Extracting Information from the Markets: A Bayesian Approach

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Disclaimer: The views expressed are the author’s and do not necessarily reflect those of the Federal Reserve Bank of Atlanta or the Federal Reserve System
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- Is it possible to reverse engineer this to extract information from prices?
- Yes, but the information retrieved is often incomplete.
- The techniques used are often quite different from those used to form prices.
Market expectations of Fed behavior

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• Market expectations is a measure of the effectiveness of Fed communication.

• This can help in devising better communication and policy.
Fed Funds Futures and Options

- The Fed Funds rate is an overnight interbank rate and is the target instrument of the Federal Reserve. The daily effective fed funds rate is reported by the Board of Governors (H.15 data series).

- Futures on the Fed Funds rate are cash settled, have been traded since 1988 and settle on the average Fed Funds rate over the month of the contract. Each contract has a notional value of $5,000,000. Being long one contract is equivalent to making a $5,000,000 loan for one month at the average Fed Funds rate over that month. Prices are quoted as 100 minus the rate. Currently, the open interest for February is almost 130,000 contracts.

- Options on the Fed Funds futures have traded since 2003. Each option is for one Fed Futures contract. Currently, the open interest for the February contract is almost 550,000 options.

- Both the futures and option contracts provide rich information on market expectations of Federal Reserve actions.
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Extracting Information from Fed Funds Futures and Options

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The Model

The price of a call or put is the discounted value of the expected payout.

\[
C_t(X, T) = e^{-r(T-t)} \int_{-\infty}^{\infty} \max(0, F_T - X) p(F_T) dF_T
\]

\[
P_t(X, T) = e^{-r(T-t)} \int_{-\infty}^{\infty} \max(0, X - F_T) p(F_T) dF_T
\]

where

- \(C_t(X, T)\) is the price of a call and \(P_t(X, T)\) is the price of a put at time \(t\) with strike \(X\) that expires at time \(T\).
- \(r\) is the interest rate from time \(t\) to \(T\).
- \(F_T\) is the futures price at time \(T\).
- \(p(\cdot)\) is the probability density of \(F_T\).
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- \( p(\cdot) \) is the probability density of \( F_T \).
- These formulas hold exactly on if the options are European and there is no risk premium.
If there is no FOMC meeting within the expiration month, the value of the futures contract will approximately be the target rate. The target rate is discreet in 25 basis point increments. If $F_{1,T}, \cdots, F_{K,T}$ are the possible target rates and $\pi_1, \cdots, \pi_K$ are the corresponding probabilities, then

$$C_t(X,T) = e^{-r(T-t)} \sum_{j=0}^{K} \max(0, F_{j,T} - X) \pi_j$$

$$P_t(X,T) = e^{-r(T-t)} \sum_{j=0}^{K} \max(0, X - F_{j,T}) \pi_j$$
The Model - continued

If we assume that the options are measured with error, then the model can be compactly written as

\[ Y = X\pi + \varepsilon \]

where \( Y \) is the \( N \)-vector of option prices, \( \pi \) is a \( K \)-vector of probabilities, \( \varepsilon \) is a \( N \)-vector of errors, and \( X \) is

\[
\begin{bmatrix}
    e^{-r(T-t)}\max(0, F_1, T - X_1) & \cdots & e^{-r(T-t)}\max(0, F_K, T - X_1) \\
    \vdots & \ddots & \vdots \\
    e^{-r(T-t)}\max(0, X_N - F_1, T) & \cdots & e^{-r(T-t)}\max(0, X_N - F_K, T)
\end{bmatrix}
\]

with \( X_i \) the strike price of the \( i^{th} \) option.
Solving the Model

\[ Y = X\pi + \varepsilon \]

is a regression model and could be solved via OLS, but \( \pi \) is a vector of probabilities and so must be non-negative and sum to one. It is easy to impose the linear restriction that the sum of the \( \pi \) must be one, but imposing the non-negativity constraint is a little more tricky in the classical regression framework.
The Bayesian Approach

The Bayesian paradigm assumes that there is a random process for generating the observed data $Y$ conditional on some set of parameters $\theta$ and a prior probability distribution for the parameters. If the likelihood, which is the density for the data generating process given $\theta$, is

$$p(Y|\theta)$$

and the prior probability density for the parameters $\theta$ is

$$p(\theta)$$

then joint density of the data and parameters and, more importantly, the probability of the parameters given the data is

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)} \propto p(Y|\theta)p(\theta)$$

This is the posterior probability of the parameters.
Advantages of the Bayesian Approach

• The close connection to probability theory allows rigorous and intuitive statements such as, “the probability that a certain parameter lies is some interval is $p$."

Advances in computer technology and Markov Chain Monte Carlo (MCMC) techniques allows for easy simulation.

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- Price of admission: A prior.
If we assume that the errors $\varepsilon = (\varepsilon_1, \cdots, \varepsilon_N)$ are normally distributed with mean zero and variance $\zeta = \text{diag}(\zeta_1, \cdots, \zeta_N)$ then the likelihood is

$$p(Y|\pi, \zeta) \propto |\zeta|^{-1/2} \exp \left( -\frac{1}{2} (Y - X\pi)'\zeta^{-1}(Y - X\pi) \right)$$
Solving the Model - continued

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- This prior is not proper on $\zeta$. Often independent inverse gamma distributions are used for the as the prior for $\zeta$. 
Simulating the Posterior - Metropolis-Hasting

- Let $p(\theta)$ be a probability density that we do not know how to directly simulate but that we can evaluate, at least up to some constant multiple.
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- Given a sample $\theta^{(1)}, \ldots, \theta^{(i)}$, draw $\theta$ from $q(\theta|\theta^{(i)})$ and compute

$$f(\theta, \theta^{(i)}) = \frac{p(\theta) \, q(\theta^{(i)}|\theta)}{p(\theta^{(i)}) \, q(\theta|\theta^{(i)})}$$
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$$f(\theta, \theta^{(i)}) = \frac{p(\theta) q(\theta^{(i)}|\theta)}{p(\theta^i) q(\theta|\theta^{(i)})}$$

- Draw $u$ from the uniform distribution on $[0, 1]$ and define

$$\theta^{(i+1)} = \begin{cases} 
\theta & \text{if } f(\theta, \theta^{(i)}) \geq u \\
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• If $q(\theta|\theta') = q(\theta'|\theta)$ then this is the Metropolis Algorithm.
MARCH MEETING  

OUTCOMES

Implied probability

- Durable Goods; New Home Sales; House Testimony by Chairman Bernake
- PPI; Case-Shiller Home Price Index; OFHEO Home Price Index

Jan-24 Jan-28 Feb-01 Feb-05 Feb-09 Feb-13 Feb-17 Feb-21 Feb-25

- 2.75%
- 3.25%
- 2.50%
- 3.00%
- 2.25%
- 2.00%
- 2.25%
- 2.75%

CPI; FOMC Minutes and Forecast
Conclusions

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- Fast computers and MCMC simulations make a Bayesian approach feasible in many situations.
- Accessible probabilistic interpretations of parameters make conveying Bayesian results easy.
- Bayesian techniques should be in every researcher’s toolbox.