Optimal Trading Strategy With Optimal Horizon

Financial Math Festival
Florida State University
March 1, 2008

Edward Qian
PanAgora Asset Management

Trading – An Integral Part of Investment Process

- **Return forecasting**
- **Portfolio construction**
- **Trading – portfolio implementation**
- **Performance attribution**

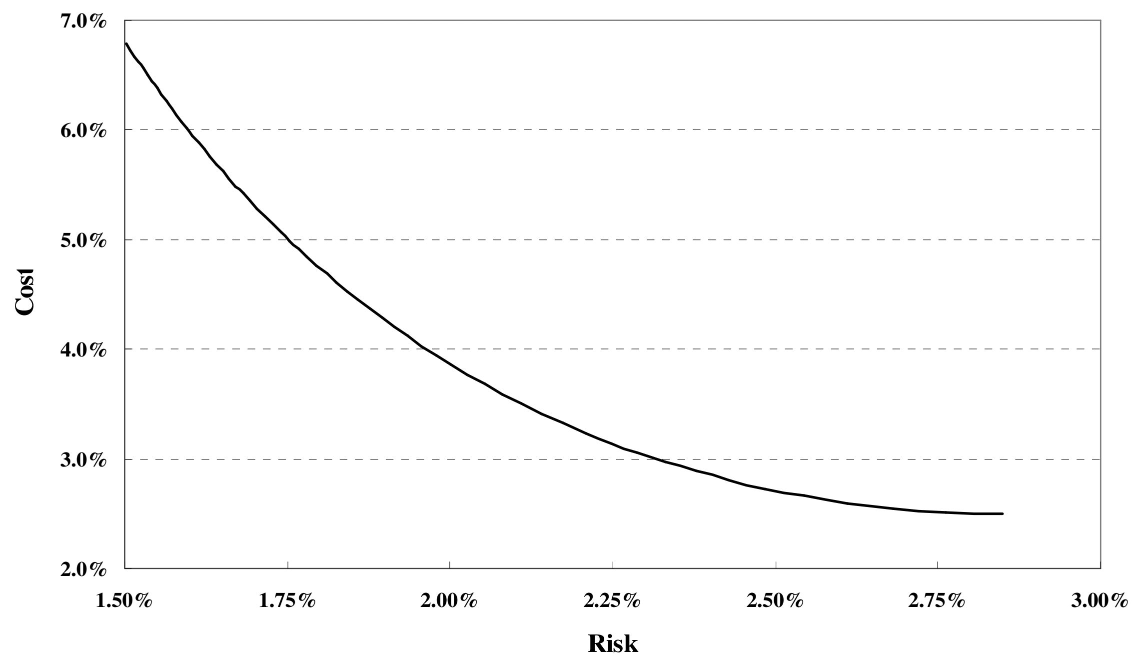
Conflicting Objectives in Trading

➢ Immediacy

- Alpha capture
- Risk reduction
- Labor costs
- Opportunity costs

➢ Costs

- Fees
- Bid/Ask spread
- Market impact



Optimal Trading Strategies

- **Optimal trading path (sequence) with minimum costs for a given level of risk**

$$h^*(t), t \in [0, T], \quad T \text{ is the trading horizon.}$$

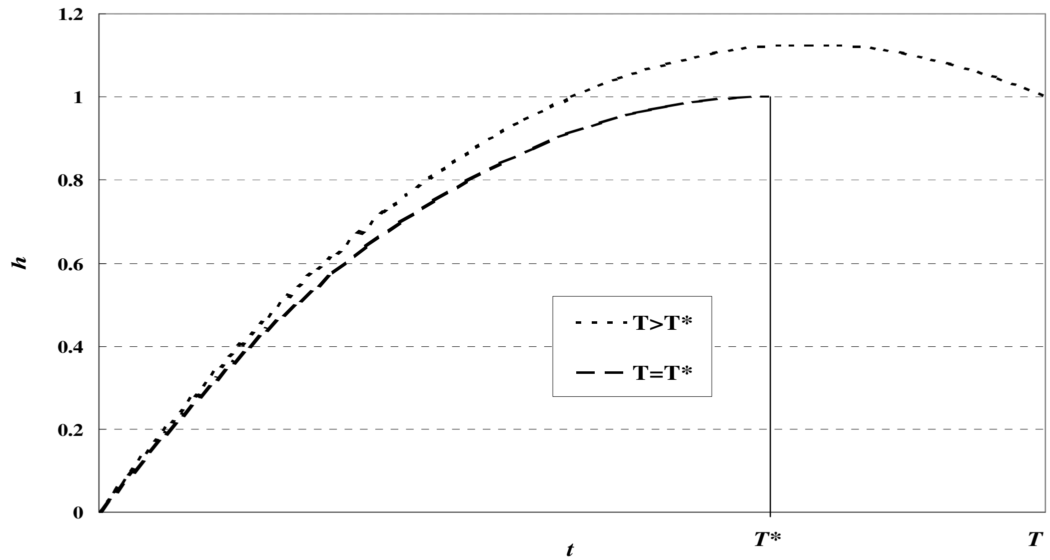
- **Previous researches (Grinold & Kahn 1999, Almgren & Chriss 2000) used a fixed horizon *T***

