

Optimal Trading Strategy With Optimal Horizon

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Trading – An Integral Part of Investment Process

- **Return forecasting**
- **Portfolio construction**
- **Trading – portfolio implementation**
- **Performance attribution**

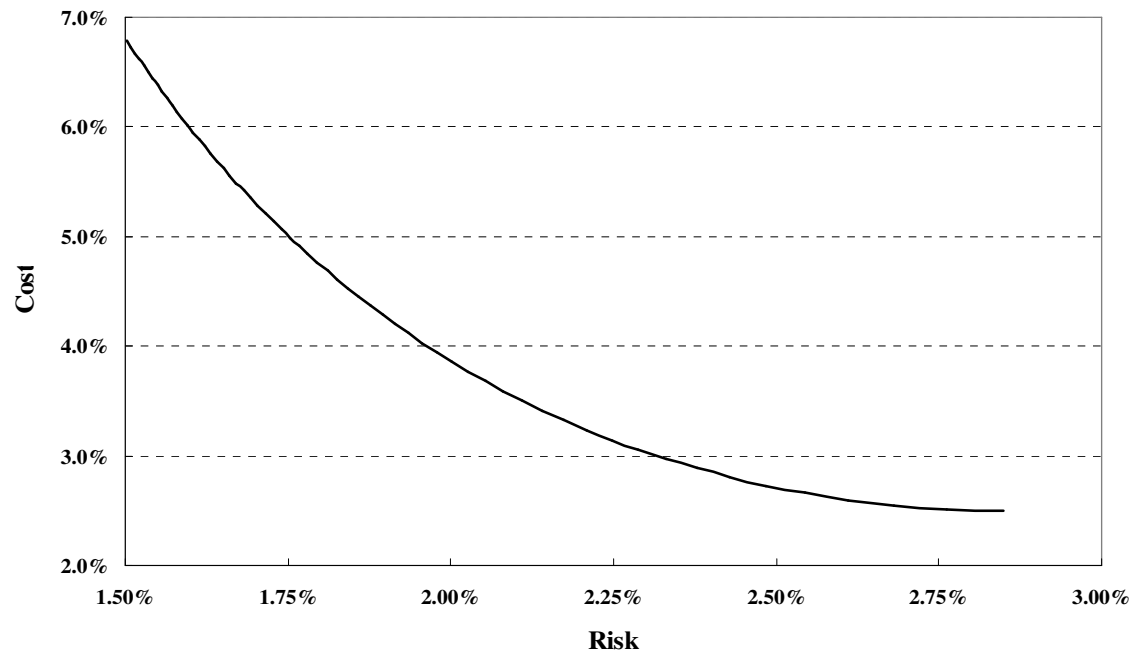
Conflicting Objectives in Trading

➤ **Immediacy**

- Alpha capture
- Risk reduction
- Labor costs
- Opportunity costs

➤ **Costs**

- Fees
- Bid/Ask spread
- Market impact



Optimal Trading Strategies

- **Optimal trading path (sequence) with minimum costs for a given level of risk**

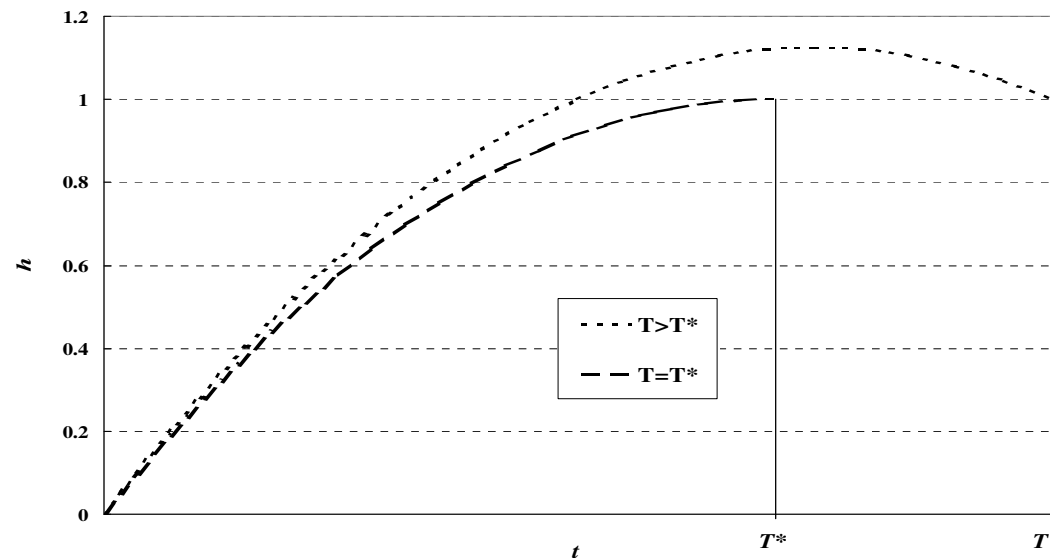
$$h^*(t), t \in [0, T], \quad T \text{ is the trading horizon.}$$

