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Personal

Born: **6/14/1960 in Turin, Italy**.
Citizenship: **Dual Italian/U.S.A.**.

Education

May 1987 **Ph.D. in Mathematics**, *Brown University*, Providence, RI, USA.
(W. Fulton, thesis advisor)
- Dissertation: On some characteristic numbers for smooth plane curves

July 1983 **Laurea in Matematica**, *Università degli Studi di Torino*, Italy.
(A. Collino, relatore)
- Tesi: Elementi di teoria delle varietà abeliane; Jacobiana associata ad una curva. Un criterio di caratterizzazione delle Jacobiane

Academic positions

2018-20 **Status-only full professor**, *University of Toronto*.
1/2012-12/2014 **Marion Bradley Brennan Professor**, *Florida State University*.
2009-present **Visiting Associate/Visitor in Mathematics**, *California Institute of Technology*.
2000-present **Professor**, *Florida State University*.
1995-2000 **Associate Professor**, *Florida State University*.
1991-1995 **Assistant Professor**, *Florida State University*.
1989-1991 **Visiting Assistant Professor**, *Oklahoma State University*.
1987-1989 **Dickson Instructor**, *University of Chicago*.

Grants and awards

2019-24 **Simons Foundation**, *Collaboration grant.*
- Project title: Characteristic classes and other invariants of algebraic schemes.

2019 **FSU**, *COFRS award, - Project title: Geometric Invariants of Schubert Varieties.*

2019 **NSF**, *award #1904591 (co-PI, conference grant):*
Facets of Algebraic Geometry.

2016–18 **NSA**, *award #H98230-16-1-0016, - Project title:*
Theory and applications of Segre classes and related intersection theoretic invariants.

2016-19 **NSF**, *award #1603247 (co-PI, conference grant):*
A Three-Workshop Series on the Mathematics and Physics of F-theory.

2015–16 **NSA**, *award #H98230-15-1-0027, - Project title:*
Segre classes as integrals over polytopes and other problems in intersection theory.

2013 **FSU**, *COFRS award. - Project title:*
Potts models, graph hypersurfaces, and Chern class identities in string theory.

2012-15 **Simons Foundation**, *Collaboration grant.*
- Project title: Algebraic geometry, singularities, and applications to physics.

2009 **FSU**, *COFRS award. - Project title: Geometry and Physics*.

2007–09 **NSA**, *award #H98230-07-1-0024,*
- Project title: Limits in Intersection Theory and Invariants of Singular Spaces.

2007–08 **FSU**, *University Teaching award*.

2004–05 **NSA**, *award #H98230-04-1-0112,*
- Project title: Blow-up algebras, characteristic classes, and enumerative geometry.

2000–01 **FSU**, *University Teaching award*.

1998 **FSU**, *Developing Scholar award*.

1995 **FSU**, *Teaching Incentive Program award*.

1995–98 **NSF**, *award #9500843, - Project title: Topics in Intersection Theory*.

1994–95 **FSU**, *University Teaching award*.

1993 **FSU**, *First Year Assistant Professor summer award*.

1987–89 **NSF**, *Postdoctoral associate (PI: W. Fulton)*
- Project title: Segre classes and intersection formulas; applications to the enumerative geometry of plane curves.

Publications

Full abstracts and links to most of the publications listed here may be found at `http://www.math.fsu.edu/~aluffi/mainhtmls/pubs.html`

— Books

3. *Algebra: Notes from the Underground*. 472 pages.
Cambridge University Press, Cambridge, 2021.

A textbook for a standard two-semester undergraduate sequence in Abstract Algebra, following a 'ring-first' approach and including a thorough treatment of modules. The book includes more than 400 exercises.

2. *Algebra: Chapter 0*. 713 pages, 2009. Second printing, 2016.
American Mathematical Society, Graduate Studies in Mathematics Volume 104

A textbook for upper level undergraduate or beginning graduate courses in Algebra, including an introduction to homological algebra. The material is presented via a categorical approach emphasizing universal properties. The book includes more than 1000 exercises.

1. *Fare Matematica — Astratto e concreto nella matematica elementare*. 144 pages, 2009.
Aracne Editrice. In Italian.

This book is aimed at the general public. It consists of a brief introduction to a few elementary themes in mathematics, emphasizing the interplay between the abstractness of the material and the concreteness of the mathematical approach to it.

— Edited Volumes

3. *Facets of Algebraic Geometry - A Collection in Honor of William Fulton's 80th Birthday, Vol.1 and Vol.2.* xiv+402 pp. & xiv+379 pp.
   Cambridge University Press, Cambridge, 2022.

   Written to honor the 80th birthday of William Fulton, the articles collected in these volumes present substantial contributions to algebraic geometry and related fields, with an emphasis on combinatorial algebraic geometry and intersection theory. Featured topics include commutative algebra, moduli spaces, quantum cohomology, representation theory, Schubert calculus, and toric and tropical geometry. The range of these contributions is a testament to the breadth and depth of Fulton's mathematical influence. The authors are all internationally recognized experts, and include well-established researchers as well as rising stars of a new generation of mathematicians. The text aims to stimulate progress and provide inspiration to graduate students and researchers in the field.

2. *Proceedings of the Special Session on Singularities and Physics, Knoxville, TN, 21-23 March 2014.* 149 pages, 2016.
   Paolo Aluffi and Mboyo Esole, editors. Journal of Singularities, Volume 15

   A Special Session on Singularities and Physics was organized for the AMS Sectional Meeting held in Knoxville, TN on March 21-23, 2014. The session focused on the theory of singularities and its interactions with different branches of theoretical physics: singularities of elliptic fibrations in string theory, renormalization issues in quantum field theory, Landau-Ginzburg models, wall-crossing phenomena, and other recent points of contact. This volume collects articles written by some of the speakers on the occasion of this meeting, expanding on the content of their talks or reporting on research originated in discussions begun at the conference.

1. *Quantum cohomology at the Mittag-Leffler Institute.* 163 pages, 1998.
   Appunti della Scuola Normale Superiore di Pisa.

   A write-up of talks given in the Fall semester of 1996 by the participants to the year on "Enumerative geometry and its interactions with theoretical physics" at the Mittag-Leffler institute in Stockholm, Sweden.

— Preprints

74. *From motivic Chern classes of Schubert cells to their Hirzebruch and CSM classes,* with Leonardo Mihalcea, Jörg Schürmann, Changjian Su. 46 pages. arXiv:2212.12509

    The equivariant motivic Chern class of a Schubert cell in a 'complete' flag manifold $X = G/B$ is an element in the equivariant K theory ring of X to which one adjoins a formal parameter y. In this paper we prove several 'folklore results' about the motivic Chern classes, including finding specializations at $y = -1$ and $y = 0$; the coefficient of the top power of y; how to obtain Chern-Schwartz-MacPherson (CSM) classes as leading terms of motivic classes; divisibility properties of the Schubert expansion of motivic Chern classes. We collect several conjectures about the positivity, unimodality, and log concavity of CSM and motivic Chern classes of Schubert cells, including a conjectural positivity of structure constants of the multiplication of Poincaré duals of CSM classes. In addition, we prove a 'star duality' for the motivic Chern classes. We utilize the motivic Chern transformation to define two equivariant variants of the Hirzebruch transformation, which appear naturally in the Grothendieck-Hirzebruch-Riemann-Roch formalism. We utilize the Demazure-Lusztig recursions from the motivic Chern class theory to find similar recursions giving the Hirzebruch classes of Schubert cells, their Poincaré duals, and their Segre versions. We explain the functoriality properties needed to extend the results to 'partial' flag manifolds G/P.

