Algebraic geometry : HW 6 solution

1. Prove that if $B$ is a domain, then $B$ is the intersection inside the quotient field of $B$ of the localizations of $B$ at maximal ideals of $B$.

Solution: Certainly $B$ is contained in the intersection inside the quotient field of $B$ of the localizations of $B$ at maximal ideals of $B$. Conversely suppose $f/g$ is an element of the quotient field of $B$ that is contained in $B_m$ for all maximal ideals $m$ of $B$. Then $g \notin m$ for all maximal ideals $m$ of $B$, and so $g$ is a unit in $B$ (if it is not, then $g$ is contained in the maximal ideal containing the ideal generated by $g$). Thus $f/g \in B$, which proves the reverse inclusion.