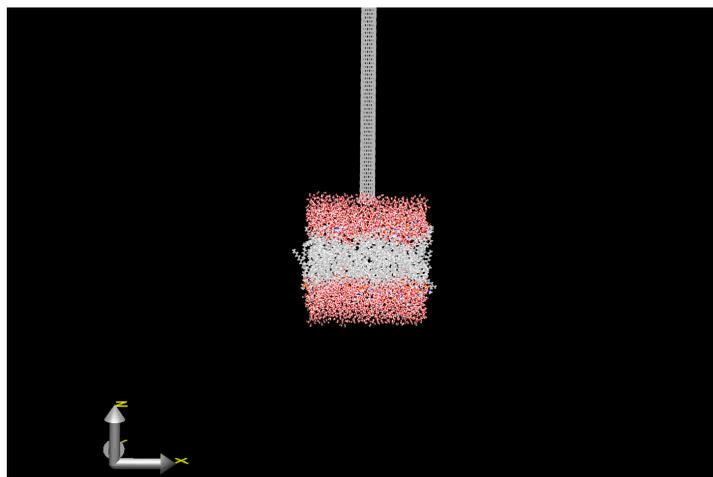
- **Berendsen** barostat maintain a constant desired pressure in the periodic box.

$$p_i = \frac{2}{3V} \left[ \frac{1}{2} \sum_{i=1}^N m_i v_i^2 - \frac{1}{2} \sum_{i=1}^N r_i \nabla U(r_i) \right]$$

