Extracting Information from the Markets: A Bayesian Approach

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The Federal Reserve Bank of Atlanta

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- Is it possible to reverse engineer this to extract information from prices?
- Yes, but the information retrieved is often incomplete.
- The techniques used are often quite different from those used to form prices.

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- Knowing market expectations aids in the prediction of market responses to Fed actions.
- Market expectations is a measure of the effectiveness of Fed communication.
- This can help in devising better communication and policy.

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- Both the futures and option contracts provide rich information on market expectations of Federal Reserve actions.

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$$\begin{array}{rcll} \textit{February} & - & 97.0200 & (\textit{vol}\ 7,697) \\ \textit{March} & - & 97.2550 & (\textit{vol}\ 15,793) \\ \textit{April} & - & 97.5250 & (\textit{vol}\ 27,575) \\ \textit{May} & - & 97.7700 & (\textit{vol}\ 15,737) \end{array}$$

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- FOMC meetings are scheduled for March 18, April 29/30, and June 24/25.

The Model

The price of a call or put is the discounted value of the expected payout.

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C_t(X,T) &= e^{-r(T-t)}\int_{-\infty}^{\infty} max(0, F_T - X)p(F_T)dF_T \\
P_t(X,T) &= e^{-r(T-t)}\int_{-\infty}^{\infty} max(0, X - F_T)p(F_T)dF_T
\end{aligned}
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where

- $C_t(X,T)$ is the price of a call and $P_t(X,T)$ is the price of a put at time t with strike X that expires at time T.
- r is the interest rate from time t to T.
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- These formulas hold exactly on if the options are European and there is no risk premium.

The Model - continued

If there is no FOMC meeting within the expiration month, the the value of the futures contract will approximately be the target rate. The target rate is discreet in 25 basis point increments. If $F_{1,T}, \cdots, F_{K,T}$ are the possible target rates and $\pi_1, \cdots, \pi_K$ are the corresponding probabilities, then

$$\begin{aligned} C_t(X,T) &= e^{-r(T-t)} \sum_{j=0}^{K} max(0, F_{j,T} - X)\pi_j \\ P_t(X,T) &= e^{-r(T-t)} \sum_{j=0}^{K} max(0, X - F_{j,T})\pi_j \end{aligned}$$

The Model - continued

If we assume that the options are measured with error, then the model can be compactly written as

$$Y = \mathbf{X}\pi + \varepsilon$$

where Y is the N-vector of option prices, π is a K-vector of probabilities, ε is a N-vector of errors, and $\mathbf{X}$ is

$$\begin{bmatrix} e^{-r(T-t)} max(0, F_{1,T} - X_1) & \cdots & e^{-r(T-t)} max(0, F_{K,T} - X_1) \\ \vdots & \ddots & \vdots \\ e^{-r(T-t)} max(0, X_N - F_{1,T}) & \cdots & e^{-r(T-t)} max(0, X_N - F_{K,T}) \end{bmatrix}$$

with X_i the strike price of the i^{th} option.

Solving the Model

$$Y = \mathbf{X}\pi + \varepsilon$$

is a regression model and could be solved via OLS, but π is a vector of probabilities and so must be non-negative and sum to one. It is easy to impose the linear restriction that the sum of the π must be one, but imposing the non-negativity constraint is a little more tricky in the classical regression framework.

The Bayesian Approach

The Bayesian paradigm assumes that there is a random process for generating the observed data Y conditional on some set of parameters θ and a prior probability distribution for the parameters. If the likelihood, which is the density for the data generating process given θ, is

$$p(Y|\theta)$$

and the prior probability density for the parameters θ is

$$p(\theta)$$

then joint density of the data and parameters and, more importantly, the probability of the parameters given the data is

$$p(\theta|Y) = p(Y|\theta)p(\theta)/p(Y) \propto p(Y|\theta)p(\theta)$$

This is the posterior probability of the parameters.

Advantages of the Bayesian Approach

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- Price of admission: A prior.

Solving the Model - continued

If we assume that the errors $\varepsilon = (\varepsilon_1, \cdots \varepsilon_N)$ are normally distributed with mean zero and variance $\zeta = diag(\zeta_1, \cdots, \zeta_N)$ then the likelihood is

$$p(Y|\pi, \zeta) \propto |\zeta|^{-1/2} exp\left(-\frac{1}{2}(Y - \mathbf{X}\pi)' \zeta^{-1}(Y - \mathbf{X}\pi)\right)$$

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- We will initially take the prior on π and ζ to be flat.
- This prior is not proper on ζ. Often independent inverse gamma distributions are used for the as the prior for ζ

