1. (bonus) Let $k$ be a field, and $R$ and $S$ be two $k$-algebras. Show that a morphism $\text{Spec } S \to \text{Spec } R$ of schemes is a morphism of schemes over $k$ (recall that $\text{Spec } k$ is often denoted by just $k$) if and only if the corresponding homomorphism of rings $R \to S$ is a $k$-algebra homomorphism.

2. (bonus) (a) Let $X \subseteq \mathbb{A}^n$ and $Y \subseteq \mathbb{A}^m$ be two affine varieties defined over a field $k$. Recall that a morphism of algebraic sets from $X$ to $Y$ is said to be defined over $k$ if the corresponding map on the affine coordinate rings is a $k$-algebra homomorphism. Suppose $\phi : X \to Y$ is a map such that for each $i$ from 1 to $m$, the $i$-th coordinate of $\phi$ (i.e., $x_i \circ \phi$, where $x_i$ denotes the $i$-th coordinate function on $\mathbb{A}^m$) is a polynomial with coefficients in $k$. Then show that $\phi$ is defined over $k$ in the sense above.

(b) Let $E$ be the elliptic curve $y^2 = x^3 - x$ defined over $\mathbb{Q}$. Show that the map $E \to E$ given by $(x, y) \mapsto (x, -y)$ is defined over $\mathbb{Q}$ (in the sense of part (a)).