Methods of Applied Mathematics 1 Qualifying Exam

August 2011

Name:

Please be careful with your sketches and show your work clearly. Be sure to read directions and to include the things that are requested. Books and notes are not allowed, but a calculator is okay.

1. For the equation:

$$\frac{dx}{dt} = \mu x - 4x^3 \ ; \ \mu \in (-\infty, \infty)$$

(a) Find equations for the fixed points as a function of the parameter μ.
(b) Sketch qualitatively different phase portraits for the whole range of values of μ.
(c) Determine the values of μ where bifurcations occur and determine the types of bifurcations.
(d) Sketch and label a bifurcation diagram. Indicate and classify all bifurcations.

2. Consider the system $\frac{d^2x}{dt^2} = x - x^3$.
(a) Convert this to a planer system and then find and classify the equilibrium points.
(b) Find a conserved energy function for the system.
(c) Use the fact that the system is conservative to sketch a phase portrait.
(d) Determine equations for the homoclinic orbits.

3. Pick four distinct functions $\phi_1(\epsilon)$,...,$\phi_4(\epsilon)$ that form an asymptotic sequence as $\epsilon \to 0$. Then pick four distinct functions $\psi_1(\epsilon)$,...,$\psi_4(\epsilon)$ all of which tend to 0 as $\epsilon \to 0$, but which do not form an asymptotic sequence. Justify your answers.

4. For small positive ϵ, find a two-term asymptotic expansion of the solution of

$$\frac{d^2x}{dt^2} + \epsilon\frac{dx}{dt} + x = 1 \quad .$$

where $x(0) = 1$ and $\frac{dx}{dt}(0) = 1$. (There is no need to correct for secular terms, just do a straight-forward expansion.)