Methods of Applied Mathematics 1 Qualifying Exam
January 2009

Name:

Please be careful with your sketches and show your work clearly. Be sure to read directions and to include the things that are requested.

1. For the problem:

$$\dot{x} = \mu x + \frac{x^3}{1 + x^2}$$

(a) Find equations for the fixed points as a function of the parameter μ.
(b) Determine the values of μ where bifurcations occur and sketch qualitatively different phase portraits as μ is varied.
(c) Sketch a bifurcation diagram and classify any bifurcations that occur (e.g., transcritical, saddle-node, subcritical or supercritical pitchfork, infinite-limit).

2. Construct phase portraits for the following system, including nullclines and steady states as well as trajectories. Classify the steady states using linear stability analysis. Do this for μ values at and on both sides of the bifurcation. What type of bifurcation takes place?

$$
\begin{aligned}
\dot{x} &= x^2 - y \\
\dot{y} &= \mu x - y
\end{aligned}
$$

3. Consider the two-dimensional system in polar coordinates with parameter $\mu \geq 0$:

$$
\begin{aligned}
\dot{r} &= r(\mu - \sin r) \\
\dot{\theta} &= 1
\end{aligned}
$$

(a) Determine the values of μ where bifurcations occur and sketch qualitatively different phase portraits as μ is varied.
(b) Classify the bifurcations and make a bifurcation diagram.

4. Find two-term asymptotic expansions for the roots of $\epsilon x^4 - x^2 + 3x - 2 = 0$.