- **Extension to optimal trading strategy with optimal horizon (Qian 2008 JOIM, Qian, Hua, Sorensen 2007)**

$$h^*(t), t \in [0, T^*].$$

Optimal Horizon - Motivation

- **Horizon is not known in advance**
  - Single stocks versus baskets
- **It is optimal along two dimensions**
- **Flip-floping in optimal trading with fixed horizon**



PanAgora

Mathematical Model - Inputs

- **Trade weight Δw and trade path** $h(t)\Delta w,\ h(0)=0$ and $h(T)=1$
- **Trade shortfall** $h(t)\Delta w - \Delta w = \Delta w\left[h(t)-1\right]$
- **Return shortfall** $f\,\Delta w\left[h(t)-1\right]dt$
- **Shortfall variance** $\sigma^2(\Delta w)^2\left[h(t)-1\right]^2 dt$
- **Fixed cost** $c\left|\Delta w\right|T,\ c>0$
- **Market impact** $\psi(\Delta w)^2\left[\dot{h}(t)\right]^2 dt\ ,\psi>0$

Mathematical Model – Objective Function

- **Find path and horizon** $h^*(t), t \in [0, T^*]$. **that maximize**

$$J = \int_0^T f \Delta w \left[h(t) - 1 \right] dt - \frac{1}{2} \lambda \int_0^T \sigma^2 (\Delta w)^2 \left[h(t) - 1 \right]^2 dt - c \left| \Delta w \right| \int_0^T dt - \psi \int_0^T (\Delta w)^2 \left[\dot{h}(t) \right]^2 dt$$

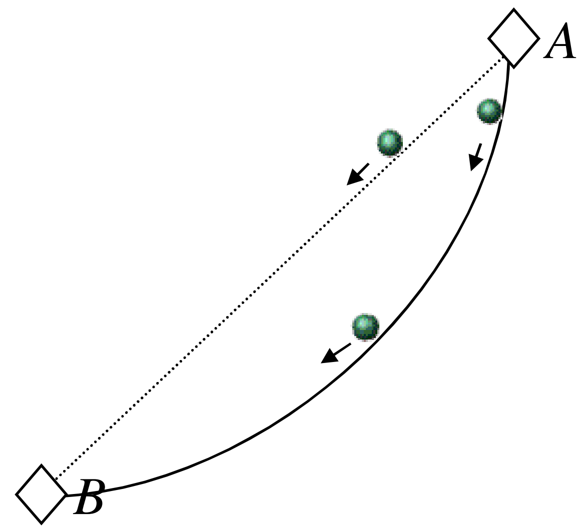
- **Similar to MV optimization that maximizes expected return for a given level of risk**

Mathematical Model – Calculus of Variation

- **Method of calculus of variation**
  - Find optimal function instead of optimal parameter
- **Ordinary differential equation for** $h(t)$
- **Boundary condition for** $h(t)$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{h}}\right] = \frac{\partial L}{\partial h}$$

$$\left. L(h,\dot{h}) - \frac{\partial L(h,\dot{h})}{\partial \dot{h}}\dot{h} \right|_{t=T} = 0$$



