Analysis Qualifying Exam

Part I: Complex Analysis

August, 2019

1. Prove that if f is entire and the imaginary part of f is bounded on $\mathbb{C}$, then f must be constant.

2. Find a conformal map from the unbounded region outside the disks $\{|z+1| \leq 1\} \cup \{|z-1| \leq 1\}$ to the upper half plane.

3. Let $\log z$ be the principal branch of the logarithm defined on $G = \{z \in \mathbb{C} | z \notin (-\infty, 0]\}$. Show that if $t > 0$, then the equation $\log z = t/z$ has exactly one root in G.

4. Show that the function $f(z) = \int_0^1 \frac{dt}{1-tz}$ is analytic on the open unit disk $\{z \in \mathbb{C} : |z| < 1\}$.

5. Find the Laurent series expansion for

$$f(z) = \frac{1}{z^3(4+z^2)}$$

   in the region $\{z : 0 < |z| < 2\}$

6. Let γ be the closed polygon $[3-i, 3+i, 1/2+i, 1/2-i, 3-i]$. Find the following integrals

$$\int_\gamma z^m/(z-1)^m, \qquad m \in \mathbb{N}$$

7. Compute the integral

$$\int_0^\pi \frac{\cos 2\theta \, d\theta}{1 - 2a\cos\theta + a^2} \quad \text{where } a^2 < 1$$