73. *Motivic Chern classes of Schubert cells, Hecke algebras, and applications to Casselman's problem,* with Leonardo Mihalcea, Jörg Schürmann, Changjian Su. 40 pages. arXiv:1902.10101. To appear in Annales Scientifiques de l'École Normale Supérieure

    Motivic Chern classes are elements in the K-theory of an algebraic variety X depending on an extra parameter y. They are determined by functoriality and a normalization property for smooth X. In this paper we calculate the motivic Chern classes of Schubert cells in the (equivariant) K-theory flag manifolds G/B. The calculation is recursive starting from the class of a point, and using the Demazure-Lusztig operators in the Hecke algebra of the Weyl group of G. The resulting classes are conjectured to satisfy a positivity property. We use the recursions to give a new proof that they are equivalent to certain K-theoretic stable envelopes recently defined by Okounkov and collaborators, thus recovering results of Fehér, Rimányi and Weber. The Hecke algebra action on the K-theory of the dual flag manifold matches the Hecke action on the Iwahori invariants of the principal series representation associated to an unramified character for a group over a nonarchimedean local field. This gives a correspondence identifying the Poincaré dual version of the motivic Chern class to the standard basis in the Iwahori invariants, and the fixed point basis to Casselman's basis. We apply this to prove two conjectures of Bump, Nakasuji and Naruse concerning factorizations, and holomorphy properties, of the coefficients in the transition matrix between the standard and the Casselman's basis.

72. *Shadows of characteristic cycles, Verma modules, and positivity of Chern-Schwartz-MacPherson classes of Schubert cells,* with Leonardo Mihalcea, Jörg Schürmann, Changjian Su. 55 pages. arXiv:1709.08697. To appear in Duke Math Journal

    We recast the theory of Chern-Schwartz-MacPherson (CSM) classes in the torus equivariant setting in terms of characteristic cycles of constructible functions. Specializing to (generalized) flag manifolds, we obtain a 'Hecke orthogonality' of CSM classes. We show that the characteristic cycle of the Verma module, restricted to the zero section, gives the CSM class of a Schubert cell. This establishes an equivalence between CSM classes and stable envelopes, reproving a result of Rimányi and Varchenko. This formulas allows us to prove a conjecture by Aluffi and Mihalcea, and to extend previous positivity results by J. Huh in the Grassmann manifold case.

— Refereed publications

71. *Lorentzian polynomials, Segre classes, and adjoint polynomials,* arXiv:2304.02043. Advances in Mathematics 437 (2024) 109440. doi:10.1016/j.aim.2023.109440

    We consider polynomials expressing the cohomology classes of subvarieties of products of projective spaces, and limits of positive real multiples of such polynomials. We study the relation between these *covolume polynomials* and Lorentzian polynomials. While these are distinct notions, we prove that, like Lorentzian polynomials, covolume polynomials have M-convex support and generalize the notion of log-concave sequences. In fact, we prove that covolume polynomials are 'sectional log-concave', that is, the coefficients of suitable restrictions of these polynomials form log-concave sequences. We observe that Chern classes of globally generated bundles give rise to covolume polynomials, and use this fact to prove that certain polynomials associated with *Segre classes* of subschemes of products of projective spaces are covolume polynomials. We conjecture that the same polynomials may be Lorentzian after a standard normalization operation. Finally, we obtain a combinatorial application of a particular case of our Segre class result. We prove that the *adjoint polynomial* of a convex polyhedral cone contained in the nonnegative orthant, and sharing a face with it, is a covolume polynomial. This implies that these adjoint polynomials are M-convex and sectional log-concave, and in fact *dually Lorentzian,* that is, Lorentzian after a certain change of variables.

70. *Motives of melonic graphs,* with Matilde Marcolli and Waleed Qaisar. arXiv:2007.08565. Ann. Inst. H. Poincaré - D (2023), 503-554. doi:10.4171/AIHPD/156

We investigate recursive relations for the Grothendieck classes of the affine graph hypersurface complements of melonic graphs. We compute these classes explicitly for several families of melonic graphs, focusing on the case of graphs with valence-4 internal vertices, relevant to CTKT tensor models. The results hint at a complex and interesting structure, in terms of divisibility relations or nontrivial relations between classes of graphs in different families. Using the recursive relations we prove that the Grothendieck classes of all melonic graphs are positive as polynomials in the class of the moduli space $\mathcal{M}_{0,4}$. We also conjecture that the corresponding polynomials are *log-concave,* on the basis of hundreds of explicit computations.

69. *Segre classes and invariants of singular varieties,* arXiv:2109.05061. In 'Handbook of Geometry and Topology of Singularities, III', Springer (2022) 419–492. doi:10.1007/978-3-030-95760-5_6

Segre classes encode essential intersection-theoretic information concerning vector bundles and embeddings of schemes. In this paper we survey a range of applications of Segre classes to the definition and study of invariants of singular spaces. We focus on several numerical invariants, on different notions of characteristic classes for singular varieties, and on classes of Lê cycles. We precede the main discussion with a review of relevant background notions in algebraic geometry and intersection theory.

68. *Positivity of Segre-MacPherson classes,* with Leonardo Mihalcea, Jörg Schürmann, Changjian Su. arXiv:1902.00762. In 'Facets of Algebraic Geometry: A Collection in Honor of William Fulton's 80th Birthday. Volume 1', Cambridge University Press (2022) 1–28. doi:10.1017/9781108877831.001

Let X be a complex nonsingular variety with globally generated tangent bundle. We prove that the signed Segre-MacPherson (SM) class of a constructible function on X with effective characteristic cycle is effective. This extends and unifies several previous results in the literature, and yields several new results. For example, we prove that Behrend's Donaldson-Thomas invariant for a closed subvariety of an abelian variety is effective; that the intersection homology Chern class of the theta divisor for a non-hyperellptic curve is signed-effective; and we prove more general effectivity results for SM classes of subvarieties which admit proper (semi-)small resolutions and for regular or affine embeddings. Among these, we mention the effectivity of (signed) Segre-Milnor classes of complete intersections if X is projective and an alternation property for SM classes of Schubert cells in flag manifolds. The latter result proves and generalizes a variant of a conjecture of Fehér and Rimányi. Finally, we extend the (known) non-negativity of the Euler characteristic of perverse sheaves on a semi-abelian variety to more general varieties dominating an abelian variety.

67. *Intersection theory, characteristic classes, and algebro-geometric Feynman rules,* with Matilde Marcolli. 36 pages. In 'Proceedings of MathemAmplitudes 2019: Intersection Theory & Feynman Integrals', Proceedings of Science (2022). doi:10.22323/1.383.0012

We review the basic definitions in Fulton-MacPherson Intersection Theory and discuss a theory of 'characteristic classes' for arbitrary algebraic varieties, based on this intersection theory. We also discuss a class of graph invariants motivated by amplitude computations in quantum field theory. These 'abstract Feynman rules' are obtained by studying suitable invariants of hypersurfaces defined by the Kirchhoff-Tutte-Symanzik polynomials of graphs. We review a 'motivic' version of these abstract Feynman rules, and describe a counterpart obtained by intersection-theoretic techniques.

66. *Newton-Okounkov bodies and Segre classes,* arXiv:1809.07344. Amer. J. Math. 143 (2021) 1505–1526. doi:10.1353/ajm.2021.0038

    Given a homogeneous ideal in a polynomial ring over the complex numbers, we adapt the construction of Newton-Okounkov bodies to obtain a convex subset of Euclidean space such that a suitable integral over this set computes the *Segre zeta function* of the ideal. That is, we extract the numerical information of the Segre class of a subscheme of projective space from an associated (unbounded) Newton-Okounkov convex set. The result generalizes to arbitrary subschemes of projective space the numerical form of a previously known result for monomial schemes.

65. *Pfaffian integrals and invariants of singular varieties,* with Mark Goresky. arXiv:1901.06312. A panorama of singularities, 1–11. Contemp. Math., 742 (2020). doi:10.1090/conm/742/14935

    Integrals of the Pfaffian form over the nonsingular part of a projective variety compute information closely related to the Mather-Chern class of the variety and to other invariants such as the local Euler obstruction along strata of its singular locus and, in the hypersurface case, Milnor numbers. We obtain simple proofs of these formulas, recovering along the way several classically known results.

