Parametric Uncertainty Quantification in the Rothermel Model with Randomized Quasi-Monte Carlo Methods

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Introduction

Rothermel's wildland surface fire model is a popular model used in wildland fire management. It has been integrated in software systems, such as FARSITE and BehavePlus. Simulations making use of a robust model are still subject to errors and uncertainty owing to the variability of the input parameters. The original model has a large number of parameters, making uncertainty quantification challenging. In this work, we use variance-based global sensitivity analysis to reduce the number of model parameters, and apply randomized quasi-Monte Carlo methods to quantify parametric uncertainties for the reduced model. The Monte Carlo estimator used in these calculations is based on a control variate approach applied to the sensitivity derivative enhanced sampling. The chaparral fuel model, selected from the Rothermel's 11 original fuel models, is studied as an example. We obtain numerical results that improve the crude Monte Carlo sampling by factors as high as three orders of magnitude.

The Rothermel model

The main output variables of the Rothermel model are the rate of fire spread (*ros* in ms^{-1}), the direction of maximum spread (*sdr* in °), the effective wind speed (*efw* in ms^{-1}), and reaction intensity (*ri* in kWm^{-2}).

***ri*:**

$$ri = \Gamma' \cdot w_n \cdot heat \cdot \eta_M \cdot \eta_S$$

where Γ' is the optimum reaction velocity, w_n is the net fuel loading, *heat* is the fuel low heat content, η_M is the moisture damping coefficient, and η_S is the mineral damping coefficient.

***ros*:**

$$ros = \frac{ri \cdot \xi \cdot (1+\Phi_c)}{\rho_b \cdot \varepsilon \cdot Q_{ig}}$$

where ξ is the propagating flux ratio, ρ_b is the ovendry bulk density, ε is the effective heating number, Q_{ig} is the heat of preignition and Φ_c is formulated as

$$\Phi_c = \sqrt{\left[\Phi_S + \Phi_w \cos\theta\right]^2 + \left[\Phi_w \sin\theta\right]^2}.$$

Here θ is the split angle between upslope direction and the direction the wind is blowing to, Φ_S and Φ_w are slope and wind factors. The direction of maximum spread and effective wind speed are given by

***sdr*:**

$$sdr = \arcsin\left(\frac{\Phi_w \sin\theta}{\Phi_c}\right)$$

***efw*:**

$$efw = \frac{1}{196.85}\left[\frac{\Phi_c}{C(\sigma)\left(\beta/\beta_{opt}\right)^{-E(\sigma)}}\right]^{1/B(\sigma)}$$

Where σ is the surface-area-to-volume ratio, *B*, *C*, *E* are some functions of σ and β is the packing ratio. The total number of parameters is 24.

Global sensitivity analysis

ANOVA-decomposition

$$f(x) = \sum_{u \subseteq \{1,\dots,d\}} f_u(x^u)$$

Variances are defined as:

$$\sigma_u^2 = \int f_u(x^u)^2\, dx^u, \qquad \sigma^2 = \int f(x)^2\, dx - f_\emptyset^2$$

Orthogonality of f_u implies

$$\sigma^2 = \sum_{u \subseteq \{1,\cdots,d\}} \sigma_u^2$$

Global sensitivity indices (Sobol' indices)

$$\underline{S}_u = \frac{1}{\sigma^2}\sum_{v \subseteq u} \sigma_v^2, \qquad \overline{S}_u = \frac{1}{\sigma^2}\sum_{v \cap u \neq \emptyset} \sigma_v^2$$

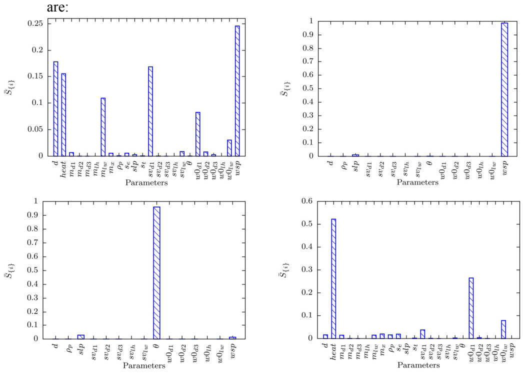
If $\overline{S}_{\{i\}}$ is relatively small, then the corresponding parameter can be frozen at its nominal value. Fixing all insignificant parameters leads to reduced models.

Global sensitivity analysis (GSA) for the Rothermel model

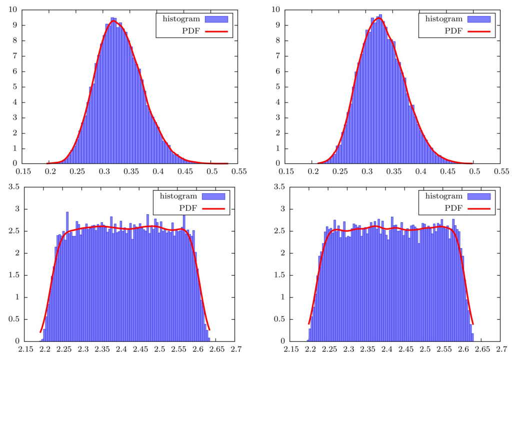
We assume that uncertainties exist in all input parameters and assign to each parameter a uniform distribution with the mean listed in the following table and the standard deviation 5% of the mean. The 5% coefficient of variation is adopted in order for both dead and living fuel damping moistures not to exceed their extinction moistures.