Mathematical Model – Equations

- **2nd order ODE**

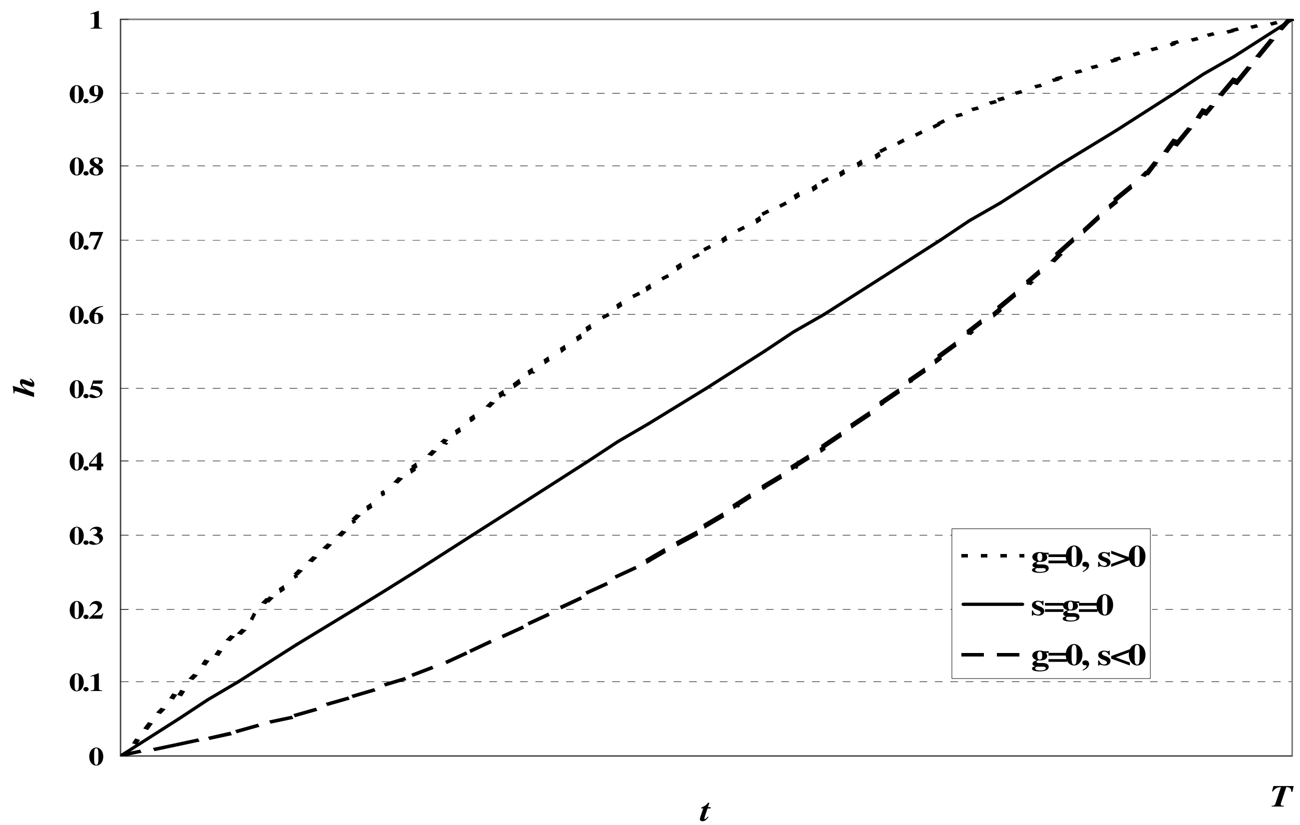
$$\ddot{h} - g^2 h = -s - g^2, \text{ with } s = \frac{f_w}{2\psi},\ g^2 = \frac{\lambda\sigma^2}{2\psi}.$$

$$\dot{h}(T) = \sqrt{\frac{c_w}{\psi}} \equiv p$$

- **Solution consists of exponential functions with parameters *s*, *g*, and *p***

Solution – No Risk Aversion

- **Three different expected returns**
  - Zero risk aversion, g=0, $s = f_w / 2\psi$



Solution – No Risk Aversion

- **Optimal horizon**

$$T^* = \frac{2\sqrt{\psi}}{\sqrt{c} + \sqrt{c+f}}$$

- **Horizon should be longer if**
  - Market impact is high
  - Fixed cost is low
  - Return is low (if it agrees with the trade)

