Complex Analysis
August 2020

Answer as many of the following questions as you can, keeping in mind that a single complete answer is usually worth more than comparable portions of several questions.

1. Let $f(z) = \frac{1}{(z-1)(z-2)}$.

   (a) Find a Laurent expansion (centered at zero) for f in the annulus $1 < |z| < 2$

   (b) Find a Laurent expansion (centered at zero) for f in the region $|z| > 2$.

2. Fix $h > 0$. Find an explicit conformal mapping from the set

$$\{z \in \mathbb{C} : \operatorname{Im}(z) > 0\} \setminus (0, ih]$$

   to the unit disk.

3. Suppose f and g are entire functions and $|f(z)| \le |g(z)|$ for all $z \in \mathbb{C}$. Prove that $f(z) = cg(z)$ for some constant c.

4. Calculate $\displaystyle\int_C \frac{8-z}{z(4-z)} \, dz$ where C is the circle of radius 7 centered at 0 and oriented clockwise.

5. Compute $\displaystyle\int_\gamma z^i \, dz$ where $z^i = \exp(i \operatorname{Log} z)$, Log denotes the principal branch and γ denotes a path from $z = -1$ to $z = 1$ below the real axis.

6. Let $f(z) = \displaystyle\int_0^1 \frac{dt}{1 - tz}$.

   (a) Show that $f(z)$ is analytic on the open unit disk.

   (b) Find a power series expansion for $f(z)$ in the disk (centered at zero).

7. Evaluate

$$\int_C \cot z \, dz$$

   where C is the circle of radius 4 centered at the origin oriented counterclockwise.