64. *The Chern-Schwartz-MacPherson class of an embeddable scheme,* arXiv:1805.11116. Forum of Mathematics, Sigma (2019), 7, 28 pages. doi:10.1017/fms.2019.25

    There is an explicit formula expressing the Chern-Schwartz-MacPherson class of a hypersurface in a nonsingular variety (in characteristic 0) in terms of the Segre class of its jacobian subscheme; this has been known for a number of years. We generalize this formula to arbitrary embeddable schemes: for every subscheme X of a nonsingular variety V, we define an associated subscheme Y of a projective bundle over V and provide an explicit formula for the Chern-Schwartz-MacPherson class of X in terms of the Segre class of Y. If X is a local complete intersection, a version of the result yields a direct expression for the Milnor class of X.

63. *The Euclidean distance degree of smooth complex projective varieties,* with Corey Harris. arXiv:1708.00024. Algebra and Number Theory 12:8 (2018) 2005–2032. doi:10.2140/ant.2018.12.2005

    We obtain several formulas for the Euclidean distance degree (ED degree) of an arbitrary nonsingular variety in projective space: in terms of Chern and Segre classes, Milnor classes, Chern-Schwartz-MacPherson classes, and an extremely simple formula equating the Euclidean distance degree of X with the Euler characteristic of an open subset of X.

62. *Projective duality and a Chern-Mather involution,* arXiv:1601.05427. Trans. Amer. Math. Soc. 370 (2018), no. 3, 1803–1822. doi:10.1090/tran/7042

    We observe that linear relations among Chern-Mather classes of projective varieties are preserved by projective duality. We deduce the existence of an explicit involution on a part of the Chow ring of projective space, encoding the effect of duality on Chern-Mather classes. Applications include Plücker formulae, constraints on self-dual varieties, generalizations to singular varieties of classical formulas for the degree of the dual and the dual defect, formulas for the Euclidean distance degree, and computations of Chern-Mather classes and local Euler obstructions for cones.

61. *The Segre zeta function of an ideal,* arXiv:1606.03098. Advances in Mathematics 320 (2017) 1201–1226. doi:10.1016/j.aim.2017.09.023

We define a power series associated with a homogeneous ideal in a polynomial ring, encoding information on the Segre classes defined by extensions of the ideal in projective spaces of arbitrarily high dimension. We prove that this power series is rational, with poles corresponding to generators of the ideal, and with numerator of bounded degree and with nonnegative coefficients. We also prove that this 'Segre zeta function' only depends on the integral closure of the ideal.

60. *Tensored Segre classes,* arXiv:1605.09393. Journal of Pure and Applied Algebra 221 (2017) 1366–1382. doi:10.1016/j.jpaa.2016.09.016

We study a class obtained from the Segre class $s(Z, Y)$ of an embedding of schemes by incorporating the datum of a line bundle on Z. This class satisfies basic properties analogous to the ordinary Segre class, but leads to remarkably simple formulas in standard intersection-theoretic situations such as excess or residual intersections. We prove a formula for the behavior of this class under linear joins, and use this formula to prove that a 'Segre zeta function' associated with ideals generated by forms of the same degree is a rational function.

59. *How many hypersurfaces does it take to cut out a Segre class?,* Journal of Algebra 471 (2017) 480–491. doi:10.1016/j.jalgebra.2016.10.003

We prove an identity of Segre classes for zero-schemes of compatible sections of two vector bundles. Applications include bounds on the number of equations needed to cut out a scheme with the same Segre class as a given subscheme of (for example) a projective variety, and a 'Segre-Bertini' theorem controlling the behavior of Segre classes of singularity subschemes of hypersurfaces under general hyperplane sections.

58. *Chern-Schwartz-MacPherson classes for Schubert cells in flag manifolds,* with L. Mihalcea. arXiv:1508.01535. Compositio Math. 152 (2016), 2603–2625. doi:10.1112/S0010437X16007685.

We obtain an algorithm computing the Chern-Schwartz-MacPherson (CSM) classes of Schubert cells in a generalized flag manifold G/B. In analogy to how the ordinary divided difference operators act on Schubert classes, each CSM class of a Schubert class is obtained by applying certain Demazure-Lusztig type operators to the CSM class of a cell of dimension one less. These operators define a representation of the Weyl group on the homology of G/B. By functoriality, we deduce algorithmic expressions for CSM classes of Schubert cells in any flag manifold G/P. We conjecture that the CSM classes of Schubert cells are an effective combination of (homology) Schubert classes, and prove that this is the case in several classes of examples. We also extend our results and conjectures to the torus equivariant setting.

57. *Segre classes as integrals over polytopes,* arXiv:1307.0830. J. Eur. Math. Soc. 18 (2016) 2849–2863. doi:10.4171/JEMS/655

We prove a formula for the Segre classes of monomial schemes as integrals over suitable Newton regions. The proof is based on the birational invariance of Segre classes, on a principalization algorithm for monomial schemes, and on careful considerations regarding simplicial decompositions of polytopes. The formula holds for *r.c. monomial schemes,* generalizing the monomial case to the singular setting and relaxing a transversality hypothesis.

56. *Multidegrees of monomial rational maps,* Publ. RIMS Kyoto Univ. 51 (2015), 635–654. doi:10.4171/PRIMS/167

We prove a formula for the multidegrees of a rational map defined by generalized monomials on a projective variety, in terms of integrals over an associated Newton region. This formula leads to an expression of the multidegrees as volumes of related polytopes, in the spirit of the classical Bernstein-Kouchnirenko theorem, but extending the scope of these formulas to more general monomial maps. We also determine a condition under which the multidegrees may be computed in terms of the characteristic polynomial of an associated matrix.

55. *Degrees of projections of rank loci,* Experimental Mathematics, 24:469–488, 2015. doi:10.1080/10586458.2015.1028689

We provide formulas for the degrees of the projections of the locus of square matrices with given rank from linear spaces spanned by a choice of matrix entries; the motivation for these computations stem from applications to 'matrix rigidity'. Surprisingly (to us), these degrees appear to match the numbers of Kekulé structures of certain 'benzenoid hydrocarbons', and arise in many other contexts with no apparent direct connection to the enumerative geometry of rank conditions.

54. *Chern classes of splayed intersections,* with E. Faber, 18 pages. Canad. J. Math. 67(2015), 1201-1218. doi:10.4153/CJM-2015-010-7

We generalize the Chern class relation for the transversal intersection of two nonsingular varieties to a relation for possibly singular varieties, under a 'splayedness' assumption. The relation is shown to hold for both the Chern-Schwartz-MacPherson class and the Chern-Fulton class. The main tool is a formula for Segre classes of splayed subschemes. We also discuss the Chern class relation under the assumption that one of the varieties is a general very ample divisor.

53. *Log canonical threshold and Segre classes of monomial schemes,* Manu. Math. (2015) 146: 1–6

We express the Segre class of a monomial scheme in projective space in terms of log canonical thresholds of associated ideals. Explicit instances of the relation amount to identities involving the classical polygamma functions.

52. *Generalized Euler characteristics, graph hypersurfaces, and Feynman periods,* Geometric, algebraic and topological methods for quantum field theory, 95–136, World Sci. Publ., Hackensack, NJ, 2014.

We give a very informal presentation of background on the Grothendieck group of varieties and on characteristic classes, and we review some recent work using these tools to study 'graph hypersurfaces'—a topic motivated by the algebro-geometric interpretation of Feynman amplitudes as periods of complements of these hypersurfaces. These notes follow closely, both in content and style, my lectures at the Summer school in Villa de Leyva, July 5-8, 2011.

51. *Splayed divisors and their Chern classes,* with E. Faber, J. London Math. Soc. (2013) 88 (2): 563-579

We obtain several new characterizations of 'splayedness' for divisors, and we consider the effect of splayedness on the Chern classes of sheaves of differential forms with logarithmic poles along splayed divisors and on the Chern-Schwartz-MacPherson classes of the complements of these divisors. We verify in several template situations a conjectural identity for Chern-Schwartz-MacPherson classes of splayed divisors and subvarieties.