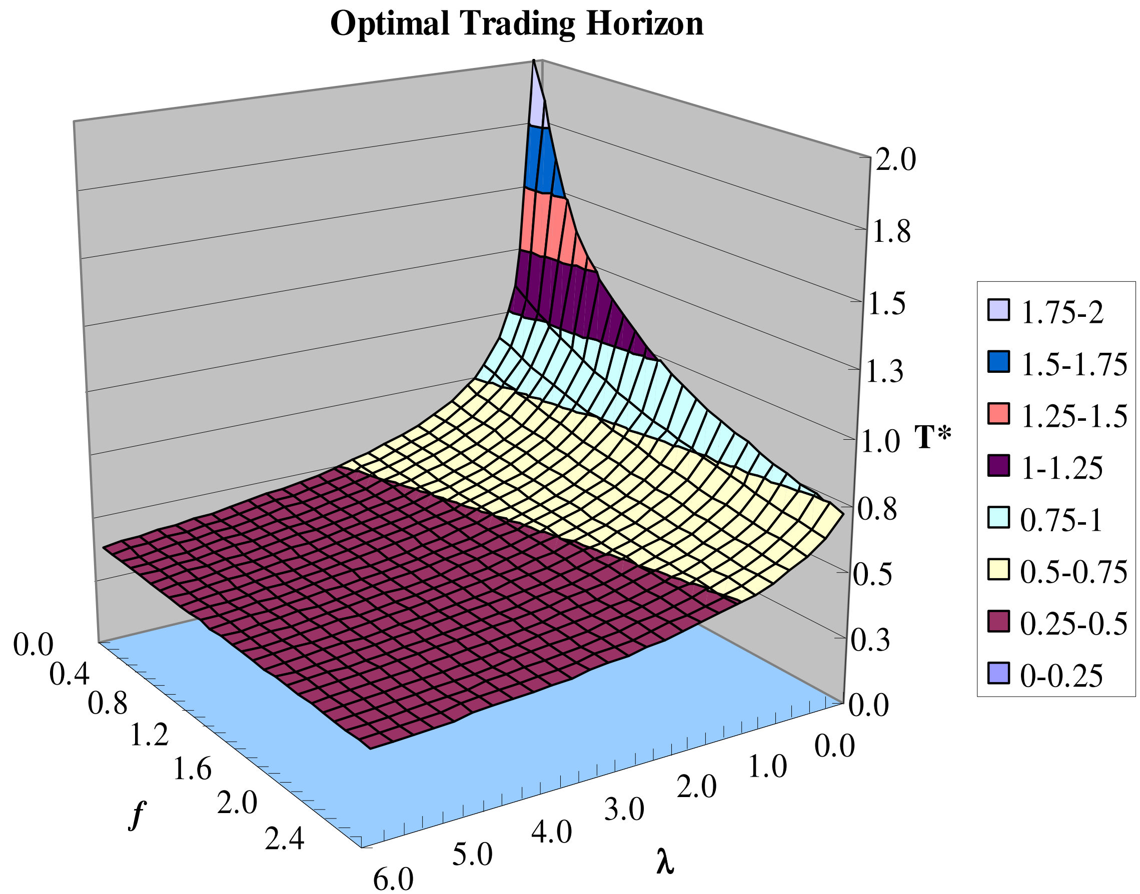
Numerical Examples – Single Stock

- **Base parameter assumption. Optimal horizon = 0.52 day**

f	1% / day
σ	4% / day
λ	2 day / %
c	0.1% / day
ψ	0.5 % day
$s = f/2\psi$	1 / day^2
$g = \sqrt{\lambda\sigma^2/2\psi}$	5.7 / day
$p = \sqrt{c/\psi}$	0.45 / day

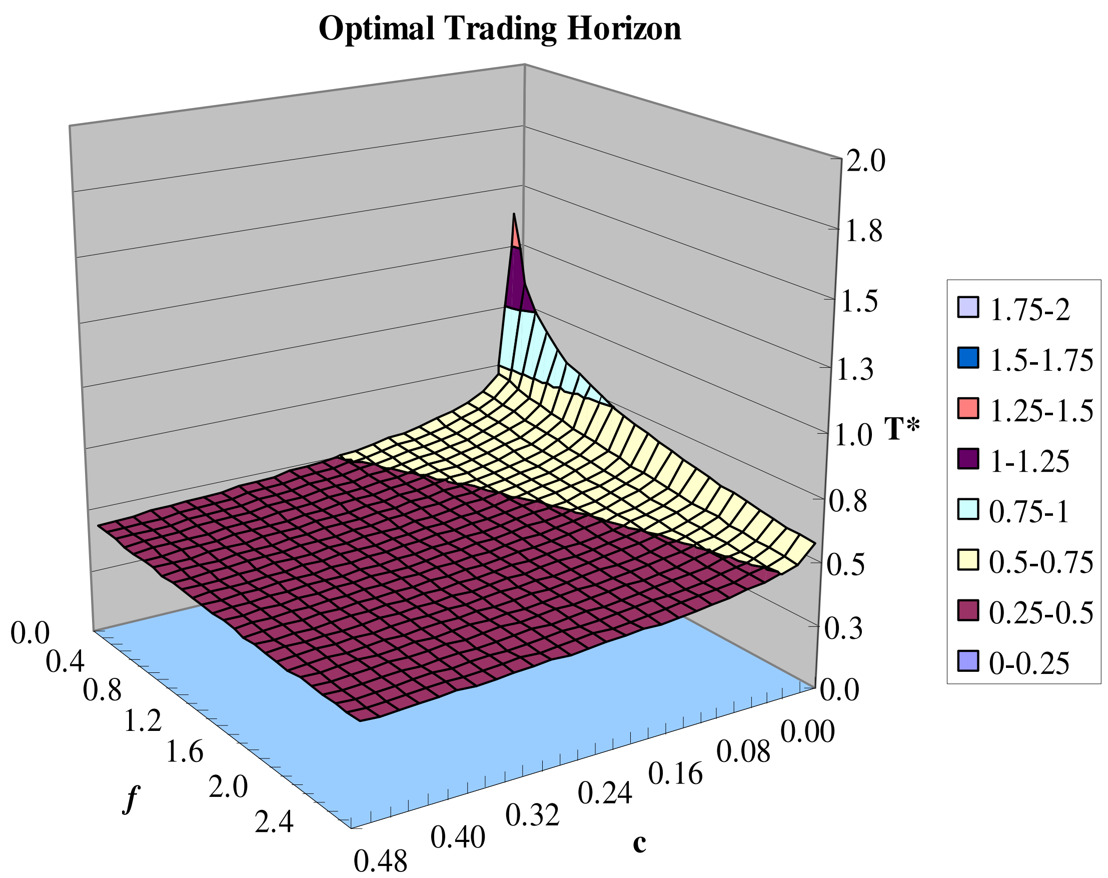
Numerical Examples – Single Stock

➢ Changing parameters – case I



Numerical Examples – Single Stock

➢Changing parameters – case II



PanAgora

Portfolios of Stocks – Objective Function and Solution

- **Find path and horizon,** $\mathbf{h}^*(t), t \in [0, T^*]$ **that maximize**

$$J = \int_0^T L(\mathbf{h}, \dot{\mathbf{h}})\, dt$$

$$L(\mathbf{h}, \dot{\mathbf{h}}) = \mathbf{f}'_{\mathbf{w}} \cdot [\mathbf{h} - \mathbf{1}] - \frac{1}{2}\lambda [\mathbf{h} - \mathbf{1}]' \boldsymbol{\Sigma}_{\mathbf{w}} [\mathbf{h} - \mathbf{1}] - c_{\mathbf{w}} - \dot{\mathbf{h}}' \boldsymbol{\Psi}_{\mathbf{w}} \dot{\mathbf{h}}$$