10/2/12

18

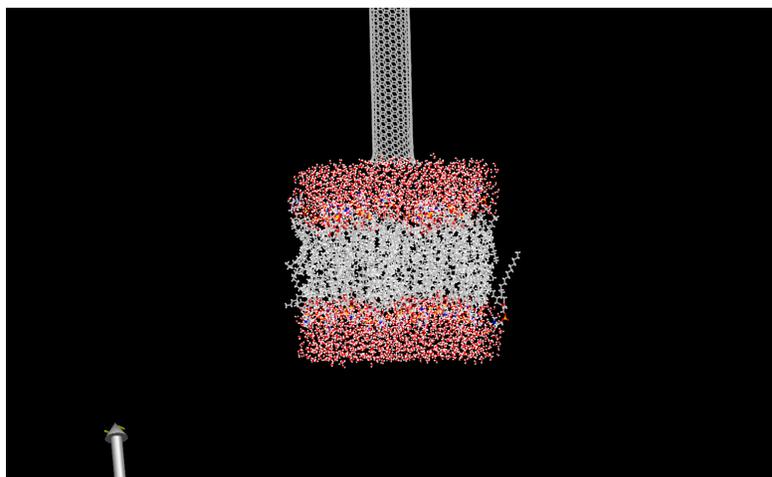
## MD 10-0



10/2/12

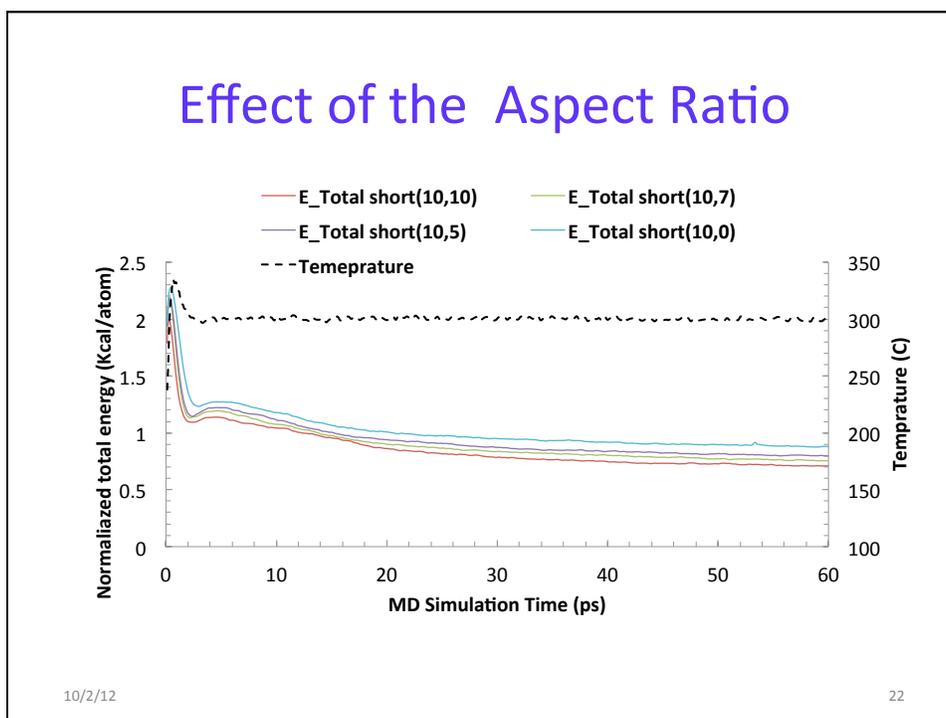
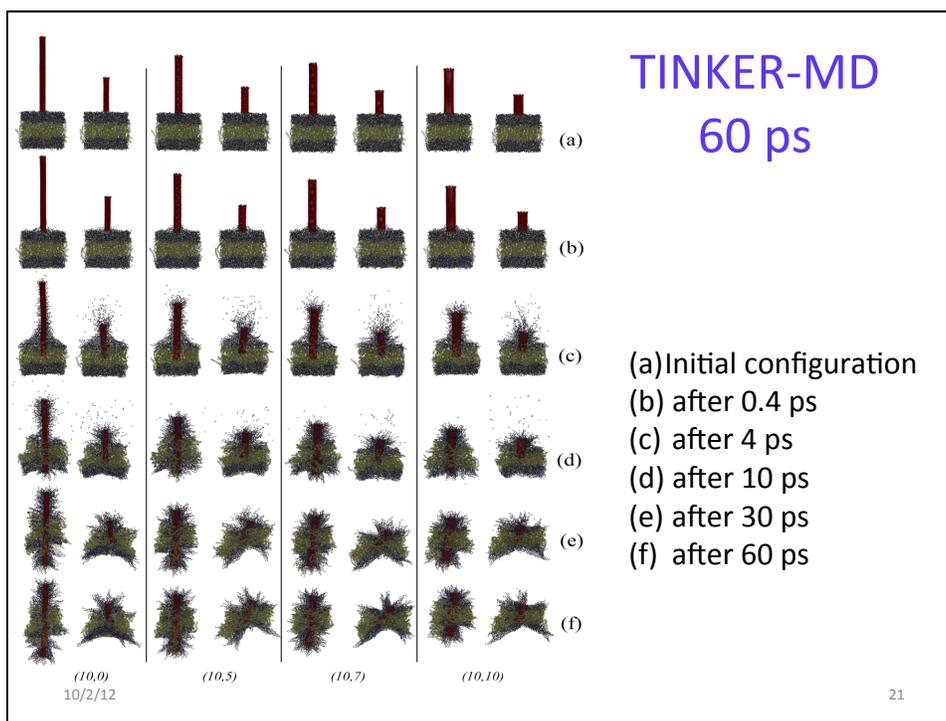
19