50. *Segre classes of monomial schemes,* Electron. Res. Announc. Math. Sci., 20 (2013) 55-70

    We propose an explicit formula for the Segre classes of monomial subschemes of nonsingular varieties, such as schemes defined by monomial ideals in projective space. The Segre class is expressed as a formal integral on a region bounded by the corresponding Newton polyhedron. We prove this formula for monomial ideals in two variables and verify it for some families of examples in any number of variables.

49. *Euler characteristics of general linear sections and polynomial Chern classes,* Rend. Circ. Mat. Palermo, (special issue) 62 (2013) 3–26.

    We give simple proofs of formulas of Dimca-Papadima and Huh for the degrees of the polar map of a homogeneous polynomial, extending these formula to any algebraically closed field of characteristic 0, and prove a conjecture of Dolgachev on 'homaloidal' polynomials in the same context. We generalize these formulas to subschemes of higher codimension in projective space, and describe a simple approach to a theory of 'polynomial Chern classes' for varieties endowed with a morphism to projective space.

48. *A motivic approach to phase transitions in Potts models,* with Matilde Marcolli. Journal of Geom. and Phys., 63 (2013) 6–31.

    We describe an approach to the study of phase transitions in Potts models based on an estimate of the complexity of the locus of real zeros of the partition function, computed in terms of the classes in the Grothendieck ring of the affine algebraic varieties defined by the vanishing of the multivariate Tutte polynomial.

47. *Grothendieck classes and Chern classes of hyperplane arrangements,* Int. Math. Res. Notices (2013) 1873-1900.

    We show that the characteristic polynomial of a hyperplane arrangement can be recovered from the class in the Grothendieck group of varieties of the complement of the arrangement, and from its Chern-Schwartz-MacPherson class. We also give relations with the Segre class of the singularity subscheme of the arrangement, and study the issue of positivity of the Chern-Schwartz-MacPherson class of an arrangement.

46. *Verdier specialization via weak factorization.* Arkiv för Matematik, 51 (2013) 1–28.

    We define a constructible function on arbitrary closed subschemes of nonsingular varieties, recovering and partially extending a construction originally introduced by Verdier. We also introduce a counterpart of the construction in a motivic group, and show that these invariants have a natural 'monodromy' decomposition.

45. *Chern classes of graph hypersurfaces and deletion-contraction relations.* Moscow Math. Jour., 12 (2012) 671–700.

    We establish a weak deletion-contraction relation for Chern classes of graph hypersurfaces, analogous to the relation for the Grothendieck class obtained in previous work, and prove corresponding multiple edge formulas. The main results are obtained as corollary of a formula for the Chern-Schwartz-MacPherson class of a transversal intersection. We also provide an alternative approach, based on Verdier specialization.

44. *Chern classes of free hypersurface arrangements,* J. Sing. 5 (2012), 19-32.

    The Chern class of the sheaf of logarithmic derivations along a simple normal crossing divisor equals the Chern-Schwartz-MacPherson class of the complement of the divisor. We extend this equality to more general divisors, which are locally analytically isomorphic to free hyperplane arrangements.

43. *Graph hypersurfaces and a dichotomy in the Grothendieck ring,* with Matilde Marcolli.
Lett. Math Phys. (2011) 95:223-232

We prove that every graph hypersurface is stably birationally equivalent to a union of points. This implies that Grothendieck classes of graph hypersurfaces span a tiny portion of the Grothendieck ring, while Belkale and Brosnan have proved that they span the whole Grothendieck ring after a suitable localization.

42. *Feynman motives and deletion-contraction relations,* with Matilde Marcolli.
In Topology of Algebraic Varieties and Singularities, Contemporary Mathematics, vol. 538, Amer. Math. Soc., Providence, RI, 2011, pp. 21-64.

We explore analogues of combinatorial deletion-contraction rules for the Feynman rules introduced in our previous work. We prove multiple edge formulas, and formulas for graphs consisting of chains of polygons. We show that the deletion-contraction relation can be lifted to the level of the category of mixed motives.

41. *Algebro-Geometric Feynman rules,* with Matilde Marcolli.
Int. J. Geom. Methods Mod. Phys. 8 (2011) 203-237

We introduce 'algebro-geometric' Feynman rules, with values in the Grothendieck groups of varieties (via motivic considerations) and in a polynomial ring (via characteristic classes), and discuss renormalization procedures and some motivic zeta functions.

40. *Parametric Feynman integrals and determinant hypersufaces,* with Matilde Marcolli.
Adv. Theor. Math. Phys. 14 (2010) 911-963

Modulo the issue of divergences, we express Feynman integrals on a graph as a period of the complement of a determinant hypersurface in a space of matrices with respect to certain loci determined by rank conditions. This provides evidence for a conjectural description of the integrals as periods of mixed Tate motives.

39. *New orientifold weak coupling limits in F-theory,* with Mboyo Esole.
J. High Energy Phys. (2010) no. 2, 1-53.

We produce new explicit examples of weak coupling limits in F-theory, obtaining new brane configurations that would not be admissible in the original framework of the limit constructed by Ashoke Sen. In our approach, limits depend on the choice of a transition from a semistable to an unstable singular fiber of an elliptic fibration. We also classify configurations of smooth D-branes satisfying the tadpole condition.

38. *Chern classes of blow-ups.*
Math Proc. of the Cambridge Phil. Soc. (2010) 148: 227-242.

We generalize a classic formula of Porteous, computing the Chern classes of a blow-up. The classic formula deals with the nonsingular case, while our generalization works whenever the center of the blow-up is regularly embedded.

37. *Limits of PGL-translates of plane curves, II,* with C. Faber.
Journal of Pure and Applied Algebra 214 (2010), 548-564.

We continue the study of degenerations of plane curves, obtaining certain numerical invariants necessary for applications to enumerative geometry.

36. *Limits of PGL-translates of plane curves, I,* with C. Faber.
Journal of Pure and Applied Algebra 214 (2010), 526-547.

We study the degenerations of plane curves under the action of one-parameter families of linear transformations.

35. *Feynman motives of banana graphs,* with Matilde Marcolli.
Comm. in Number Theory and Physics (2009) 1-57.

We study an infinite family of graphs known as 'banana graphs' from an algebro-geometric, intersection-theoretic viewpoint, with the goal of obtaining information about corresponding Feynman integrals.

34. *Chern class identities from tadpole matching in type IIB and F-theory,*
with Mboyo Esole.
J. High Energy Phys. (2009) no. 3, 032, 29 pp.

We prove a family of Chern class identities for elliptic fibrations, conjectured on the basis of considerations in string theory.

33. *Chern classes of Schubert cells and varieties,* with Leonardo Constantin Mihalcea.
J. Algebraic Geom. 18 (2009), 37-61.

We give explicit formulas for the Chern-Schwartz-MacPherson classes of all Schubert varieties in the Grassmannian of d-planes in a vector space, and conjecture that these classes are effective. We prove this is the case for (very) small values of d.

32. *Une nouvelle preuve de la concordance des classes définies par M.-H. Schwartz et par R. MacPherson,* with Jean-Paul Brasselet.
Bull. Soc. math. France 136 (2008) 159-166.

We give a very short proof of the identity between different definitions of Chern classes for singular varieties.

31. *Celestial integration, stringy invariants, and Chern-Schwartz-MacPherson classes,*
in 'Real and Complex Singularities', J.-P. Brasselet, M.A. Soares Ruas Eds., Birkhäuser (2007) 1–14.

We survey the notion of integration on the category of modifications of a variety, that we defined in 28., and several applications.

30. *Limits of Chow groups, and a new construction of Chern-Schwartz-MacPherson classes.*
Pure Appl. Math. Q., Robert MacPherson special issue, part II, 2 (2006), 915–942.

We introduce a new 'proChow functor', and use it to give a new construction of Chern-Schwartz-MacPherson classes, providing a new proof of a conjecture of Deligne and Grothendieck.

29. *Classes de Chern pour variétés singulières, revisitées,*
C. R. Math. Acad. Sci. Paris 342 (2006), no. 6, 405–410.

We summarize a new construction of Chern-Schwartz-MacPherson classes, and use it to give very short proofs of (generalizations of) two known results on these classes.

28. *Modification systems and integration in their Chow groups,*
Selecta Mathematica 11 (2005) 155-202.

