Algebraic geometry : HW 6 solution

1. Prove that if B is a domain, then B is the intersection inside the quotient field of B of the localizations of B at maximal ideals of B.

Solution: Certainly B is contained in the intersection inside the quotient field of B of the localizations of B at maximal ideals of B. Conversely suppose f/g is an element of the quotient field of B that is contained in $B_{\mathfrak{m}}$ for all maximal ideals $\mathfrak{m}$ of B. Then $g \notin \mathfrak{m}$ for all maximal ideals $\mathfrak{m}$ of B, and so g is a unit in B (if it is not, then g is contained in the maximal ideal containing the ideal generated by g). Thus $f/g \in B$, which proves the reverse inclusion.