- **A system of second order linear ODE's, which can be solved numerically**

$$\mathbf{h}(t) = \sum_{i=1}^{2N} e^{p_i t} \mathbf{a} + \mathbf{q}(t)$$

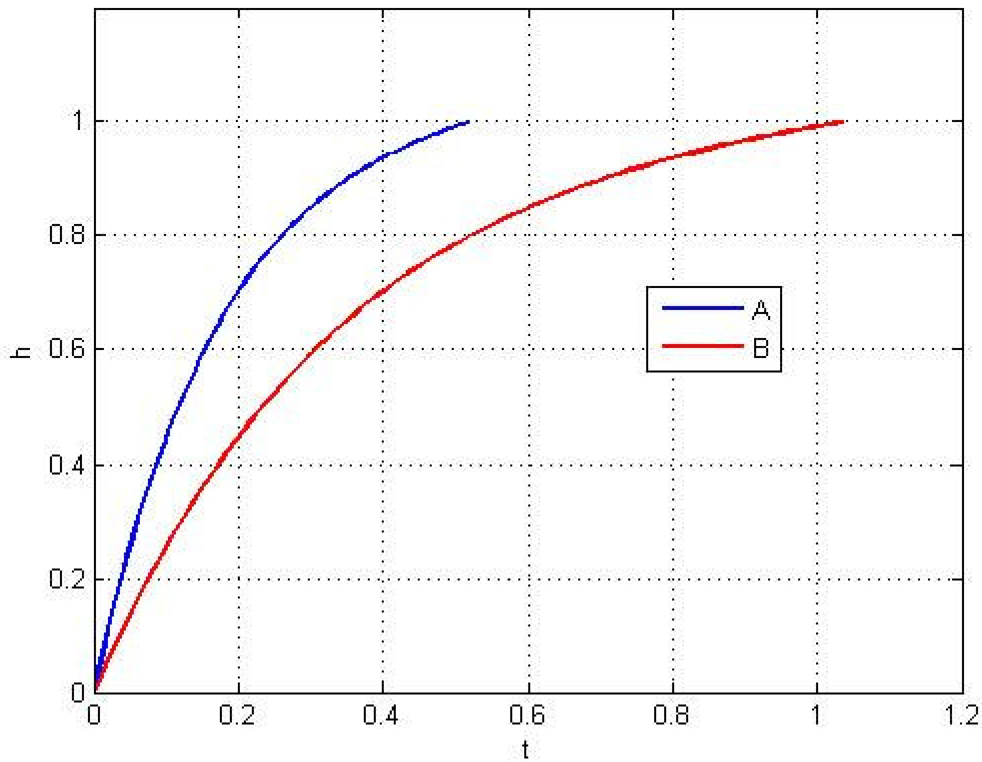
Numerical Examples – Two Stocks

- **Base parameter assumption**

	Stock A	**Stock B**
f	1% / day	1% / day
σ	4% / day	4% / day
λ	2 day / %	2 day / %
c	0.1% / day	0.1% / day
ψ	**0.5 % day**	**2 % day**
$s = f/2\psi$	1 / day^2	0.25 / day^2
$g = \sqrt{\lambda\sigma^2/2\psi}$	5.7 / day	2.8 / day
$p = \sqrt{c/\psi}$	0.45 / day	0.22 / day
T^*	**0.52 day**	**1.04 day**