We introduce a formal integration on the category of modifications of a given variety. This leads to new invariants, including 'stringy' Chern classes, and is closely related to the theory of Chern-Schwartz-MacPherson classes.

27. *Lectures on characteristic classes of singular varieties,*
in 'Topics in cohomological studies of algebraic varieties', Birkhäuser (2005) 1–32.

Notes for a cycle of five lectures given at the Banach Center, Warsaw, in April 2002.

26. *Chern classes of birational varieties,*
International Mathematics Research Notices 63 (2004), 3367-3377.

We prove that the Chern classes of birational varieties 'with the same canonical class' are images of the same class in a resolution of indeterminacies of any birational map between them. This is an analog of a celebrated theorem of Batyrev, asserting the equality of the Betti numbers of varieties satisfying the same condition.

25. *Shadows of blow-up algebras,*
Tohoku Math. J. (2) 56 (2004), 593-619.

We introduce a new 'blow-up algebra' interpolating between the Rees and Symmetric algebras of an ideal, and use it to give a new construction of the Chern-Schwartz-MacPherson class of a hypersurface in a nonsingular variety.

24. *Inclusion-exclusion and Segre classes, II,*
Contemp. Math. 324 (2003) 51–61.

We pursue the exploration of 'inclusion-exclusion' principles in the theory of Segre classes, by proving such a principle holds for another variation on the notion of Segre class. This is used to provide a simple computation of the classes introduced in our previous work, in certain special (but representative) cases.

23. *Inclusion-exclusion and Segre classes,*
Comm. Algebra 31 (2003) 3619–3630.

We propose a variation of the notion of Segre class, by forcing a naive 'inclusion-exclusion' principle to hold. We establish several general properties of the new class, and obtain an expression for the Milnor class of an arbitrary scheme in terms of this class.

22. *Interpolation of characteristic classes of singular hypersurfaces,* joint with Jean-Paul Brasselet,
Advances in Math 180 (2003) 692-704.

We show that Chern-Schwartz-MacPherson's classes of a hypersurface 'interpolate' two other notions of characteristic classes, provided that a regularity condition on the singular locus is satisfied. We apply this result to obtain a lift of Chern-Schwartz-MacPherson's classes to intersection homology for the same class of hypersurfaces.

21. *Computing characteristic classes for projective schemes,*
Journal of Symbolic Computation 35 (2003) 3-19.

We present an algorithm (implemented in `Macaulay2`) computing several classes associated to a projective scheme, given its defining homogeneous ideal.

20. *Linear orbits of arbitrary plane curves,* joint with Carel Faber,
Michigan Math. Jour., 48 (2000), 1-37.

We obtain an algorithm computing the degree of the closure of the linear orbit of an arbitrary plane curve, and give explicit formulas for plane curves with irreducible singularities.

19. *Plane curves with small linear orbits II,* joint with Carel Faber,
International Journal of Mathematics, 11 (2000) 591–608.

The completion of the study of plane curves with positive dimensional stabilizer under the action of the group of projective linear transformations of the plane. In this work we analyze curves obtained as unions of lines.

18. *Plane curves with small linear orbits I,* joint with Carel Faber,
Annales de l'institut Fourier, 50 (2000) 151–196.

A study of the enumerative geometry of the orbits of most plane curves with positive dimensional stabilizer under the action of the group of projective linear transformations of the plane.

17. *Differential forms with logarithmic poles and Chern-Schwartz-MacPherson classes of singular varieties,*
Comptes Rendus de l'Académie des Sciences, Série I, 329 (1999), 619–624.

We prove a formula for Schwartz-MacPherson's chern class of a singular variety in terms of the classes of the sheaf of differential forms with logarithmic poles along the components of a divisor arising in the resolution of singularities of the variety. As an application, we obtain a formula for the class of a singular hypersurface in terms of the Mather-Chern class of a suitable sheaf.

16. *Weighted Chern-Mather classes and Milnor classes of hypersurfaces,*
in 'Singularities and Arrangements, Sapporo-Tokyo 1998', Advanced Studies in Pure Mathematics 29, p.1–20.

We extend the notion of Chern-Mather class to possibly nonreduced schemes, and show how this new notion can be used to compute the difference between Schwartz-MacPherson's chern class and the class of the virtual tangent bundle of a singular hypersurface of a nonsingular variety.

15. *Chern classes for singular hypersurfaces,*
Trans. Amer. Math. Soc. 351 (1999) 3989–4026.

A new formula relating Fulton's Chern class and MacPherson's Chern class of a possibly singular hypersurface in a nonsingular variety is proved, and applied to Segre class and Chern class computations.

14. *Characteristic classes of discriminants and enumerative geometry,*
Comm. in Algebra 26(10) (1998) 3165–3193.

We show how computing characteristic classes of strata of the discriminant of a linear system on a nonsingular variety would lead to enumerative results, and carry out these computations in low codimensions. Also, we compute the classes for all strata of the discriminant of d-tuples of points on the projective line, and of cubic curves in the projective plane.

13. *Singular schemes of hypersurfaces,*
Duke Math. Journal, 80 (1995) 325–351.

A class is introduced generalizing the notion of Milnor number to possibly nonisolated hypersurface singularities. This is used to recover several results in duality theory, as well as producing new constraints for a scheme to be the jacobian scheme of a hypersurface in a smooth variety.

12. *A blow-up construction and graph coloring,*
Discrete Math., 145 (1995) 11–35.

A nonsingular algebraic variety is constructed, encoding the incidence information of a given graph G, and is used to obtain the chromatic polynomial of G in terms of intersection-theoretic information.

11. *A remark on the Chern class of a tensor product,* with Carel Faber,
Manu. Math. 88 (1995) 85–86.

A short note showing that the $(r+1)$-st Chern class of a rank-r element α in the Grothendieck group of vector bundles over a scheme does not change if α is tensored by a line bundle.

10. *MacPherson's and Fulton's classes of hypersurfaces,*
International Mathematics Research Notices (1994), 455–465.

It is shown that MacPherson's Chern class of a hypersurface in a smooth variety agrees, at least numerically, with a construction inspired by Fulton's canonical class of a scheme.

9. *Linear orbits of d-tuples of points in P^1,*
with Carel Faber, Jour. für die R. und Ang. Math. 444 (1993), 205–220.

   Orbit closures of sets of points of the projective line under the action of the automorphism group of the latter are studied in terms of their degree and multiplicity along their boundary.

8. *Multiplicities of discriminants,* with Fernando Cukierman,
Manu. Math. 78 (1993), 245–258.

   The multiplicity of the discriminant of a line bundle over a nonsingular variety at a section is computed in terms of the Segre class of the jacobian scheme of the section.

7. *Linear orbits of smooth plane curves,*
with Carel Faber, Journal of Alg. Geom. 2 (1993) 155–184.

   A desingularization of the orbit closures of plane curves under the action of the automorphism group of the plane is constructed and used to study such orbits.

6. *Two characteristic numbers for smooth plane curves of any degree,*
Trans. of the Amer. Math. Soc., vol. 329 (1992), 73–96.

   A sequence of blow-ups over projective spaces parametrizing plane curves is used to obtain enumerative results concerning smooth plane curves of arbitrary degree.

5. *Some characteristic numbers for nodal and cuspidal plane curves of any degree,*
Manu. Math. 72 (1991), 425–444.

   A compactification of the space of reduced plane curves is used to compute characteristic numbers for singular plane curves of arbitrary degree.

4. *The enumerative geometry of plane cubics II: nodal and cuspidal cubics,*
Math. Annalen 289 (1991), 543–572.

   A variety of complete cubics is used to study the enumerative geometry of several families of irreducible singular plane cubics.

3. *How many smooth plane cubics with given j-invariant are tangent to 8 lines in general position?,*
Contemporary Mathematics 123 (1991), 15–29.

   Formulas are obtained for the characteristic numbers of families parametrized by hypersurfaces in the projective space of plane cubics.

2. *The enumerative geometry of plane cubics I: smooth cubics,*
Trans. of the Amer. Math. Soc., vol. 317 (1990), 501–539.