- **Previous researches (Grinold & Kahn 1999, Almgren & Chriss 2000) used a fixed horizon T**
- **Extension to optimal trading strategy with optimal horizon (Qian 2008 JOIM, Qian, Hua, Sorensen 2007)**

$$h^*(t), t \in [0, T^*].$$

Optimal Horizon - Motivation

- **Horizon is not known in advance**
 - Single stocks versus baskets
- **It is optimal along two dimensions**
- **Flip-floping in optimal trading with fixed horizon**



Mathematical Model - Inputs

- **Trade weight Δw and trade path** $h(t)\Delta w$, $h(0) = 0$ and $h(T) = 1$
- **Trade shortfall** $h(t)\Delta w - \Delta w = \Delta w[h(t) - 1]$
- **Return shortfall** $f\Delta w[h(t) - 1]dt$
- **Shortfall variance** $\sigma^2(\Delta w)^2[h(t) - 1]^2 dt$
- **Fixed cost** $c|\Delta w|T$, $c > 0$
- **Market impact** $\psi(\Delta w)^2[\dot{h}(t)]^2 dt$, $\psi > 0$

Mathematical Model – Objective Function

➤ Find path and horizon $h^*(t), t \in [0, T^*]$ that maximize

$$J = \int_0^T f \Delta w [h(t) - 1] dt - \frac{1}{2} \lambda \int_0^T \sigma^2 (\Delta w)^2 [h(t) - 1]^2 dt - c |\Delta w| \int_0^T dt - \psi \int_0^T (\Delta w)^2 [\dot{h}(t)]^2 dt$$

➤ Similar to MV optimization that maximizes expected return for a given level of risk

Mathematical Model – Calculus of Variation

➤ **Method of calculus of variation**

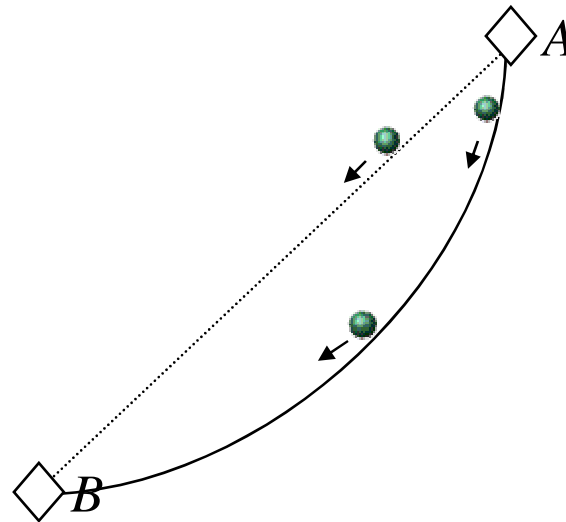
- Find optimal function instead of optimal parameter

➤ **Ordinary differential equation for $h(t)$**

➤ **Boundary condition for $h(t)$**

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{h}} \right] = \frac{\partial L}{\partial h}$$

$$L(h, \dot{h}) - \frac{\partial L(h, \dot{h})}{\partial \dot{h}} \dot{h} \bigg|_{t=T} = 0$$



Mathematical Model – Equations

➤ 2nd order ODE

$$\ddot{h} - g^2 h = -s - g^2, \text{ with } s = \frac{f_w}{2\psi}, \quad g^2 = \frac{\lambda\sigma^2}{2\psi}.$$

