COMPLEX PRELIMINARY EXAM Department of Mathematics Florida State University

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1 Part 1

1. Calculate the Laurent series about $z = 0$ for $\frac{1}{1+z^2}$ when $|z| > 1$.

2. Calculate the image of the strip $\{z : \pi/2 < Imz < \pi\}$ under the mapping e^{iz}.

3. Suppose $\Omega \subset \mathbb{C}$ is open and $f : \Omega \to \mathbb{C}$ is analytic and nonconstant. If $g : f(\Omega) \to \mathbb{R}$ is harmonic, then show that $g(f)$ is harmonic in Ω.

4. Using residues, calculate the improper integral $\int_{-\infty}^{\infty} \frac{\cos 3x}{(1+x^2)^2} dx$.

2 Part 2

5. Suppose that $|z_0| < 1$. Show that

$$|\frac{z - z_0}{1 - \bar{z}_0 z}| \leq 1$$

when $|z| \leq 1$. Show that equality occurs if and only if $|z| = 1$.

6. Construct a conformal mapping of the strip $\{z : 0 < Imz < 1\}$ onto the unit disk $\{z : |z| < 1\}$.

7. Construct an entire function with a single zero at the square root of each positive integer, $\sqrt{k}$, where $k = 1, 2, 3, ...$