   A variety of complete plane cubics is constructed and employed to obtain a thorough analysis of the enumerative geometry of smooth cubics.

1. *The characteristic numbers for smooth plane cubics,*
Proceedings of 'Algebraic Geometry, Sundance 1986', Springer Lecture Notes 1311, 1–8.

   The characteristic numbers for the family of smooth plane cubics are computed, verifying results of Maillard and Zeuthen.

Ph.D. students

- **Grayson Jorgenson, Ph.D. 2020**
  Dissertation: *Secant Indices, Duality Defect, and Generalizations of the Segre Zeta Function*
- **Xiping Zhang, Ph.D. 2018**
  Dissertation: *Characteristic Classes and Local Invariants of Determinantal Varieties and a Formula for Equivariant Chern-Schwartz-MacPherson Classes of Hypersurfaces*
- **Corey Harris, Ph.D. 2017**
  Dissertation: *Effective methods in intersection theory and combinatorial algebraic geometry*
- **William Adams, Ph.D. 2015**
  Dissertation: *Lagrangian Specialization via Log Resolutions and Schwartz-MacPherson Chern Classes*
- **Xia Liao, Ph.D. 2013**
  Dissertation: *Chern classes of sheaves of logarithmic vector fields for free divisors*
- **James Fullwood, Ph.D. 2012**
  Dissertation: *On elliptic fibrations and F-theory compactifications of string vacua*
- **Judson Stryker, Ph.D. 2011**
  Dissertation: *Chern-Schwartz-MacPherson classes of graph hypersurfaces and Schubert varieties*
- **Dimitre Tzigantchev, Ph.D. 2006**
  Dissertation: *Predegree polynomials of plane configurations in projective space*
- **Deborah Jones, Ph.D. 2003**
  Dissertation: *Intersection numbers of divisors in graph varieties*

— Current doctoral students:

- **Franquiz Caraballo-Alba.**
- **Brandon Story.**

— Doctoral students co-directed:

- **Yunyi Shen, Ph.D. 2017.** Major professor: Prof. Matilde Marcolli (Caltech).
  Dissertation: *Arithmetic aspects of noncommutative geometry: Motives of noncommutative tori and phase transitions on $GL(N)$ and Shimura varieties systems*
- **Dan Li, Ph.D. 2012.** Major professor: Prof. Matilde Marcolli (Caltech).
  Dissertation: *Periods and motives: applications in mathematical physics*

Invited talks (last 5 years)

2023 **University of Florida (online)**, *Gainesville, RI*, Algebra seminar, (J. Booher).
Segre classes and Lorentzian polynomials

2023 **University of Kiel**, *Kiel, Germany*, Conference on Characteristic Classes and Singular Spaces, (T. Essig, I. Pallarés, G. Peñafort, M. Zach).
Segre classes and Lorentzian polynomials

2023 **Brown University**, *Providence, RI*, Algebra seminar, (D. Abramovich).
Segre classes and Lorentzian polynomials

2022 **Mittag-Leffler Institute (online)**, *Stockholm, Sweden*, Moduli of Curves in Stockholm: A conference on the occasion of Carel Faber's 60th birthday, (J. Bergström, O. Bergvall, L. Halvard Halle, N. Pagani, D. Petersen).
Segre classes, old and new

2021 **SISSA, Italy (online)**, Algebraic Geometry Seminar, (A. Ricolfi).
Newton-Okounkov bodies and Segre classes

2021 **Columbus, Ohio (online)**, Degeneracy loci and applications, (D. Anderson, J. Kiers).
Chern Classes of Embeddable Schemes

2019 **Padua, Italy**, MathemAmplitudes 2019: Intersection Theory & Feynman Integrals, (H. Frellesvig, S. Laporta, M. K. Mandal, P. Mastrolia, S. Mizera).
Characteristic classes in Intersection Theory

2019 **Curso de Doctorado**, *Universidad de Valladolid*, Valladolid, Spain, (D. Camazon, A. Campillo, S. Encinas).
Introduction to intersection theory and characteristic classes for singular varieties. 4 lectures.

2019 **Ohio State University**, *Columbus, OH*, Algebraic Geometry Seminar, (D. Anderson).
Newton-Okounkov bodies and Segre classes

2019 **ASGARD Math 2019**, *University of Oslo*, Oslo, Norway, (C. Harris, F. Rincon, K. Shaw).
Characteristic classes of singular varieties. 3 lectures.

2019 **Honolulu, HI**, AMS meeting #1147, Special Session on Real and Complex Singularities, (G. Ishikawa, D. Trotman, L. Wilson).
Newton-Okounkov bodies and Segre classes

2019 **Kagoshima, Japan**, 14th Algebra-Analysis-Geometry Workshop, (T. Ohmoto, S. Kimura, O. Saeki, K. Takeuchi, H. Ishida, M. Murakami).
Newton-Okounkov bodies and Segre classes

2018 **University of Michigan**, *Ann Arbor, MI*, Algebraic Geometry Seminar, (W. Fulton, M. Mustață).
Newton-Okounkov bodies and Segre classes

2018 **Northeastern University**, *Boston, MA*, Geometry, Physics, and Representation Theory Seminar, (I. Martino).
Newton-Okounkov bodies and Segre classes

2018 **Simons Foundation**, *New York, NY*, MPS Conference on Singularities: Geometric, Topological, and Analytic Aspects—in honor of S. Cappell, (M. Banagl, G. Friedman, S. Weinberger, R. Young).
Chern Classes of Embeddable Schemes

2018 **Boston, MA**, AMS meeting #1139, Special session on Singularities of Spaces and Maps, (T. Gaffney, D. Massey).
Segre zeta functions

2018 **Worldwide center of Mathematics**, *Cambridge, MA*, (D. Massey).
Segre classes and other intersection-theoretic invariants. 3 lectures.

2018 **Nashville, TN**, AMS meeting #1138, Special session on Algebraic Geometry, Representation Theory, and Applications, (S. Kumar, J.M. Landsberg, L. Oeding).
Degrees of projections of rank loci

2018 **University of North Carolina**, *Chapel Hill, NC*, Algebra seminar, (R. Rimanyi).
Segre zeta functions

Visits (≥ 1 month)

Jan.-Aug. 2023 **California Institute of Technology**.
Jan.-Aug. 2022 **California Institute of Technology**.
Aug.'20-Aug.'21 **California Institute of Technology**.
May-July 2020 **University of Toronto**.
May-Aug. 2019 **University of Toronto**.

Jan.-June 2018	**University of Toronto**.
May-June 2017	**California Institute of Technology**.
May'15-July'16	**California Institute of Technology**.
May-July 2014	**California Institute of Technology**.
June-July 2013	**California Institute of Technology**.
Jan.-July 2012	**California Institute of Technology**.
Summer 2011	**California Institute of Technology**.
Summer 2010	**California Institute of Technology**.
July 2009	**Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
Jan.-April 2009	**Mathematical Sciences Research Institute**, *Berkeley*.
Sep.-Dec. 2008	**California Institute of Technology**.
May-July 2008	**Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
June-July 2007	**Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
April-May 2007	**Mittag-Leffler Institut**, *Stockholm*, Sweden.
Jan.-Mar. 2007	**Institut de Mathématiques de Luminy**, *France*.
June-July 2006	**Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
June-July 2005	**Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
May 2005	**Institut de Mathématiques de Luminy**, *France*.
May-July 2004	**Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
May-July 2003	**Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
Sep. 01-July 02	**Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
May-July 2001	**Institut de Mathématiques de Luminy**, *France*.
July 2000	**Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
Sep.-Dec. 1999	**Harvard University**.
Sep. 96-Apr. 97	**Mittag-Leffler Institut**, *Stockholm*, Sweden.
June-July 1996	**Mathematisches Forschungsinstitut**, *Oberwolfach*, Germany.
May 1996	**University of Chicago**.
May 1995	**University of Chicago**.
May 1994	**University of Chicago**.
Jan.-June 1993	**Mathematical Sciences Research Institute**, *Berkeley*.
Jan.-June 1992	**Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
May-July 1990	**Mathematisches Institut**, *Erlangen*, Germany.

