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Personal

Born: **6/14/1960 in Turin, Italy.**

Citizenship: **Dual Italian/U.S.A..**

Education

May 1987 **Ph.D. in Mathematics**, *Brown University*, Providence, RI, USA.
(W. Fulton, thesis advisor)

- Dissertation: On some characteristic numbers for smooth plane curves

July 1983 **Laurea in Matematica**, *Università degli Studi di Torino*, Italy.

(A. Collino, relatore)

- Tesi: Elementi di teoria delle varietà abeliane; Jacobiana associata ad una curva.
Un criterio di caratterizzazione delle Jacobiane

Academic positions

2018-present **Status-only full professor**, *University of Toronto*.

1/2012-12/2014 **Marion Bradley Brennan Professor**, *Florida State University*.

2009-present **Visiting Associate/Visitor in Mathematics**, *California Institute of Technology*.

2000-present **Professor**, *Florida State University*.

1995-2000 **Associate Professor**, *Florida State University*.

1991-1995 **Assistant Professor**, *Florida State University*.

1989-1991 **Visiting Assistant Professor**, *Oklahoma State University*.

1987-1989 **Dickson Instructor**, *University of Chicago*.

Grants and awards

2016-18 **NSA**, award #H98230-16-1-0016, - Project title:
Theory and applications of Segre classes and related intersection theoretic invariants.

2016-19 **NSF**, award #1603247 (co-PI, conference grant):
A Three-Workshop Series on the Mathematics and Physics of F-theory.

2015-16 **NSA**, award #H98230-15-1-0027, - Project title:
Segre classes as integrals over polytopes and other problems in intersection theory.

2013 **FSU**, *COFRS* award. - Project title:
Potts models, graph hypersurfaces, and Chern class identities in string theory.

2012-17 **Simons Foundation**, *Collaboration grant*.
- Project title: *Algebraic geometry, singularities, and applications to physics.*

2009 **FSU**, *COFRS* award. - Project title: *Geometry and Physics.*

- 2007–09 **NSA**, award #H98230-07-1-0024,
- Project title: *Limits in Intersection Theory and Invariants of Singular Spaces.*
- 2007–08 **FSU**, University Teaching award.
- 2004–05 **NSA**, award #H98230-04-1-0112,
- Project title: *Blow-up algebras, characteristic classes, and enumerative geometry.*
- 2000–01 **FSU**, University Teaching award.
- 1998 **FSU**, Developing Scholar award.
- 1995 **FSU**, Teaching Incentive Program award.
- 1995–98 **NSF**, award #9500843, - Project title: *Topics in Intersection Theory.*
- 1994–95 **FSU**, University Teaching award.
- 1993 **FSU**, First Year Assistant Professor summer award.
- 1987–89 **NSF**, Postdoctoral associate (PI: W. Fulton)
- Project title: *Segre classes and intersection formulas; applications to the enumerative geometry of plane curves.*

Publications

Full abstracts and links to most of the publications listed here may be found at <http://www.math.fsu.edu/~aluffi/mainhtmls/pubs.html>

— BOOKS / PROCEEDINGS VOLUMES

4. *Proceedings of the Special Session on Singularities and Physics, Knoxville, TN, 21-23 March 2014.* 149 pages, 2016.
Paolo Aluffi and Mboyo Esole, editors. Journal of Singularities, Volume 15
A Special Session on Singularities and Physics was organized for the AMS Sectional Meeting held in Knoxville, TN on March 21-23, 2014. The session focused on the theory of singularities and its interactions with different branches of theoretical physics: singularities of elliptic fibrations in string theory, renormalization issues in quantum field theory, Landau-Ginzburg models, wall-crossing phenomena, and other recent points of contact. This volume collects articles written by some of the speakers on the occasion of this meeting, expanding on the content of their talks or reporting on research originated in discussions begun at the conference.
3. *Algebra: Chapter 0.* 713 pages, 2009.
American Mathematical Society, Graduate Studies in Mathematics Volume 104
A textbook for upper level undergraduate or beginning graduate courses in Algebra, including an introduction to homological algebra. The material is presented via a categorical approach emphasizing universal properties. The book includes more than 1000 exercises.
2. *Fare Matematica — Astratto e concreto nella matematica elementare.* 144 pages, 2009.
Aracne Editrice. In Italian.
This book is aimed at the general public. It consists of a brief introduction to a few elementary themes in mathematics, emphasizing the interplay between the abstractness of the material and the concreteness of the mathematical approach to it.

1. *Quantum cohomology at the Mittag-Leffler Institute*. 163 pages, 1998. Appunti della Scuola Normale Superiore di Pisa.

A write-up of talks given in the Fall semester of 1996 by the participants to the year on “Enumerative geometry and its interactions with theoretical physics” at the Mittag-Leffler institute in Stockholm, Sweden.

— PREPRINTS

66. *Newton-Okounkov bodies and Segre classes*, 17 pages.