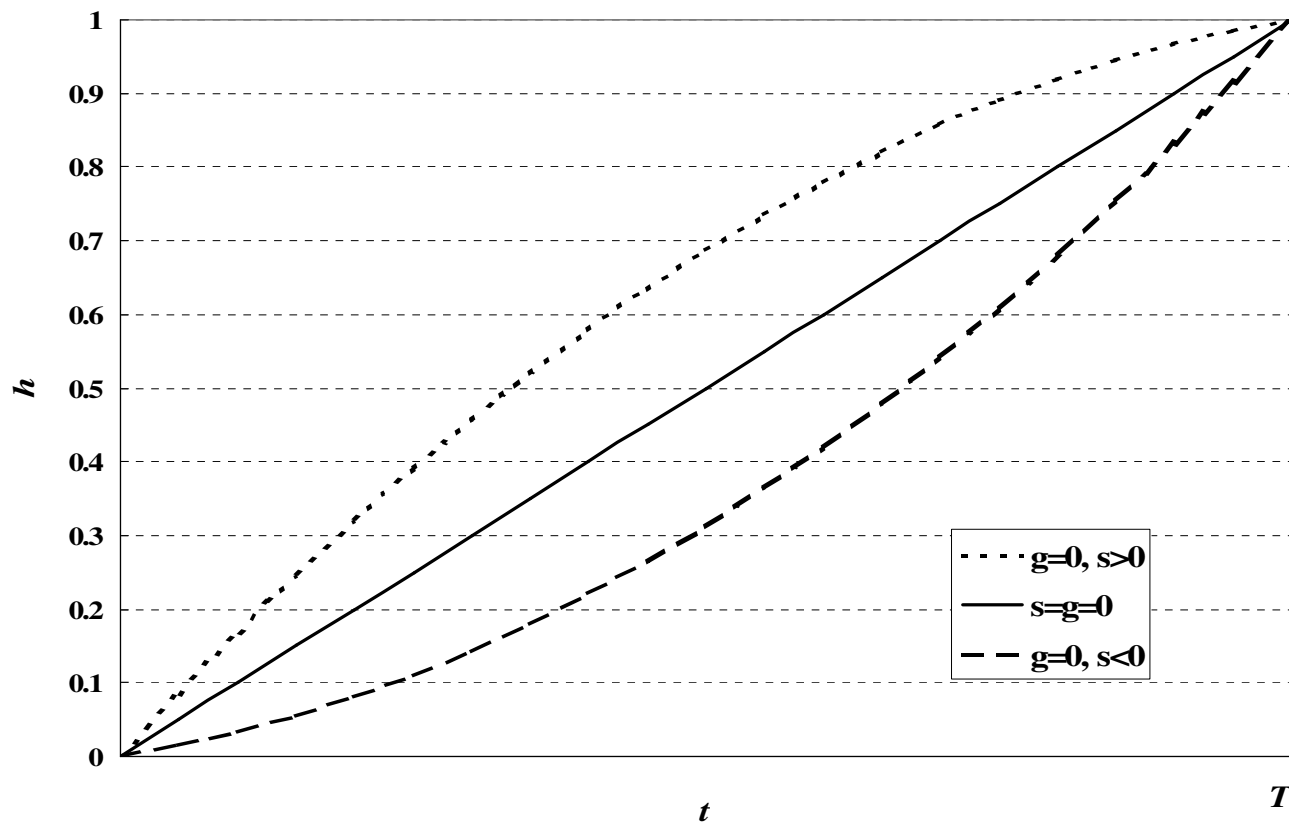
$$\dot{h}(T) = \sqrt{\frac{c_w}{\psi}} \equiv p$$

➤ **Solution consists of exponential functions with parameters s , g , and p**

Solution – No Risk Aversion

➤ Three different expected returns

- Zero risk aversion, $g=0$, $s = f_w/2\psi$



Solution – No Risk Aversion

➤ **Optimal horizon**

$$T^* = \frac{2\sqrt{\psi}}{\sqrt{c} + \sqrt{c+f}}$$

➤ **Horizon should be longer if**

- Market impact is high
- Fixed cost is low
- Return is low (if it agrees with the trade)

