Optimal Trading Strategy
With Optimal Horizon

Financial Math Festival
Florida State University
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Trading – An Integral Part of Investment Process

- Return forecasting
- Portfolio construction
- Trading – portfolio implementation
- Performance attribution
Conflicting Objectives in Trading

- **Immediacy**
  - Alpha capture
  - Risk reduction
  - Labor costs
  - Opportunity costs

- **Costs**
  - Fees
  - Bid/Ask spread
  - Market impact
**Optimal Trading Strategies**

- Optimal trading path (sequence) with minimum costs for a given level of risk
  \[ h^* (t), t \in [0, T], \quad T \text{ is the trading horizon.} \]

- Previous researches (Grinold & Kahn 1999, Almgren & Chriss 2000) used a fixed horizon \( T \)

- Extension to optimal trading strategy with optimal horizon (Qian 2008 JOIM, Qian, Hua, Sorensen 2007)
  \[ h^* (t), t \in [0, T^*]. \]
Optimal Horizon - Motivation

- Horizon is not known in advance
  - Single stocks versus baskets
- It is optimal along two dimensions
- Flip-floping in optimal trading with fixed horizon
**Mathematical Model - Inputs**

- **Trade weight** $\Delta w$ and trade path $h(t)\Delta w$, $h(0) = 0$ and $h(T) = 1$
- **Trade shortfall** $h(t)\Delta w - \Delta w = \Delta w[h(t) - 1]$

- **Return shortfall** $\int \Delta w[h(t) - 1] \, dt$
- **Shortfall variance** $\sigma^2 (\Delta w)^2 \left[ h(t) - 1 \right]^2 \, dt$
- **Fixed cost** $c|\Delta w|T$, $c > 0$
- **Market impact** $\psi (\Delta w)^2 \left[ \dot{h}(t) \right]^2 \, dt$, $\psi > 0$
Mathematical Model – Objective Function

➢ Find path and horizon \( h^*(t), t \in [0, T^*] \). that maximize

\[
J = \int_0^T f \Delta w \left[ h(t) - 1 \right] dt - \frac{1}{2} \lambda \int_0^T \sigma^2 (\Delta w)^2 \left[ h(t) - 1 \right]^2 dt - c|\Delta w| \int_0^T dt - \psi \int_0^T (\Delta w)^2 \left[ h(t) \right]^2 dt
\]

➢ Similar to MV optimization that maximizes expected return for a given level of risk
Mathematical Model – Calculus of Variation

- Method of calculus of variation
  - Find optimal function instead of optimal parameter

- Ordinary differential equation for \( h(t) \)

- Boundary condition for \( h(t) \)

\[
\frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{h}} \right] = \frac{\partial L}{\partial h}
\]

\[
L(h, \dot{h}) - \frac{\partial L}{\partial \dot{h}} \dot{h} \bigg|_{t=T} = 0
\]
Mathematical Model – Equations

2\textsuperscript{nd} order ODE

\[ \ddot{h} - g^2h = -s - g^2, \quad \text{with} \quad s = \frac{f_w}{2\psi}, \quad g^2 = \frac{\lambda \sigma^2}{2\psi}. \]

\[ \dot{h}(T) = \sqrt{\frac{c_w}{\psi}} \equiv p \]

Solution consists of exponential functions with parameters \( s, g, \) and \( p \)
Solution – No Risk Aversion

➢ Three different expected returns
  • Zero risk aversion, $g=0$, $s = \frac{f_w}{2\psi}$
Solution – No Risk Aversion

Optimal horizon

\[ T^* = \frac{2\sqrt{\psi}}{\sqrt{c + \sqrt{c + f}}} \]

Horizon should be longer if

- Market impact is high
- Fixed cost is low
- Return is low (if it agrees with the trade)
**Numerical Examples – Single Stock**

- **Base parameter assumption. Optimal horizon = 0.52 day**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>1% / day</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>4% / day</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>2 day / %</td>
</tr>
<tr>
<td>( c )</td>
<td>0.1% / day</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.5 % day</td>
</tr>
<tr>
<td>( s = f / 2\psi )</td>
<td>1 / day²</td>
</tr>
<tr>
<td>( g = \sqrt{\lambda \sigma^2 / 2\psi} )</td>
<td>5.7 / day</td>
</tr>
<tr>
<td>( p = \sqrt{c / \psi} )</td>
<td>0.45 / day</td>
</tr>
</tbody>
</table>
Numerical Examples – Single Stock

Changing parameters – case I

Optimal Trading Horizon

- $T^*$
- $f$
- $\lambda$

Legend:
- 1.75-2
- 1.5-1.75
- 1.25-1.5
- 1.0-1.25
- 0.75-1
- 0.5-0.75
- 0.25-0.5
- 0-0.25
Numerical Examples – Single Stock

Changing parameters – case II

Optimal Trading Horizon

1.75-2
1.5-1.75
1.25-1.5
1-1.25
0.75-1
0.5-0.75
0.25-0.5
0-0.25
**Portfolios of Stocks – Objective Function and Solution**

- **Find path and horizon, $h^*(t), t \in [0, T^*]$ that maximize**
  
  $$J = \int_0^T L(h, \dot{h}) dt$$

  $$L(h, \dot{h}) = f'_w [h - 1] - \frac{1}{2} \lambda [h - 1]' \Sigma_w [h - 1] - c_w - h' \Psi_w h$$

- **A system of second order linear ODE’s, which can be solved numerically**

  $$h(t) = \sum_{i=1}^{2N} e^{Pi t} a + q(t)$$
### Numerical Examples – Two Stocks

#### Base parameter assumption

<table>
<thead>
<tr>
<th></th>
<th>Stock A</th>
<th>Stock B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>1% / day</td>
<td>1% / day</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>4% / day</td>
<td>4% / day</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>2 day / %</td>
<td>2 day / %</td>
</tr>
<tr>
<td>$c$</td>
<td>0.1% / day</td>
<td>0.1% / day</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.5 % day</td>
<td>2 % day</td>
</tr>
<tr>
<td>$s = f / 2\psi$</td>
<td>1 / day$^2$</td>
<td>0.25 / day$^2$</td>
</tr>
<tr>
<td>$g = \sqrt{\lambda \sigma^2 / 2\psi}$</td>
<td>5.7 / day</td>
<td>2.8 / day</td>
</tr>
<tr>
<td>$p = \sqrt{c / \psi}$</td>
<td>0.45 / day</td>
<td>0.22 / day</td>
</tr>
<tr>
<td>$T^*$</td>
<td>0.52 day</td>
<td>1.04 day</td>
</tr>
</tbody>
</table>
Numerical Examples – Two Stocks

Individual paths
Numerical Examples – Two Stocks

- Combined path – zero correlations
**Numerical Examples – Two Stocks**

- Combined path – non-zero return correlations

![Graphs showing combined paths with different return correlations](image-url)
Numerical Examples – Two Stocks

➢ Combined path – non-zero impact correlations

![Graph showing combined paths with non-zero impact correlations](image)
Numerical Examples – Two Stocks

- Combined path – non-zero correlations

![Graphs showing combined paths for two stocks with non-zero correlations.](image)
Numerical Examples – Two Stocks

- Combined path – non-zero correlations
Summary

- There is often an optimal trading horizon with optimal trading strategy.

- Our analytic solution shows the optimal horizon depends on:
  - Expected return, stock volatility, fixed cost, market impact
  - Correlations play a significant role for stock portfolios

Further research

- Portfolio constraints: dollar neutral, sector neutral
- No reverse trading
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