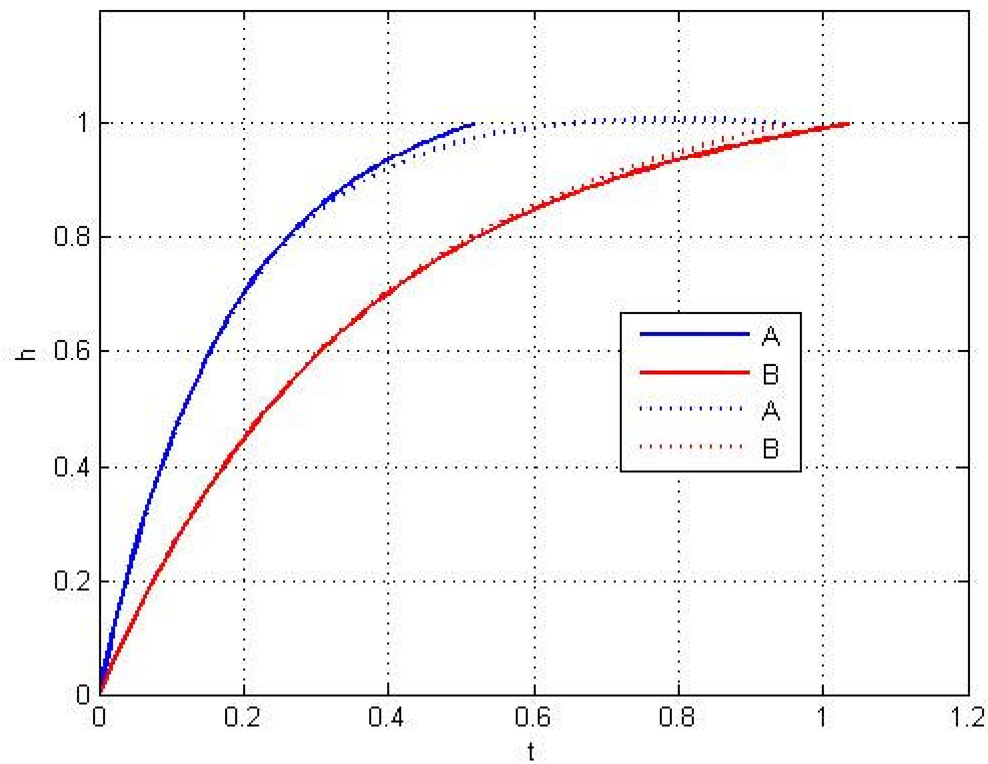
Numerical Examples – Two Stocks

- **Individual paths**



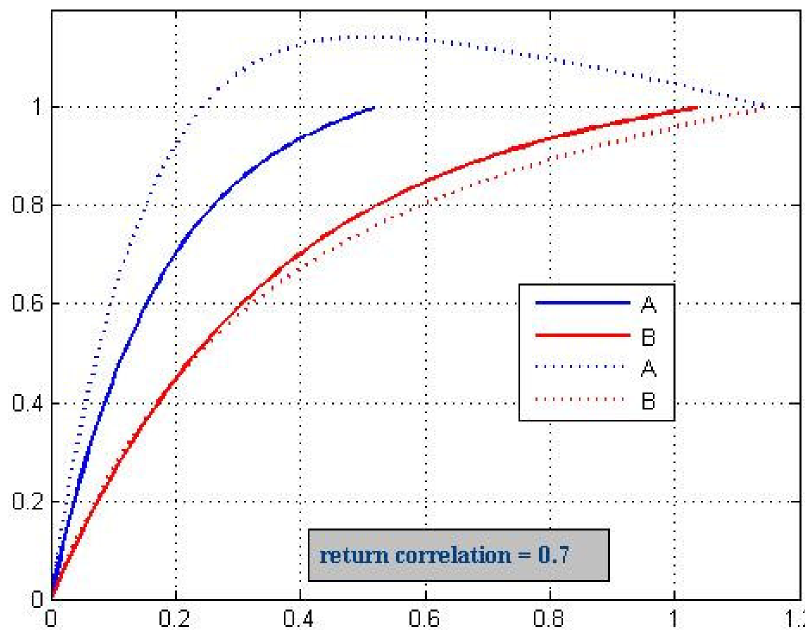
Numerical Examples – Two Stocks

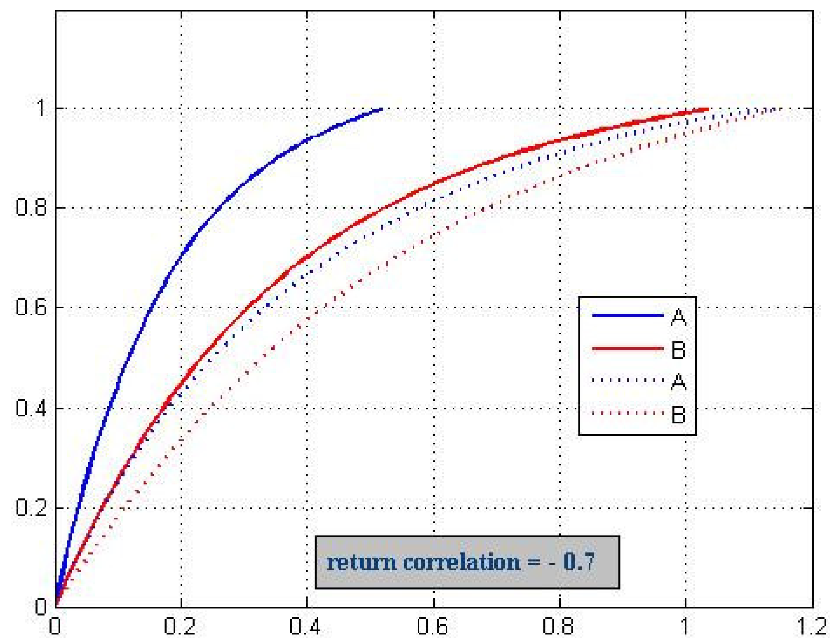
- **Combined path – zero correlations**