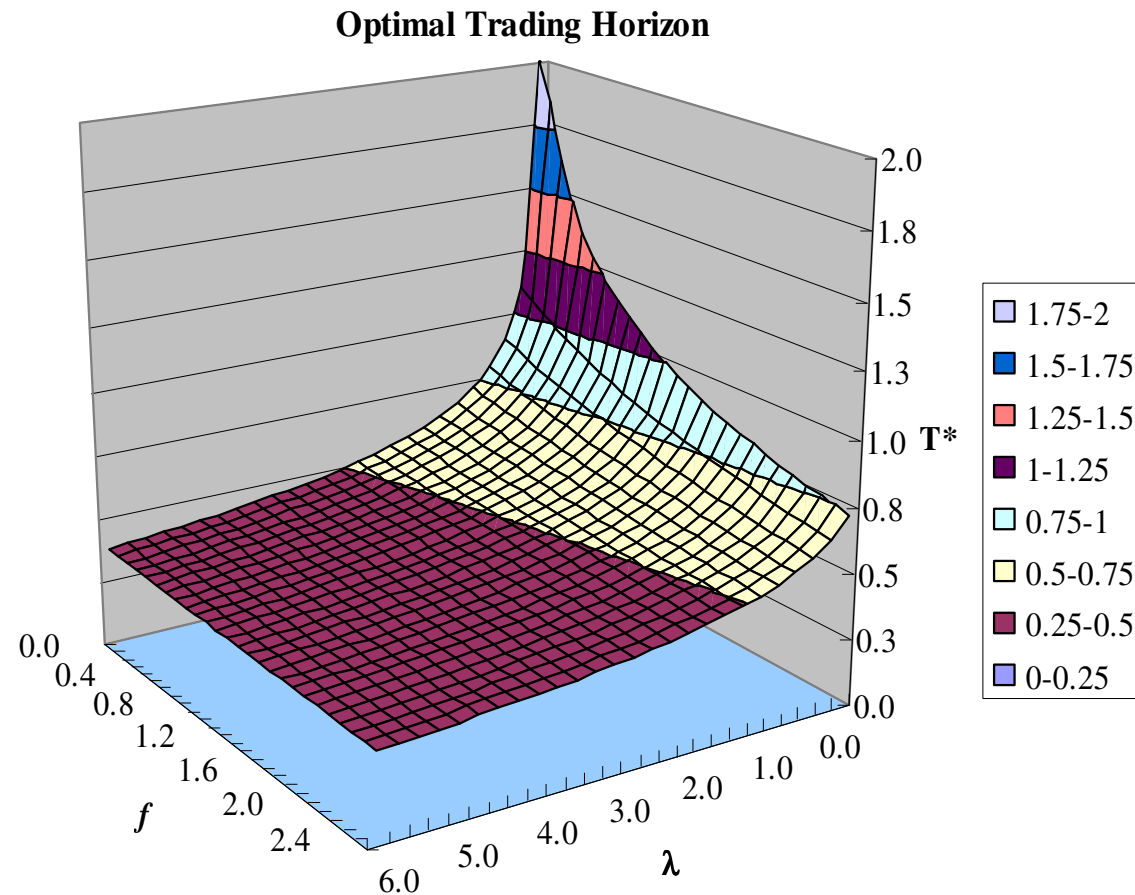
Numerical Examples – Single Stock

➤ **Base parameter assumption. Optimal horizon = 0.52 day**

f	1% / day
σ	4% / day
λ	2 day / %
c	0.1% / day
ψ	0.5 % day
$s = f/2\psi$	1 / day ²
$g = \sqrt{\lambda\sigma^2/2\psi}$	5.7 / day
$p = \sqrt{c/\psi}$	0.45 / day

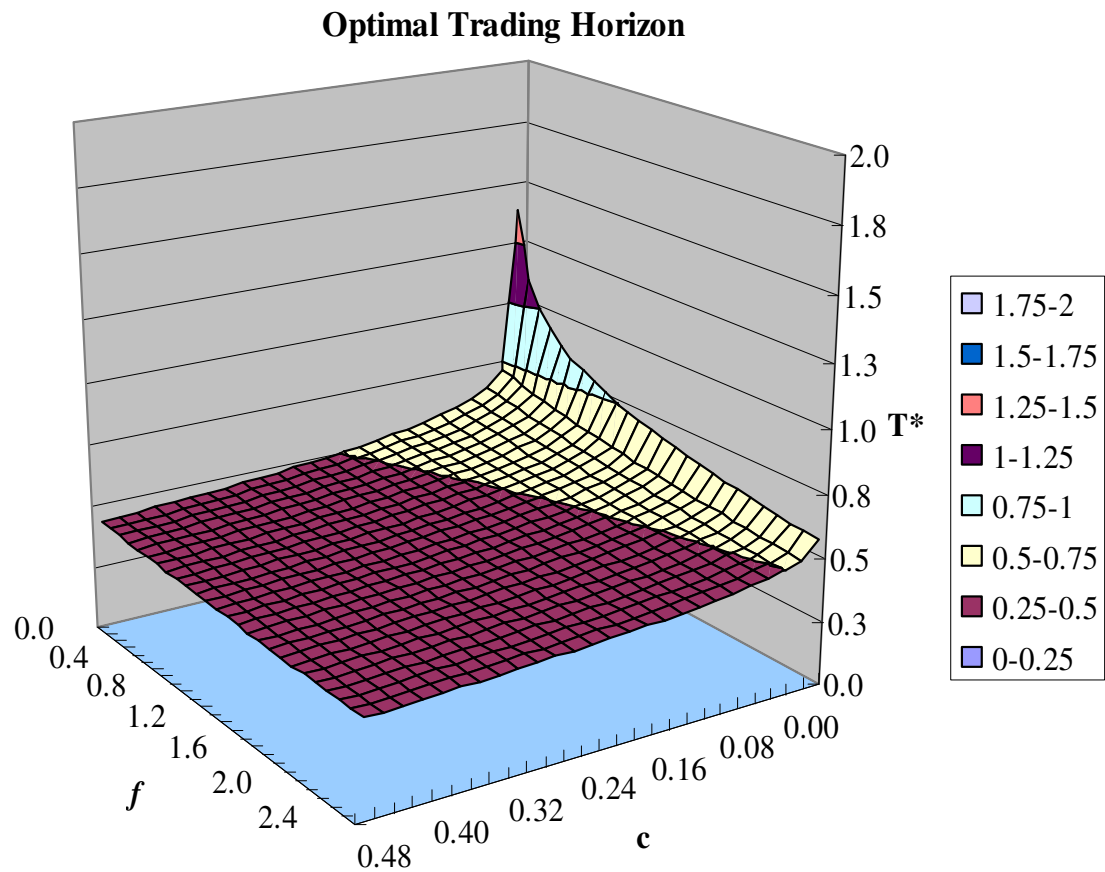
Numerical Examples – Single Stock

➤ Changing parameters – case I



Numerical Examples – Single Stock

➤ Changing parameters – case II



Portfolios of Stocks – Objective Function and Solution

➤ Find path and horizon, $\mathbf{h}^*(t), t \in [0, T^*]$ that maximize

$$J = \int_0^T L(\mathbf{h}, \dot{\mathbf{h}}) dt$$

$$L(\mathbf{h}, \dot{\mathbf{h}}) = \mathbf{f}'_{\mathbf{w}} \cdot [\mathbf{h} - \mathbf{1}] - \frac{1}{2} \lambda [\mathbf{h} - \mathbf{1}]' \Sigma_{\mathbf{w}} [\mathbf{h} - \mathbf{1}] - c_{\mathbf{w}} - \dot{\mathbf{h}}' \Psi_{\mathbf{w}} \mathbf{h}$$

