1 Various remarks and comments at the beginning of the lecture regarding previous lectures

Remark 1.1. Restriction maps need not be injective.
Germs of sections of vector bundles form a sheaf (See Hartshorne Ex II.1.?)
Regular functions on a variety form a sheaf with the additional condition that \( \mathcal{F}(\emptyset) = \{0\} \).

2 Discussion about topics for the courses over the next two semesters

Material omitted here. (It will probably appear in the syllabus when the next course is taught.)

3 Proj

Recall: If \( A \) is a ring, we define Spec\( A \) which is the analog of an affine variety.
What is the analog of a projective variety in the theory of schemes?
(One answer is to use the functor \( t: \text{varieties} \rightarrow \text{schemes} \).)
Let \( S \) be a graded ring, i.e. there exists a decomposition
\[
S = \bigoplus_{d \geq 0} S_d
\]
as a direct sum of abelian groups such that \( \forall d, e \geq 0 \ S_d S_e \subseteq S_{d+e} \).

Example 3.1. \( S = \mathbb{k}[x_0, \ldots, x_n] \ S_d = \mathbb{k} \text{ linear combinations of elements of } S \text{ of degree } d \).

Definition 3.2. An element of \( S_d \) is said to be a homogeneous element of \( S \) of degree \( d \).
An ideal \( a \) of \( S \) is said to be homogeneous if \( a = \bigoplus_{d \geq 0} (a \cap S_d) \) or equivalently if \( a \) can be generated by homogeneous elements. \( (\forall f \in a, \text{ if } f = \sum f_d \text{ then } f_d \in a.) \)

Example 3.3. \( (x^2 + y^2) \subseteq \mathbb{k}[x, y] \) is not homogeneous.
Example 3.4. \( (x, y^2 + xy) \) is homogeneous.

Definition 3.5. \( S_+ \) denotes the ideal \( \bigoplus_{d \geq 0} S_d \) and is called the irrelevant ideal.

Example 3.6. In \( k[x, y, z] \), \( S_+ = (x, y, z) \).

Definition 3.7. The set Proj \( S \) consists of homogeneous prime ideals \( \mathfrak{p} \) that do not contain \( S_+ \).