Given a homogeneous ideal in a polynomial ring over \mathbb{C} , we adapt the construction of Newton-Okounkov bodies to obtain a convex subset of Euclidean space such that a suitable integral over this set computes the *Segre zeta function* of the ideal. That is, we extract the numerical information of the Segre class of a subscheme of projective space from an associated (unbounded) Newton-Okounkov convex set. The result generalizes to arbitrary subschemes of projective space the numerical form of a previously known result for monomial schemes.

65. *The Chern-Schwartz-MacPherson class of an embeddable scheme*, 20 pages. arXiv:1805.11116

There is an explicit formula expressing the Chern-Schwartz-MacPherson class of a hypersurface in a nonsingular variety (in characteristic 0) in terms of the Segre class of its jacobian subscheme; this has been known for a number of years. We generalize this formula to arbitrary embeddable schemes: for every subscheme X of a nonsingular variety V , we define an associated subscheme Y of a projective bundle over V and provide an explicit formula for the Chern-Schwartz-MacPherson class of X in terms of the Segre class of Y . If X is a local complete intersection, a version of the result yields a direct expression for the Milnor class of X .

64. *Shadows of characteristic cycles, Verma modules, and positivity of Chern-Schwartz-MacPherson classes of Schubert cells*, with Leonardo Mihalcea, Jörg Schürmann, Changjian Su. 40 pages. arXiv:1709.08697.

We recast the theory of Chern-Schwartz-MacPherson (CSM) classes in the torus equivariant setting in terms of characteristic cycles of constructible functions. Specializing to (generalized) flag manifolds, we obtain a ‘Hecke orthogonality’ of CSM classes. We show that the characteristic cycle of the Verma module, restricted to the zero section, gives the CSM class of a Schubert cell. This establishes an equivalence between CSM classes and stable envelopes, reproving a result of Rimányi and Varchenko. This formula allows us to prove a conjecture by Aluffi and Mihalcea, and to extend previous positivity results by J. Huh in the Grassmann manifold case.

63. *The Euclidean distance degree of smooth complex projective varieties*, with Corey Harris. 25 pages. arXiv:1708.00024. To appear in *Algebra and Number Theory*.

We obtain several formulas for the Euclidean distance degree (ED degree) of an arbitrary nonsingular variety in projective space: in terms of Chern and Segre classes, Milnor classes, Chern-Schwartz-MacPherson classes, and an extremely simple formula equating the Euclidean distance degree of X with the Euler characteristic of an open subset of X .

62. *Projective duality and a Chern-Mather involution*, arXiv:1601.05427. Trans. Amer. Math. Soc. 370 (2018), no. 3, 1803–1822. doi:10.1090/tran/7042

We observe that linear relations among Chern-Mather classes of projective varieties are preserved by projective duality. We deduce the existence of an explicit involution on a part of the Chow ring of projective space, encoding the effect of duality on Chern-Mather classes. Applications include Plücker formulae, constraints on self-dual varieties, generalizations to singular varieties of classical formulas for the degree of the dual and the dual defect, formulas for the Euclidean distance degree, and computations of Chern-Mather classes and local Euler obstructions for cones.

61. *The Segre zeta function of an ideal*, arXiv:1606.03098. Advances in Mathematics 320 (2017) 1201–1226. doi:10.1016/j.aim.2017.09.023

We define a power series associated with a homogeneous ideal in a polynomial ring, encoding information on the Segre classes defined by extensions of the ideal in projective spaces of arbitrarily high dimension. We prove that this power series is rational, with poles corresponding to generators of the ideal, and with numerator of bounded degree and with nonnegative coefficients. We also prove that this ‘Segre zeta function’ only depends on the integral closure of the ideal.

60. *Tensored Segre classes*, arXiv:1605.09393. Journal of Pure and Applied Algebra 221 (2017) 1366–1382. doi:10.1016/j.jpaa.2016.09.016

We study a class obtained from the Segre class $s(Z, Y)$ of an embedding of schemes by incorporating the datum of a line bundle on Z . This class satisfies basic properties analogous to the ordinary Segre class, but leads to remarkably simple formulas in standard intersection-theoretic situations such as excess or residual intersections. We prove a formula for the behavior of this class under linear joins, and use this formula to prove that a ‘Segre zeta function’ associated with ideals generated by forms of the same degree is a rational function.

59. *How many hypersurfaces does it take to cut out a Segre class?*, Journal of Algebra 471 (2017) 480–491. doi:10.1016/j.jalgebra.2016.10.003

We prove an identity of Segre classes for zero-schemes of compatible sections of two vector bundles. Applications include bounds on the number of equations needed to cut out a scheme with the same Segre class as a given subscheme of (for example) a projective variety, and a ‘Segre-Bertini’ theorem controlling the behavior of Segre classes of singularity subschemes of hypersurfaces under general hyperplane sections.

58. *Chern-Schwartz-MacPherson classes for Schubert cells in flag manifolds*, with L. Mihailescu. arXiv:1508.01535. Compositio Math. 152 (2016), 2603–2625. doi:10.1112/S0010437X16007685.