➤ A system of second order linear ODE's, which can be solved numerically

$$\mathbf{h}(t) = \sum_{i=1}^{2N} e^{p_i t} \mathbf{a} + \mathbf{q}(t)$$

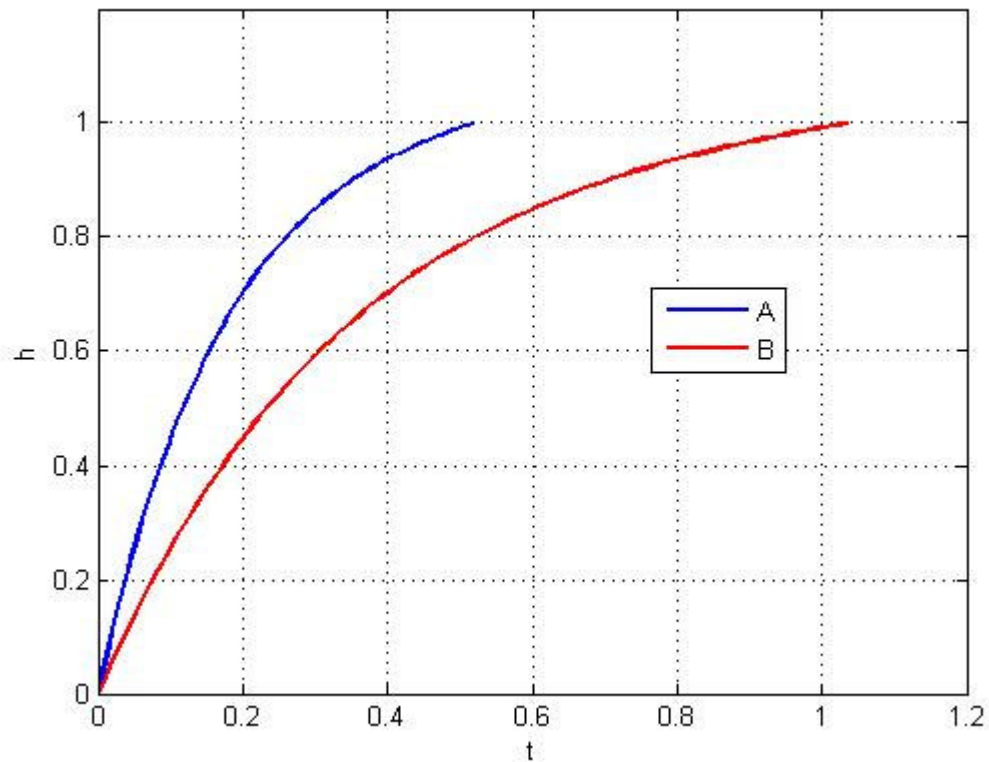
Numerical Examples – Two Stocks

➤ Base parameter assumption

	Stock A	Stock B
f	1% / day	1% / day
σ	4% / day	4% / day
λ	2 day / %	2 day / %
c	0.1% / day	0.1% / day
ψ	<u>0.5 % day</u>	<u>2 % day</u>
$s = f / 2\psi$	1 / day ²	0.25 / day ²
$g = \sqrt{\lambda\sigma^2 / 2\psi}$	5.7 / day	2.8 / day
$p = \sqrt{c/\psi}$	0.45 / day	0.22 / day
T^*	<u>0.52 day</u>	<u>1.04 day</u>

