Categorical group invariants of 3-manifolds

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Abstract

For a given class $\mathcal{G}$ of groups, a closed topological n-manifold M^n is of $\mathcal{G}$-category $\leq k$ if it can be covered by k open subsets such that for each path-component W of the subsets the image of its fundamental group $\pi_1(W) \to \pi(M^n)$ belongs to $\mathcal{G}$. The smallest number k such that M^n admits such a covering is the $\mathcal{G}$-category, $cat_{\mathcal{G}}(M^n)$. For $n = 3$, M^3 has $\mathcal{G}$-category ≤ 4. We characterize all closed 3-manifolds of $\mathcal{G}$-category 1, 2, and 3 for various classes $\mathcal{G}$.