

MODULAR FORMS/FUNCTIONS AND THEIR RELATION WITH ARITHMETIC FUNCTIONS IN NUMBER THEORY

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ABSTRACT

The function $r_k(n)$ is an arithmetic function that gives the number of ways in which a number $n \in \mathbb{N}$ can be written as a sum of k squares. The function $\sigma_k(n)$ is another arithmetic function and its value is the sum of the k power of the divisors of n , for $n \in \mathbb{N}$. We will prove that the Jacobi theta function is a Modular form of weight $\frac{1}{2}$ generated by $\Gamma_0(2)$ and that the Eisenstein series is a modular form of weight $2k$ generated by $\text{SL}(2, \mathbb{Z})$. Also we will show how this two modular forms relate to *sum of k squares* function and the *sigma* function.

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