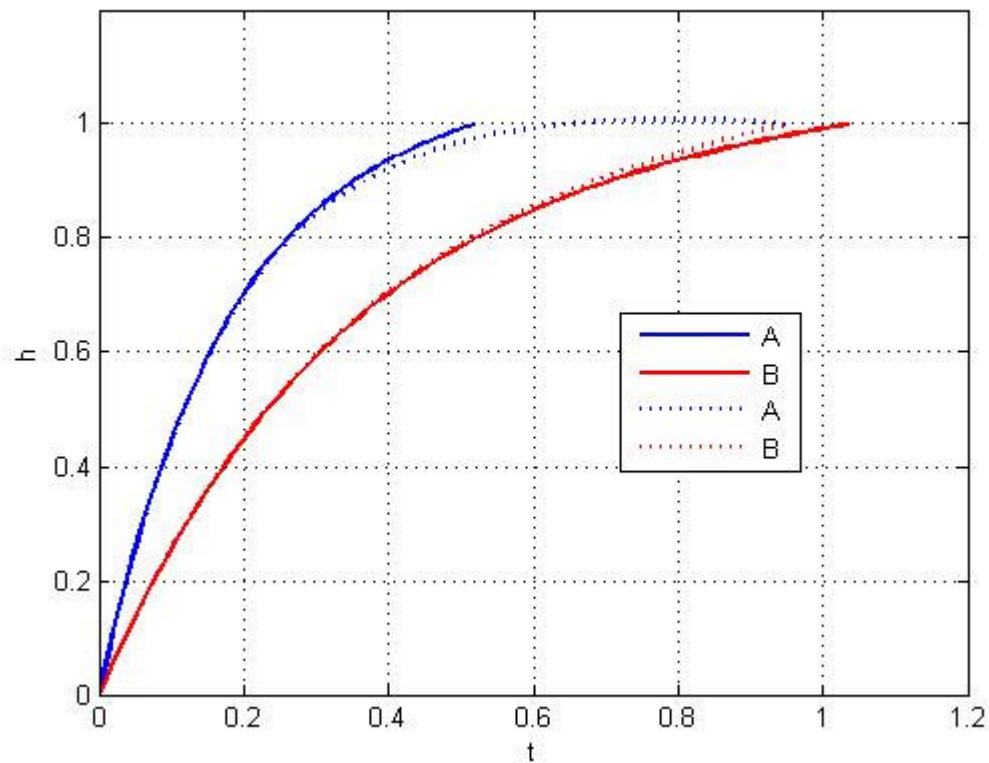
Numerical Examples – Two Stocks

➤ Individual paths



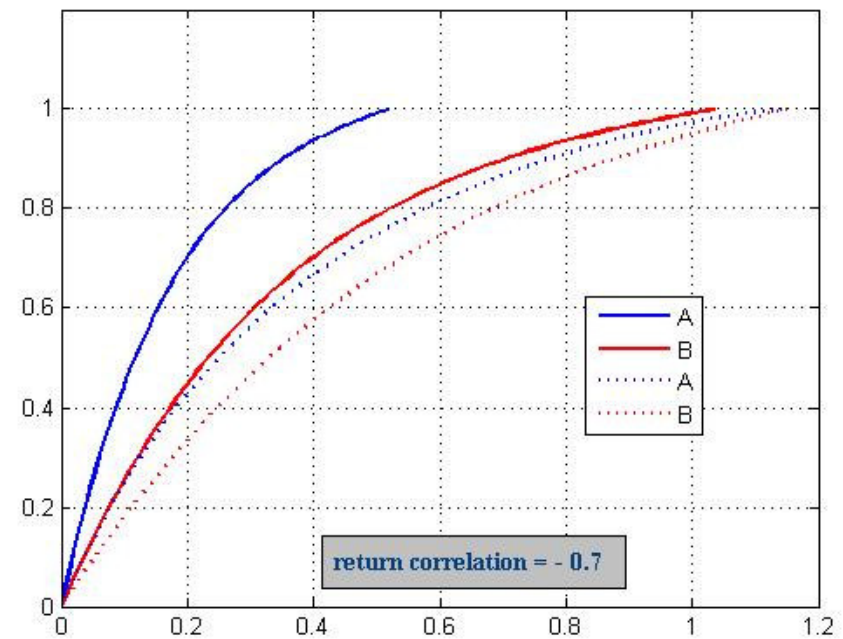
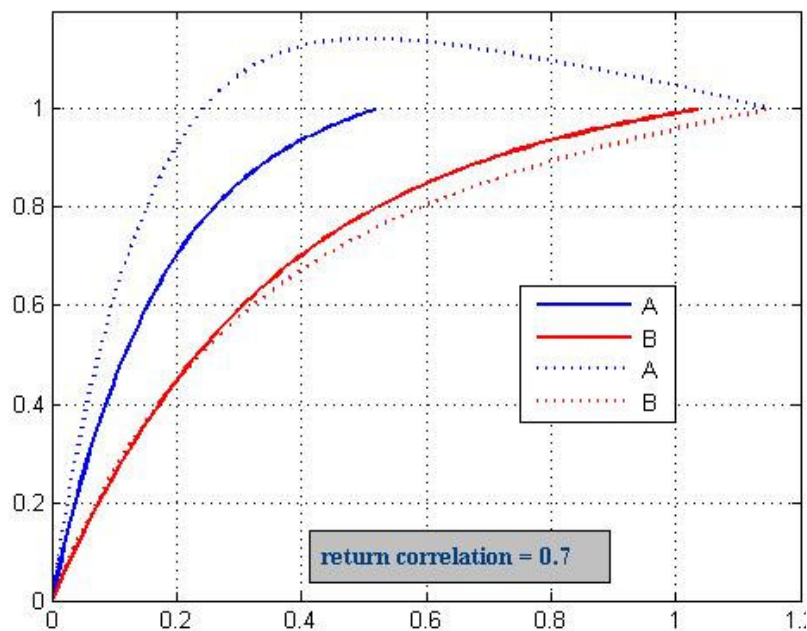
Numerical Examples – Two Stocks