Parameter	Symbol	Value	Unit
fuel bed depth	d	1.83	m
low heat content	*heat*	18622.0	$kJkg^{-1}$
1-h fuel moisture	m_{d1}	8.0	%
10-h fuel moisture	m_{d2}	8.0	%
100-h fuel moisture	m_{d3}	8.0	%
live herbaceous fuel moisture	m_{lh}	150.0	%
live woody fuel moisture	m_{lw}	150.0	%
moisture of extinction	m_x	20	%
particle density	ρ_p	512.5	kgm^{-3}
effective mineral content	s_e	1.0	%
slope	*slp*	14.04	°
total mineral content	s_t	5.55	%
1-h surface area/vol ratio	sv_{d1}	6562.0	m^2m^{-3}
10-h surface area/vol ratio	sv_{d2}	358.0	m^2m^{-3}
100-h surface area/vol ratio	sv_{d3}	98.0	m^2m^{-3}
live herb surface area/vol ratio	sv_{lh}	4921.0	m^2m^{-3}
live woody surface area/vol ratio	sv_{lw}	4921.0	m^2m^{-3}
direction of wind vector	θ	45	°
1-h fuel load	$w0_{d1}$	1.12	kgm^{-2}
10-h fuel load	$w0_{d2}$	0.90	kgm^{-2}
100-h fuel load	$w0_{d3}$	0.45	kgm^{-2}
live herbaceous fuel load	$w0_{lh}$	0	kgm^{-2}
live woody fuel load	$w0_{lw}$	1.12	kgm^{-2}
midflame wind speed	wsp	2.3	ms^{-1}

The upper global sensitivity indices for *ros, efw*, *sdr*, and *ri* (left-right, up-down) are:



To support the results of GSA, the figures below qualitatively contrast for each output the histogram of the full model with the dimension-reduced model (left-right: full model and reduced model; up-down: *ros*, *efw*).



Uncertainty quantification

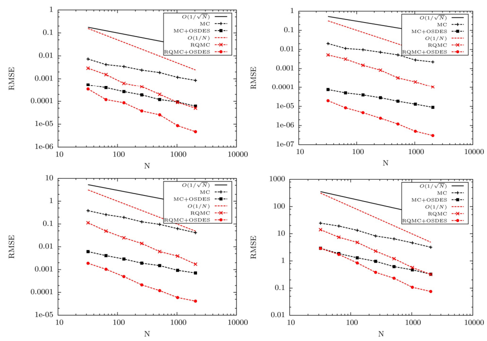
Uncertainty quantification (UQ) is performed for each output with the randomized quasi-Monte Carlo method (random-start scrambled Halton sequence) coupled with optimized sensitivity derivative enhanced sampling.

Optimized sensitivity derivative enhanced sampling (OSDES)

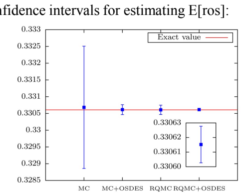
$$\Theta_{f,n}^{OSDES} = \frac{1}{N}\sum_{i=1}^{N}\left(f(x_i) - \beta^*\left(J^{(n)}(x_i) - E\left[J^{(n)}(x)\right]\right)\right)$$

Where $J^{(n)}$ denotes the *n*th order Taylor series and $\beta^* = \frac{\mathrm{cov}(J^{(n)}, f)}{V(J^{(n)})}$.

The following sampling techniques are compared: crude Monte Carlo method (MC), Monte Carlo method coupled with OSDES (MC+OSDES), standard RQMC (RQMC), RQMC coupled with OSDES (RQMC+OSDES). The convergence behaviors for estimating the first moments are shown as follows: (left-right, up-down: *ros*, *efw*, *sdr*, *ri*)



The 95% confidence intervals for estimating E[ros]:



Conclusions

Effective wildland fire management requires fast prediction of potential or ongoing fire. Mathematical models built for predicting fire behavior are based on a number of input fire environment parameters, which are inevitably subject to uncertainties. we proposed using global sensitivity analysis to reduce model complexity, and use optimized SDES, a control variate Monte Carlo approach, together with random-start Halton sequences, a randomized quasi-Monte Carlo method, to simulate the reduced model. Our proposed method improves standard Monte Carlo simulation error by factors as high as three orders of magnitude when applied to the parametric uncertainty quantification of the Rothermel model at a computational overhead of less than 10%. This makes our proposed method significantly more efficient than the crude Monte Carlo sampling.

References

Jimenez E et al. (2008) Quantifying parametric uncertainty in the Rothermel model. *Inter J Wildland Fire* **17**, 638-649.

Cao Y et al. (2006) A variance reduction method based on sensitivity derivatives. *Appl Numer Math* **56**, 800-813.

Sobol' I (2001) Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates. *Math Comput Simulat* 55, 271-280.

Ökten G (2009) Generalized von Neumann-Kakutani transformation and random-start scrambled Halton sequences. *J Complexity* 25, 318-331.