Algebraic geometry and elliptic curves : HW 1

1. If $P = (x, y)$ is a point on the elliptic curve $y^2 = x^3 + ax + b$ (assume that the characteristic of the field is not 2 or 3) other than the point at infinity and such that $y \neq 0$, and then show that

$$2P = \left(\left(\frac{3x^2 + a}{2y} \right)^2 - 2x, -y + \left(\frac{3x^2 + a}{2y} \right) \left(3x - \left(\frac{3x^2 + a}{2y} \right)^2 \right) \right).$$

2. Consider the elliptic curve $y^2 = x^3 + 1$ over $\mathbf{Q}$. Find the sum of the points $(-1, 0)$ and $(0, 1)$. Also find $2(2, 3)$.

3. Check that $y^2 = x^3 - x$ is an elliptic curve over $\mathbf{F}_5$. Find the number of points of the corresponding projective elliptic curve over $\mathbf{F}_5$.