Numerical Examples – Two Stocks

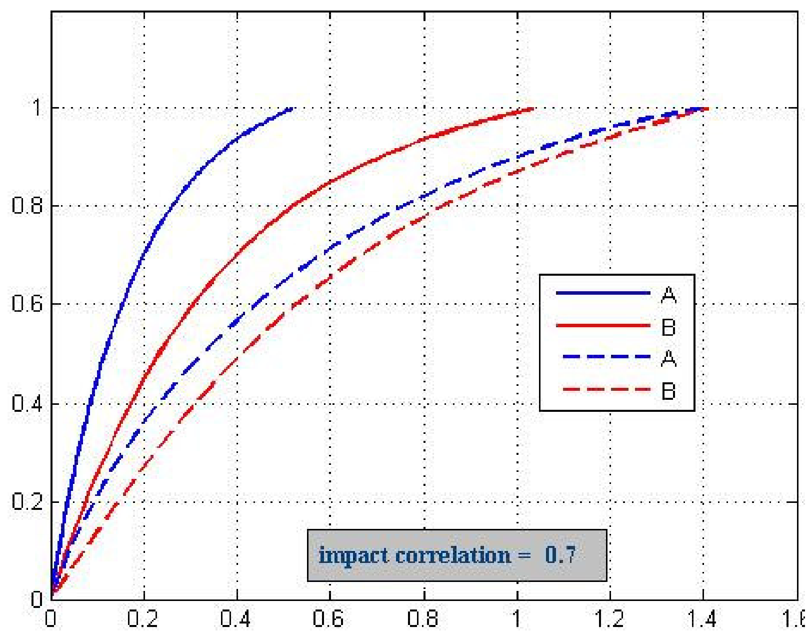
➢Combined path – non-zero return correlations

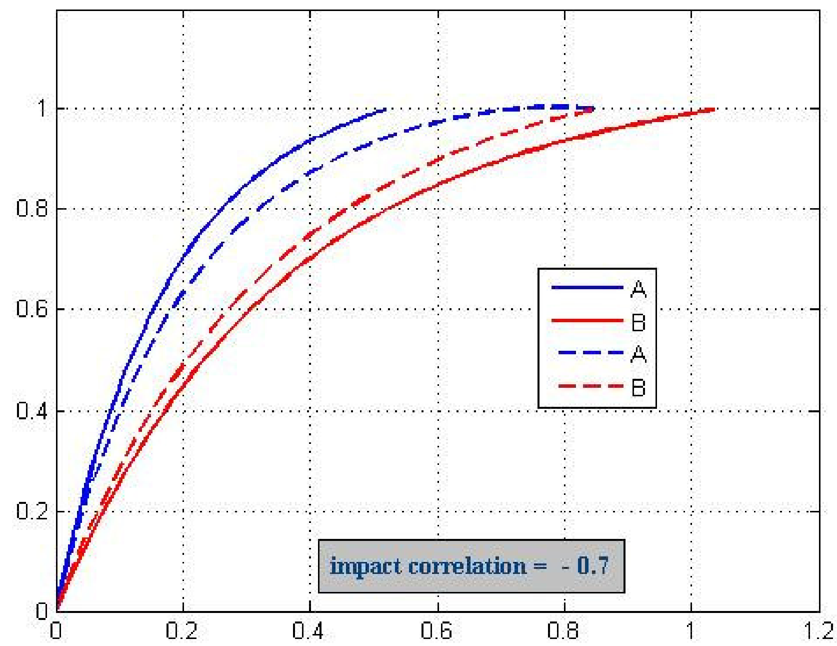




Numerical Examples – Two Stocks

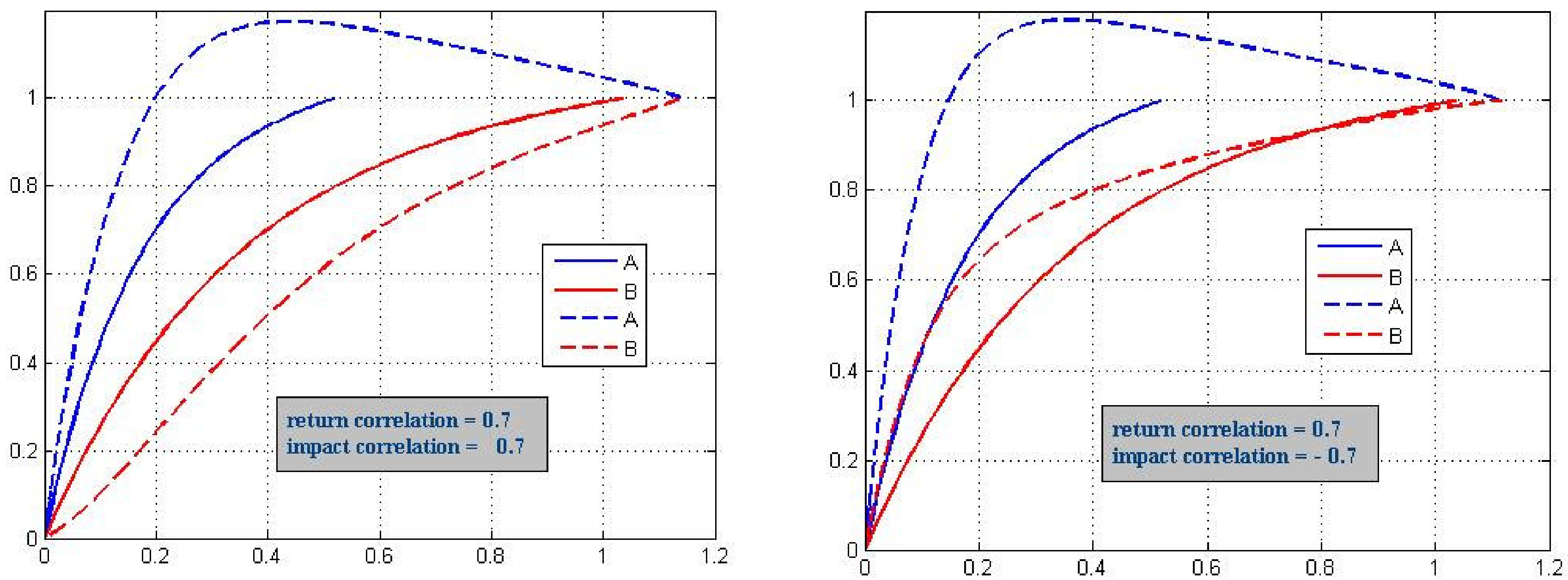
➢Combined path – non-zero impact correlations