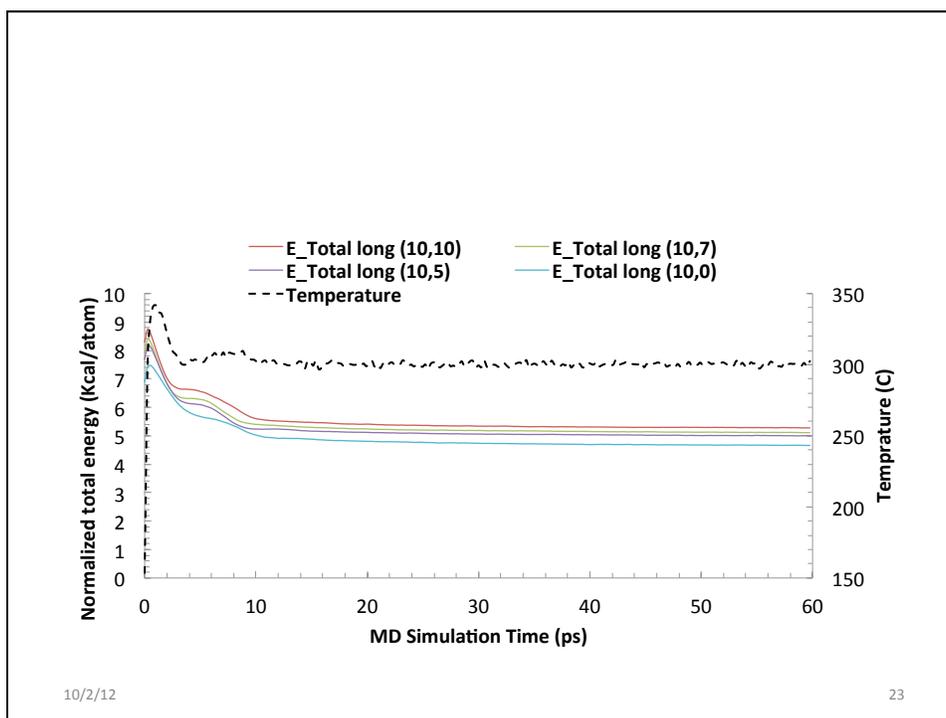
## MD10-10



10/2/12

20





10/2/12

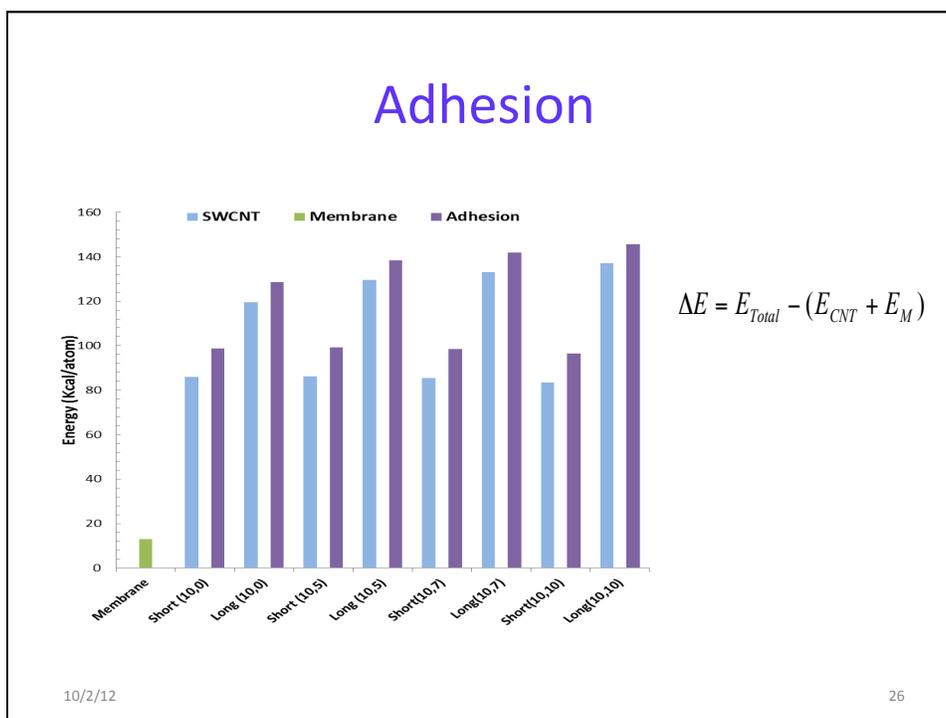
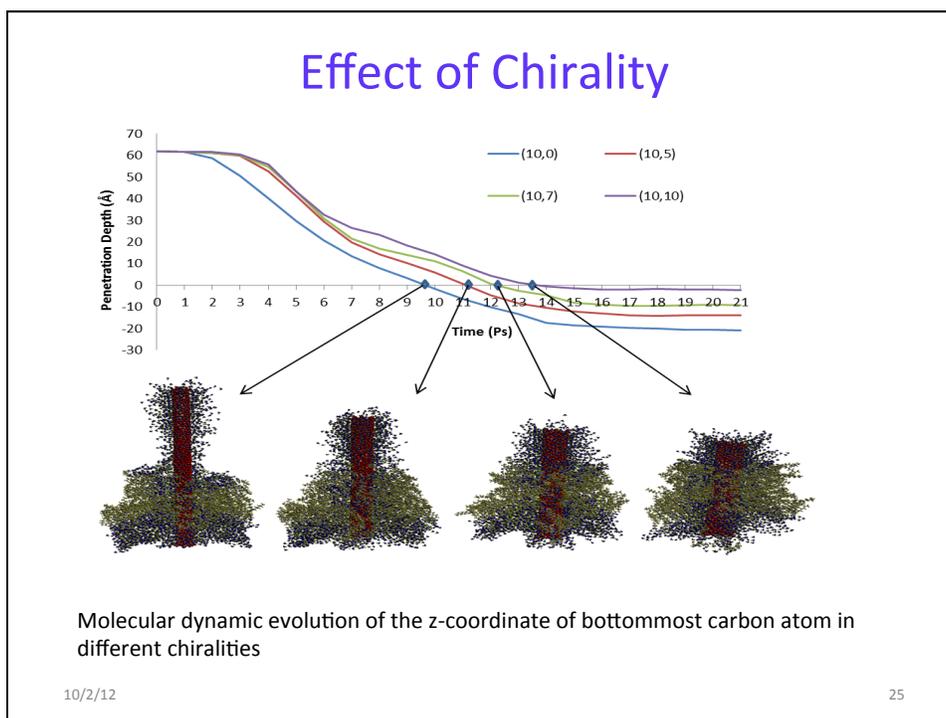
23

## Finding

- Holding the chirality fixed, longer nanotubes find their ways through the membrane faster with less rotation from the vertical position.
- On the contrary, tubes with length shorter than thickness of membrane (i.e.  $\sim 60 \text{ \AA}$ ) are more likely to rotate in the membrane while trafficking via endocytosis mechanism.

10/2/12

24



## Conclusions

- At earliest time the mechanism of interaction between the SWCNT and the membrane is pure penetration. As time evolves penetration is accompanied by endocytosis and almost halfway through 60 ps, endocytosis becomes the dominant form of interaction.
- CNTs with length shorter than the thickness of the membrane- undergo significant rotation during the endocytosis stage.
- Lower chirality (higher aspect ratio) assisted the SWCNT to cross through the membrane faster.
- As lesser of the SWCNT energy was consumed to establish adhesion with the membrane it was capable of translocating through the membrane from the other side faster than the other nanotubes

10/2/12

27

## Publications

**NANO** LETTERS

R. Zeineldin , M. Al-Haik and Laurie G. Hudson, " Role of Polyethylene Glycol Integrity in Specific Receptor Targeting of Carbon Nanotubes to Cancer Cells, " Nano Letters, 2009, 9 (2), pp 751–757.

**Langmuir**

A. Alipour Skandani , R. Zeineldin , and M. Al-Haik , " Effect of Chirality and Length on the Penetrability of Single-Walled Carbon Nanotubes into Lipid Bilayer Cell Membranes, " Langmuir, 2012, 28 (20), pp 7872–7879

10/2/12

28