# Calculus I : Final

Show your steps: you may not get any credit if the steps to the correct answer are missing. To ensure full credit, draw a box around the answer(s) to each problem. The exam is out of 33 points. Good luck!

1. (3 points) Use a linear approximation (or differentials) to estimate $\sqrt{25.01}$.

2. (3 points) A spherical balloon is being inflated so that its volume increases at the rate of 10 centimeter cubed per second. How fast is its radius increasing (in centimeters per second) when the radius is 10 centimeters? Recall that the volume of a sphere of radius $r$ is $\frac{4}{3}\pi r^3$. Note that your answer may involve $\pi$.

3. (3 points) Find the derivative of $x^{\sqrt{x}}$ (Hint: you may use logarithmic differentiation).

4. (3 points) Find $\lim_{x\to 0^+} \sqrt{x}\ln x$.

5. (4 points) For the function $f(x) = x^4 - 6x^2$, find the values of $x$ where $f(x)$ has local maxima, local minima, and inflection points (recall that an inflection point is one where the function changes concavity). Be sure to justify your answers and write them at the bottom of your answer as follows:
   (a) Local maximum values are at $x =$ (if none, leave blank).
   (b) Local minimum values are at $x =$ (if none, leave blank).
   (c) Inflection points are are at $x =$ (if none, leave blank).

6. (2 points) A particle is moving such that its distance from the origin at time $t$ is given by $s(t)$. If $s''(t) = t$, $s(0) = 0$, and $s(1) = 1$, find $s(t)$ (as a function of $t$) (recall that $s'(t)$ is the velocity of the particle).

7. (3 points) Find the area of the region enclosed by the curve $y = x^2 + x$ and the line $y = 2x$. A diagram is attached.

8. (4 points) Find the volume of the solid obtained by rotating the region enclosed by the curve $y = x^2 + x$ and the line $y = 2x$ about the line $x = -1$. A diagram is attached. In case you cannot solve the problem, be sure to show your steps using the two methods that may be tried in this situation (you may get some partial credit even if a method cannot be carried out to give the final answer).

9. (4 points) A cylindrical metal tank is to hold 2 cubic meters of water. Find the dimensions of the tank that will minimize the cost of the metal used. Recall that for a cylinder of radius $r$ and height $h$, the volume is $\pi r^2 h$ and the surface area is $2\pi r^2 + 2\pi rh$. Note that the cost of the metal used is proportional to the surface area. Be sure to justify why your answer gives the minimum.

10. Compute the following integrals:
    (a) (2 points) $\int \frac{x^3}{1+x^2}\, dx$
    (b) (2 points) $\int \frac{x^2}{1+x^2}\, dx$