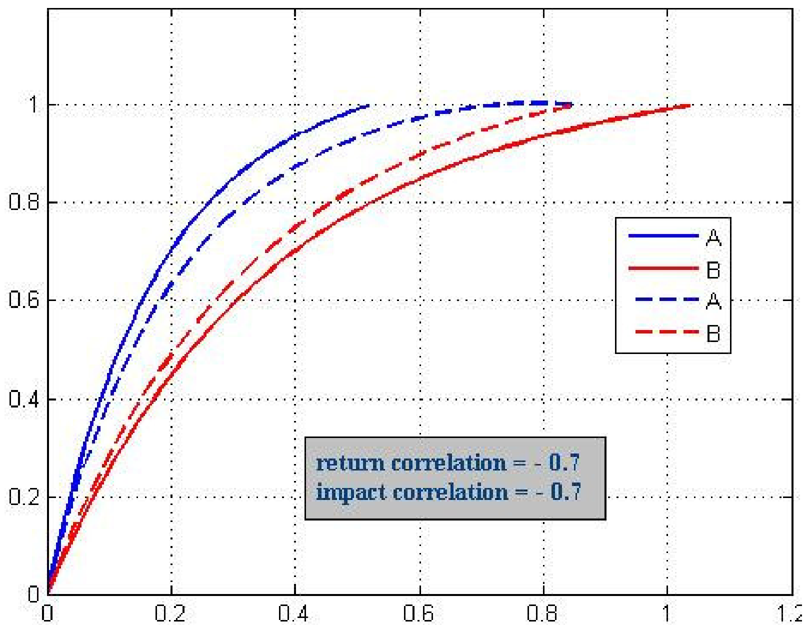
Numerical Examples – Two Stocks

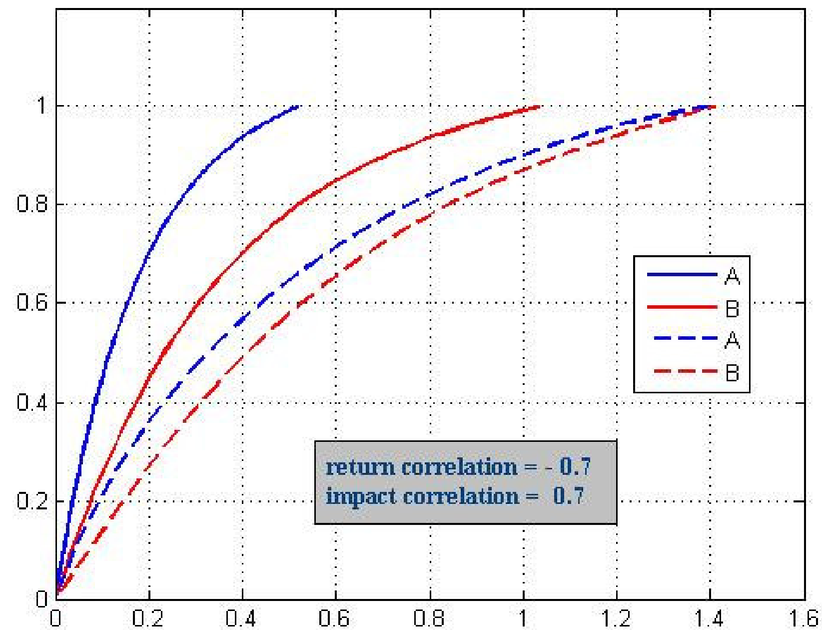
➢Combined path – non-zero correlations



Numerical Examples – Two Stocks

➢Combined path – non-zero correlations





Summary

- **There is often an optimal trading horizon with optimal trading strategy**
- **Our analytic solution shows the optimal horizon depends on**
  - Expected return, stock volatility, fixed cost, market impact
  - Correlations play a significant role for stock portfolios
- **Further research**
  - Portfolio constraints: dollar neutral, sector neutral
  - No reverse trading