Simulating the Posterior - Metropolis-Hasting

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- Given a sample $\theta^{(1)}, \cdots \theta^{(i)}$, draw θ from $q(\theta|\theta^{(i)})$ and compute

$$f(\theta, \theta^{(i)}) = \frac{p(\theta)}{p(\theta^{i})} \frac{q(\theta^{(i)}|\theta)}{q(\theta|\theta^{(i)})}$$

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$$\theta^{(i+1)} = \begin{cases} \theta & \text{if } f(\theta, \theta^{(i)}) \geq u \\ \theta^{(i)} & \text{if } f(\theta, \theta^{(i)}) < u \end{cases}$$

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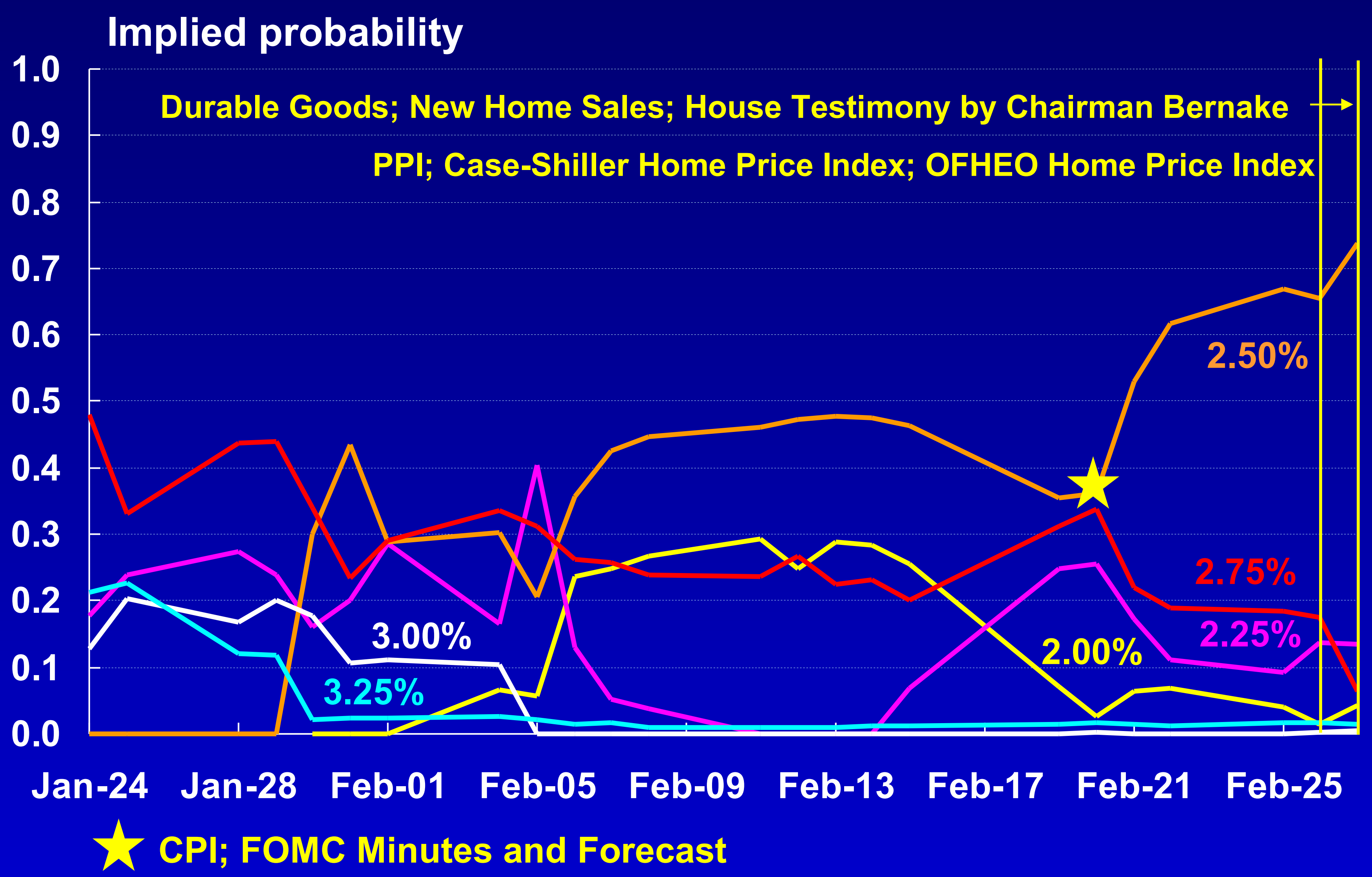
- If $q(\theta|\theta') = q(\theta'|\theta)$ then this is the Metropolis Algorithm.

MARCH MEETING OUTCOMES



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- Fast computers and MCMC simulations make a Bayesian approach feasible in many situations.
- Accessible probabilistic interpretations of parameters make conveying Bayesian results easy.
- Bayesian techniques should be in every researcher's toolbox.