Algebraic geometry : HW 3

1. Write the equation of the tangent to the elliptic curve $E$ given by $y^2 z = x^3 - xz^2$ in $\mathbb{P}^2$ at the point $[0 : 1 : 0]$. Show that this tangent does not intersect the curve in any other point.

2. If $T$ is a subset of $k[x_0, \ldots, x_n]$ consisting of homogeneous polynomials, show that $Z(T) = Z((T))$, where $(T)$ is the ideal generated by $T$. Recall that if $I$ is a homogeneous ideal, then $Z(I)$ is the set of points $P$ in $\mathbb{P}^2$ such that $f(P) = 0$ for all homogeneous polynomials $f$ in $I$. 