Service (to the profession)

- Managing Editor, *Journal of Singularities,* 2014—present (Duties shared with Javier Fernández de Bobadilla and Victor Goryunov)
- Associate Editor, *Journal of Singularities,* 2009–14 (David Massey, Editor-in-Chief)
- Editor, *ISRN Geometry,* 2011–12

- Refereeing/preliminary evaluations of papers for publications: Acta Mathematica; Advances in Mathematics (2); Advances in Applied Mathematics (2); Algebraic Combinatorics; Annales de l'Institut Fourier (2); Annales scientifiques de l'École normale supérieure; Annali dell'Università di Ferrara; Arkiv for Matematik; Aspects of Mathematics; Asian Journal of Mathematics; American Journal of Mathematics (2); Birkhäuser; Bulletin of the American Math Society; Bulletin of the London Math Society (2); Canadian Journal of Mathematics; Canadian Mathematical Bulletin (2); Central European Journal of Mathematics; Communications in Algebra (3); Communications in Pure and Applied Mathematics; Compositio Mathematica (4); Comptes Rendus (2); Contemporary Mathematics (4); Duke Math Journal; Forum of Mathematics, Sigma; Geometriae Dedicata; Geometry and Topology (6); Handbook of Singularities; Impanga Lecture Notes; IMRN (7); International Journal of Algebra and Computation; Inventiones Mathematicae (2); Journal of Algebra (2); Journal of Algebraic Geometry (4); Journal of Complexity; Journal of Geometry; Journal of Geometry and Physics (2); Journal of Mathematics and Music; Journal of Pure and Applied Algebra (2); Journal of Singularities (2); Journal of Software for Algebra and Geometry (3); Journal of the Australian Math Society; Journal of Topology (3); K-theory; Le Matematiche; Mathematica Scandinavica; MEGA 2013 (3); Mathematics of Computer Science; Mathematische Zeitschrift (2); Michigan Mathematical Journal (1+1); Proceedings of the American Mathematical Society (3); Pure and Applied Math Quarterly (2); Rendiconti del circolo matematico di Palermo; Selecta Mathematica (1+1); SIAM Journal on Applied Algebra and Geometry; Springer Lecture Notes; Topology and its applications; Transactions of the American Mathematical Society (6); Transformation groups.
- Grant reviewing: N.S.A. (4), N.S.F. (25); Swedish Research Council (8); Dutch NWO 'Veni' and 'Vidi' programs.
- NSF panelist, (2003).
- Co-organizer (with Ettore Aldrovandi and Eriko Hironaka) of the Special Session on Algebraic Geometry and Topology at the Sectional Meeting #994 of the AMS, Tallahassee, March 12-13, 2004.
- Mathematical Reviews (77 reviews)
- Outside Ph.D. thesis examiner for Dmitry Kerner, Tel-Aviv University, 2007
(Thesis: 'Enumeration of singular hypersurfaces'; E. Shustin, Ph.D. advisor)
- Outside evaluator of the mathematics program at Bethune-Cookman University, May 2009
- Co-organizer (with Mboyo Esole) of the Special Session on Singularities and Physics at the Sectional Meeting #1097 of the AMS, Knoxville, TN, March 21-23, 2014.
- Scientific committee, INPANGA '15, Będlewo, Poland.
- Evaluator, IMPAN International Fellowships, 2015.
- Co-organizer (with Lara Anderson, Mboyo Esole, Shing-Tung Yau) of a three-workshop series on the mathematics and physics of F-theory: Virginia Tech (2016), Harvard (2018), Florida State (2019).
- NSF panelist, (2017).
- Co-organizer (with D. Anderson, M. Hering, M. Mustata, S. Payne) of 'Facets of Algebraic Geometry', a conference in honor of W. Fulton's 80th birthday, Ann Arbor, MI, October 18-20, 2019.
- Co-organizer (with L. Mihalcea) of the Special Session on Singularities and Characteristic Classes at the AMS National Meeting, Denver, CO, January 15-18, 2020.
- Outside Ph.D. thesis examiner for Irma Pallarés Torres, Basque Center for Applied Mathematics, 2021.
- Distinguished speaker at Pi Mu Epsilon induction ceremony, 2023

Service (departmental/university)

— Director of pure mathematics: 2005-07 (through Fall '07)

Duties included: revision of graduate curriculum; organization of graduate and undergraduate courses; graduate student recruitment and advising; membership in the executive, faculty evaluation, and hiring committees.

— Co-director of pure mathematics: Spring 2015.

— Committees:

- Brennan Professor selection committee: 2014, 2015, 2018, 2021
- Chair nomination (mathematics): 2004-05 (chair); 2011-12; 2020-21
- Chair nomination (physics): 2021-22
- Chair nomination (statistics): 2007-08
- COFRS evaluation: 2011, 2014, 2015, 2019, 2021
- Colloquium: 1998-99
- Curriculum: 1993-96; 2022-3
- Doctoral preliminary examination: 1999-2001; 2002-03; 2003-05 (chair); 2009-10; 2014-15; 2017-18; 2020-1; 2022-3
- Election: 2002-04
- Executive: 1999-2000; 2005-07 (through fall '07); 2012-18
- Facilities and technology (self-study): 1992-93
- Faculty evaluation: 2000-01; 2002 (fall); 2004 (spring); 2004-07 (through fall '07); spring 2013-15; fall 2017-19; fall 2022
- FSU math society: 1995-96
- Graduate: 2006-07, 2010, 2015
- Graduate student recruitment and financial aid: 1997-2001; 2002-05
- GPC subcommittee, department member, 2013
- GPC subcommittee evaluating the Accounting department, chair, 2021
- Hiring: 1999-2000; 2004-05; 2017-8; 2022-3
- Honors Policy: 2021
- iChange team: 2019-21
- Library: 1994-96, 1997-2001
- Mathematics Research Enhancement Program (chair): 2019-present
- Preliminary examination policy (chair): 1999-2000
- Pure mathematics: 2004-05 (chair)
- Quality Enhancement Review: 2003-04, 2012-13 (chair), 2019
- Retirement: 2003
- Science Area Promotion & Tenure Committee: 2017
- Study Abroad Scholarship Committee: 2016-19
- SPP peer evaluation: 2011
- TIP peer evaluation: 1994-95, 1998-99
- University Senate: 2000-01; 2006-08; 2019-present
- Undergraduate major recruitment and retention: Fall 2004 (chair)
- Visibility (e-print series): 2000-2022
- Department representative as 'Academic Sponsor' of the Mathematical Sciences Research Institute, 2020-present

— STUDENT COMMITTEES:

