Methods of Applied Mathematics I: MAP 5165 –Qualifier Exam Spring 2008

Work out all problems. No notes, books or calculators are allowed.

1. (25) For the following ODE, find one term of the inner and outer expansions and match them:

$$\varepsilon \frac{d^2 u}{dx^2} + 2\frac{du}{dx} - 3u = 0$$

with boundary conditions: $u(0) = 1$ and $u(1) = 2$ with $0 < \varepsilon \ll 1$.

2. (25) Consider the nonlinear dynamical system

$$\dot{x} = x(3 - x - y), \qquad \dot{y} = y(2 - x - y)$$

i) Find all fixed points.
ii) Determine whether each fixed point is linearly stable or not and draw its local phase portrait.

3. (25) Consider the algebraic equation

$$\varepsilon(x^2 + x) - 1 = 0, \qquad 0 < \varepsilon \ll 1$$

i) Find the exact roots.
ii) Use Taylor series expansion to find three terms approximation to the roots $x_{1,2}(\varepsilon)$ as $\varepsilon \to 0$.
iii) Use singular perturbation methods to find three terms approximation to the roots.

4. (25) (a) Consider the dynamical system

$$\dot{x} = 1 + rx + x^2.$$

Sketch all the qualitatively different vector fields that occurs as r is varied. Determine what type of bifurcation (of fixed points) occurs at the critical value $r = r_c$ and determine the value(s) of r_c.
(b) Show that the dynamical system

$$\dot{x} = x - rx(1 - x),$$

undergoes a transcritical bifurcation. Determine the value(s) of r at which the bifurcation happens.