➤ Combined path – zero correlations



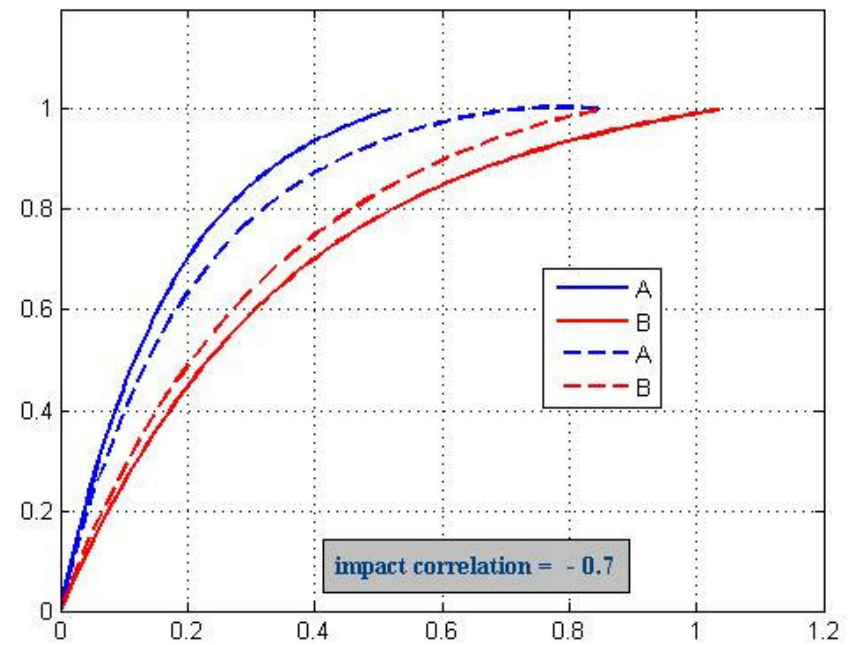
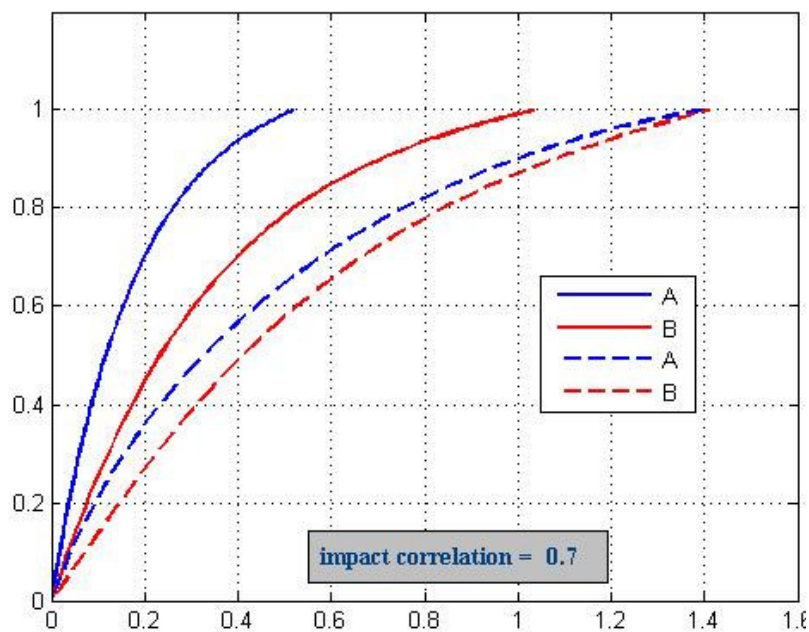
Numerical Examples – Two Stocks

➤ Combined path – non-zero return correlations



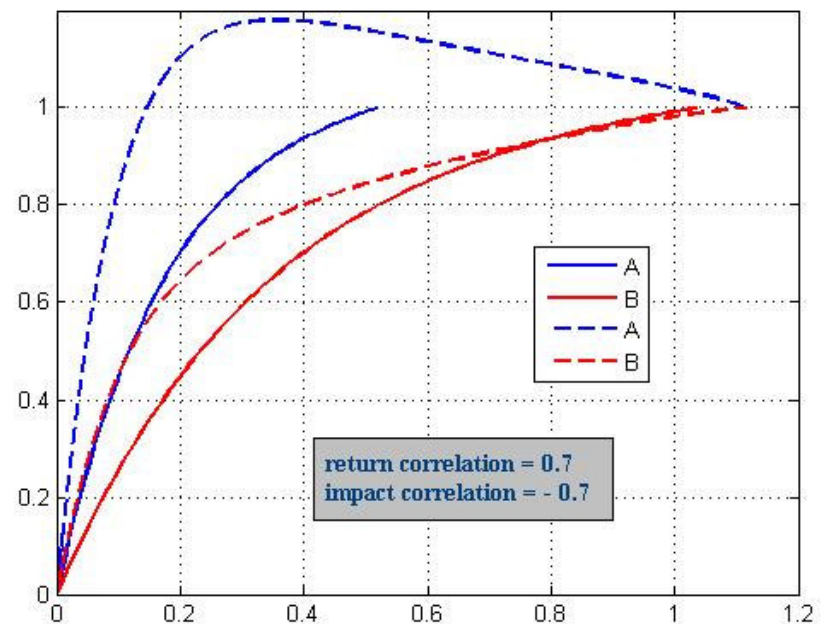
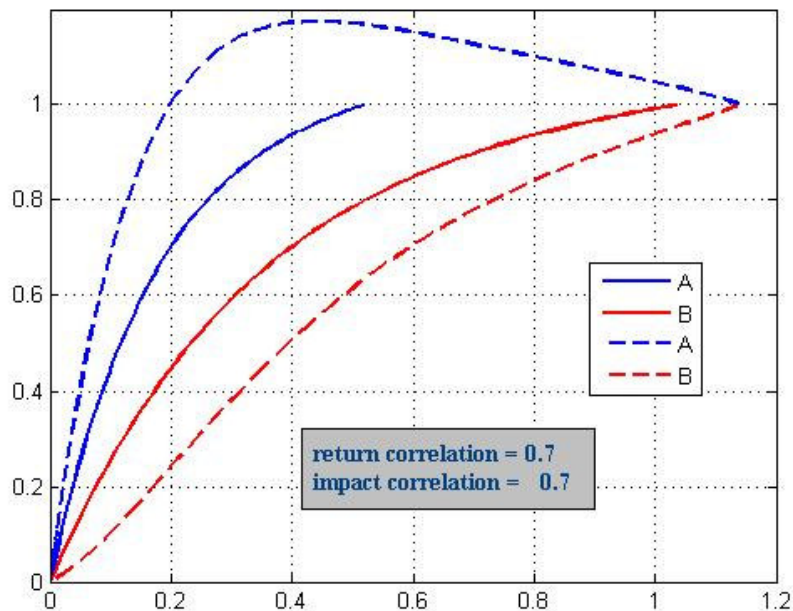
Numerical Examples – Two Stocks