- *Advanced topics examination/Ph.D. candidacy:*
  W. Adams (Apr. 98, chair); D. Jones (Dec. 98, chair); B. Goforth (Apr. 99); C. Stockwell (Apr. 99); D. Tzigantchev (Jan. 03, chair); A. Novocin (Feb. 06); R. Todd (Oct. 06); G. Levy (Nov. 06); E. Tatar (Dec. 06); Y. Lebedev (Nov. 07); Q. Yuan (Nov. 07); Y. Cha (Jan. 08); J. Stryker (Apr. 08, chair); D. Li (Aug. 08); T. Fang (Nov. 09); X. Liao (Nov. 09, chair); I. Kunwar (Mar. 10); I. Dungan (Apr. 10); J. Fullwood (Apr. 10, chair); Y. Shen (Aug. 11); S. Aksoy (Oct. 12); M. Aktas (Dec. 13); C. Harris (Dec. 13, chair); E. Imamoglu (Jan. 14); L. Sparaco (Apr. 14); X. Zhang (Dec. 14, chair); Y. Valdes (Oct. 15); M. Niemeier (Dec. 16); G. Jorgenson (Oct. 17, chair); S. Aldossari (Jan. 18); S. Anbouhi (Apr. 19); Y. Zhou (Apr. 19); A. Malik (Apr. 21); F. Caraballo-Alba (Dec. 21, chair); H. Badri Bou KedBey (Dec. 22).
- *Master's committee:*
  J. de Marco (2004, chair).
- *Honors thesis committee:*
  —R. Orlando, 'A Study of Physics Beyond the Standard Model' (H. Prosper, chair; 2018).
  —C. Mooneyham, 'Classical Topology and Combinatorial Group Theory' (E. Klassen, chair; 2020)
- *Ph.D. committees (mathematics):*
  —B. Goforth (April 2005, dissertation: 'Description and analysis of a two-variable version of the NTRU crypto system'. Sam Huckaba, major professor).
  —A. Novocin (April 2008, dissertation: 'Factoring Univariate Polynomials over the Rationals'. Mark van Hoeij, major professor).
  —Y. Lebedev (November 2008, dissertation: 'Openmath library for computing on Riemann surfaces'. Mika Seppälä, major professor).
  —G. Levy (December 2009, dissertation: 'Solutions of Second Order Recurrence Relations'. Mark van Hoeij, major professor).
  —E. Tatar (May 2010, dissertation: 'Picard 2-Stacks and length 3 complexes of abelian sheaves'. Ettore Aldrovandi, major professor).
  —Y. Cha (December 2010, dissertation: 'Closed form solutions of linear difference equations'. Mark van Hoeij, major professor).
  —Q. Yuan (March 2012, dissertation: 'Finding all Bessel type solutions for linear differential equations with rational function coefficients'. Mark van Hoeij, major professor).
  —T. Fang (October 2012, dissertation: 'Solving linear differential equations in tterms of hypergeometric functions by 2-descent'. Mark van Hoeij, major professor).
  —I. Dungan (April 2014, dissertation: 'N-butterflies: modeling weak morphisms of strict N-groups'. Ettore Aldrovandi, major professor).
  —M. Aktas (April 2017, dissertation: 'Topology of N-Gonal Curve Complements'. Eriko Hironaka and Washington Mio, major professors).
  —E. Imamoglu (May 2017, dissertation: 'Algorithms for solving linear differential equations with rational function coefficients'. Mark van Hoeij, major professor).
  —W. Xu (November 2017, dissertation: 'Third order A-hypergeometric functions'. Mark van Hoeij, major professor).
  —Y. Valdes (April 2018, dissertation: 'The 1-type of algebraic K-Theory as a multifunctor'. Ettore Aldrovandi, major professor).

- *Ph.D. committees (mathematics), continued:*
—M. Niemeier (June 2020, dissertation: 'Central extensions of simplicial groups and presheaves of simplicial groups'. Ettore Aldrovandi, major professor).
—S. Aldossari (July 2020, dissertation: 'Algorithms for Simplifying Differential Equations'. Mark van Hoeij, major professor).
—Y. Zhou (April 2022, dissertation: 'Algorithms for factoring linear recurrence operators'. Mark van Hoeij, major professor).
—S. Anbouhi (June 2022, dissertation: 'Universal Mappings and the Metric Geometry of Functional Data'. Washington Mio, major professor).
- *Ph.D. committees (university representative):*
—Fernando Febres Cordero (Physics, 2004 prospectus; 2007 Ph.D. dissertation: 'Next-to-leading-order corrections to weak boson production with a massive quark jet pair at hadron colliders'. Laura Reina, major professor)
—Farrooh Fattoyev (Physics, 2009 prospectus; 2011 Ph.D. dissertation: 'Sensitivity of neutron star properties to the equation of state'. Jorge Pieckarevicz, major professor)
—Paulo Rottmann (Physics, 2011 Ph.D. dissertation: 'Z-sum approach to loop integrals'. Laura Reina, major professor)
—Benjamin Thayer (Physics, 2012 Ph.D. dissertation: 'On-shell methods applied to exotic Higgs production at hadronic colliders'. Laura Reina, major professor)
—Heribertus Hartanto (Physics, 2010 prospectus; 2013 Ph.D. dissertation: 'Top-Quark associated production with one hard photon at hadron colliders'. Laura Reina, major professor)
—Steve Honeywell (Physics, 2014 prospectus; 2017 Ph.D. dissertation: 'Automated one-loop QCD and electroweak calculations with NLOX'. Laura Reina, major professor)
—Lewis Jeter (Music, 2016 preliminary exam. Cliff Callender and Nancy Rogers, major professors)

- *Ph.D. committees (university representative, continued):*
—Lauren Hartburg (Music, 2019 preliminary exam; 2021 Ph.D. dissertation: 'Uniform triadic transformation spaces: structures and sequences in the triadic music of Alfred Schnittke'. Evan Jones, major professor)
—Jennifer Diane Harding (Music, 2021 Ph.D. dissertation: 'Applications of the discrete Fourier transform to music analysis'. Michael Buchler, major professor)
—Irene Roman (Physics, 2022 prospectus. Fernando Cordero, major professor)

Teaching, Florida State University (1991-present)

— Graduate:
- *Abstract algebra I:* Fall 1995; Fall 2003
- *Abstract algebra II:* Spring 1996
- *Groups, Rings, Vector Spaces I:* Fall 1994; Fall 2002; Fall 2004; Fall 2005; Fall 2009; Fall 2013; Fall 2016; Fall 2019; Fall 2022
- *Groups, Rings, Vector Spaces II:* Spring 1995; Spring 2003; Spring 2005; Spring 2006; Spring 2010; Spring 2014; Spring 2017; Spring 2020
- *Groups, Rings, Vector Spaces III:* Fall 2005; Fall 2006; Fall 2010; Fall 2014; Fall 2017; Fall 2020; Fall 2023
- *Algebraic geometry (topics course):* Fall 1993; Spring 1994; Fall 1997; Spring 1998; Fall 2000; Spring 2006; Spring 2014; Spring 2015; Spring 2017; Spring 2019; Fall 2019; Fall 2021
- *Homological and commutative algebra (topics course):* Spring 2011
- *Euler calculus (topics course):* Fall 2012

- *Algebra seminar:* Fall 1992; Spring 2000; Fall 2000; Fall 2018
- *Algebraic geometry seminar:* Fall 1998; Spring 1999; Spring 2006; Fall 2009; Spring 2010; Fall 2010, Spring 2011

— Undergraduate:

- *Applied Linear Algebra I:* Fall 2007
- *Calculus I:* Fall 1991, 2 sections
- *Calculus II:* Spring 1995
- *Calculus III:* Fall 1992, 2 sections; Fall 1994; Spring 1996; Fall 1998; Spring 2000; Fall 2000; Fall 2002; Fall 2003
- *College geometry:* Fall 1998
- *Complex variables:* Fall 2016
- *Discrete mathematics I:* Fall 1993; Fall 1995
- *Discrete mathematics II:* Spring 1994
- *Introduction to abstract algebra I:* Fall 1997; Fall 2000; Fall 2003; Fall 2004; Fall 2006; Fall 2007; Fall 2014; Fall 2017; Fall 2018; Fall 2021; Fall 2023
- *Introduction to abstract algebra II:* Spring 1998; Spring 2005; Spring 2008; Spring 2015; Spring 2019
- *Introduction to advanced mathematics:* Spring 1999; Spring 2010; Fall 2010; Fall 2011; Fall 2012; Fall 2013; Fall 2020; Fall 2022
- *Introduction to real analysis:* Fall 2009
- *Mathematical logic:* Spring 2011
- *Number theory:* Spring 2020

Teaching, other institutions

— Brown University (1984–87):

- Elementary Calculus I, II; Intermediate Calculus I, II.

— The University of Chicago (1987-89):

- Honors Calculus, I, II, III; Abstract Algebra I, II, III; Introduction to Algebraic Geometry.

— Oklahoma State University (1989-91):

- Business Calculus, Calculus II, Differential Equations, Linear Algebra, Combinatorics, Complex Analysis.

— KTH, Stockholm, Sweden (January-April 1997):

- Intersection theory and enumerative geometry.

— California Institute of Technology :

- Homological algebra (Fall 2008).
- Algebraic geometry (Winter 2021).
- Introduction to Abstract Algebra (Winter 2022, Winter 2023).