Elliptic curves : HW 3

Note: In any problem below, you may assume results from an earlier problem even if you do not prove it. The bonus problems are optional, but may be used to offset lost points on any of the homeworks.

Let C denote the curve $y = x^2$ in $\mathbf{A}^2$.

1. Check that C is smooth.

2. (bonus) Show that at a point $P = (x_0, y_0)$ on C, a uniformizer is $x - x_0$ (recall that a uniformizer is a generator of the maximal ideal of the local ring at P). Also, if $x_0 \neq 0$, then $y - y_0$ is also a uniformizer at P.

3. Find the order of vanishing of y at $O = (0, 0)$.

4. Find the divisor of the function y on C.

5. Find the divisor of the differential dy on C.

Let C' denote the curve $yz = x^2$ in $\mathbf{P}^2$ (the projectivization of C).

5. (bonus) Check that C' is nonsingular at the point at infinity (denote this point by ∞).

6. (bonus) Find a uniformizer at ∞ on C'.

7. (bonus) Show that the divisor of the differential $d(x/z)$ on C' is -2∞.

8. Show that the genus of C' is 0.