➤ Combined path – non-zero impact correlations



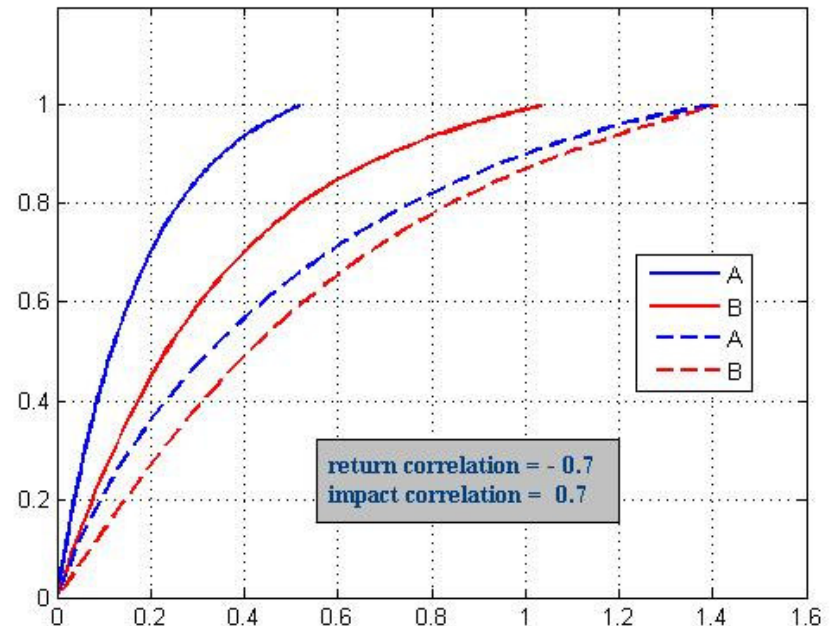
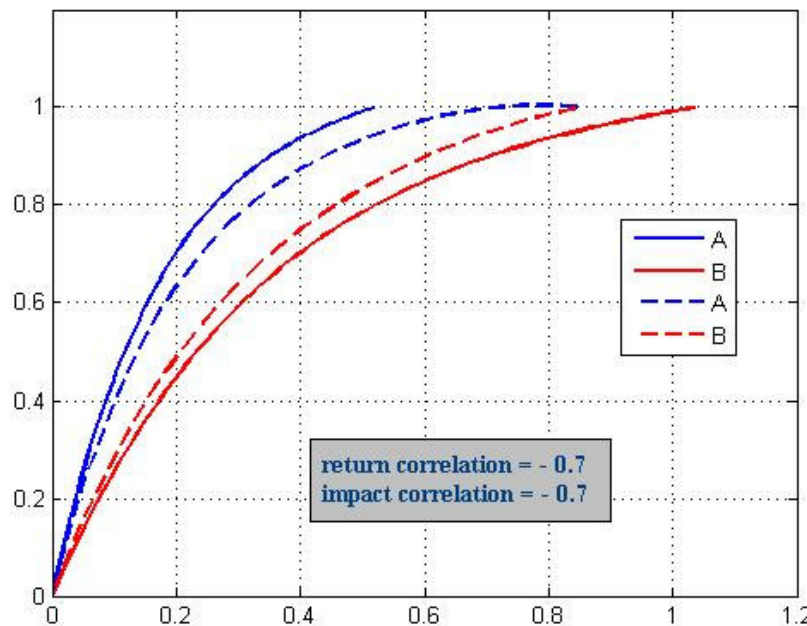
Numerical Examples – Two Stocks

➤ Combined path – non-zero correlations



Numerical Examples – Two Stocks

➤ Combined path – non-zero correlations



Summary

- **There is often an optimal trading horizon with optimal trading strategy**
- **Our analytic solution shows the optimal horizon depends on**
 - Expected return, stock volatility, fixed cost, market impact
 - Correlations play a significant role for stock portfolios
- **Further research**
 - Portfolio constraints: dollar neutral, sector neutral
 - No reverse trading



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