We obtain an algorithm computing the Chern-Schwartz-MacPherson (CSM) classes of Schubert cells in a generalized flag manifold G/B . In analogy to how the ordinary divided difference operators act on Schubert classes, each CSM class of a Schubert class is obtained by applying certain Demazure-Lusztig type operators to the CSM class of a cell of dimension one less. These operators define a representation of the Weyl group on the homology of G/B . By functoriality, we deduce algorithmic expressions for CSM classes of Schubert cells in any flag manifold G/P . We conjecture that the CSM classes of Schubert cells are an effective combination of (homology) Schubert classes, and prove that this is the case in several classes of examples. We also extend our results and conjectures to the torus equivariant setting.

57. *Segre classes as integrals over polytopes*, arXiv:1307.0830.
 J. Eur. Math. Soc. 18 (2016) 2849–2863. doi:10.4171/JEMS/655
- We prove a formula for the Segre classes of monomial schemes as integrals over suitable Newton regions. The proof is based on the birational invariance of Segre classes, on a principalization algorithm for monomial schemes, and on careful considerations regarding simplicial decompositions of polytopes. The formula holds for *r.c. monomial schemes*, generalizing the monomial case to the singular setting and relaxing a transversality hypothesis.
56. *Multidegrees of monomial rational maps*, Publ. RIMS Kyoto Univ. 51 (2015), 635–654. doi:10.4171/PRIMS/167
- We prove a formula for the multidegrees of a rational map defined by generalized monomials on a projective variety, in terms of integrals over an associated Newton region. This formula leads to an expression of the multidegrees as volumes of related polytopes, in the spirit of the classical Bernstein-Kouchnirenko theorem, but extending the scope of these formulas to more general monomial maps. We also determine a condition under which the multidegrees may be computed in terms of the characteristic polynomial of an associated matrix.
55. *Degrees of projections of rank loci*, Experimental Mathematics, 24:469–488, 2015. doi:10.1080/10586458.2015.1028689
- We provide formulas for the degrees of the projections of the locus of square matrices with given rank from linear spaces spanned by a choice of matrix entries; the motivation for these computations stem from applications to ‘matrix rigidity’. Surprisingly (to us), these degrees appear to match the numbers of Kekulé structures of certain ‘benzenoid hydrocarbons’, and arise in many other contexts with no apparent direct connection to the enumerative geometry of rank conditions.
54. *Chern classes of splayed intersections*, with E. Faber, 18 pages. Canad. J. Math. 67(2015), 1201-1218. doi:10.4153/CJM-2015-010-7
- We generalize the Chern class relation for the transversal intersection of two non-singular varieties to a relation for possibly singular varieties, under a ‘splayedness’ assumption. The relation is shown to hold for both the Chern-Schwartz-MacPherson class and the Chern-Fulton class. The main tool is a formula for Segre classes of splayed subschemes. We also discuss the Chern class relation under the assumption that one of the varieties is a general very ample divisor.
53. *Log canonical threshold and Segre classes of monomial schemes*, Manu. Math. (2015) 146: 1–6
- We express the Segre class of a monomial scheme in projective space in terms of log canonical thresholds of associated ideals. Explicit instances of the relation amount to identities involving the classical polygamma functions.
52. *Generalized Euler characteristics, graph hypersurfaces, and Feynman periods*, Geometric, algebraic and topological methods for quantum field theory, 95–136, World Sci. Publ., Hackensack, NJ, 2014.
- We give a very informal presentation of background on the Grothendieck group of varieties and on characteristic classes, and we review some recent work using these tools to study ‘graph hypersurfaces’—a topic motivated by the algebro-geometric interpretation of Feynman amplitudes as periods of complements of these hypersurfaces. These notes follow closely, both in content and style, my lectures at the Summer school in Villa de Leyva, July 5-8, 2011.

51. *Splayed divisors and their Chern classes*, with E. Faber, J. London Math. Soc. (2013) 88 (2): 563-579
- We obtain several new characterizations of 'splayedness' for divisors, and we consider the effect of splayedness on the Chern classes of sheaves of differential forms with logarithmic poles along splayed divisors and on the Chern-Schwartz-MacPherson classes of the complements of these divisors. We verify in several template situations a conjectural identity for Chern-Schwartz-MacPherson classes of splayed divisors and subvarieties.
50. *Segre classes of monomial schemes*, Electron. Res. Announc. Math. Sci., 20 (2013) 55-70
- We propose an explicit formula for the Segre classes of monomial subschemes of non-singular varieties, such as schemes defined by monomial ideals in projective space. The Segre class is expressed as a formal integral on a region bounded by the corresponding Newton polyhedron. We prove this formula for monomial ideals in two variables and verify it for some families of examples in any number of variables.
49. *Euler characteristics of general linear sections and polynomial Chern classes*, Rend. Circ. Mat. Palermo, (special issue) 62 (2013) 3-26.
- We give simple proofs of formulas of Dimca-Papadima and Huh for the degrees of the polar map of a homogeneous polynomial, extending these formula to any algebraically closed field of characteristic 0, and prove a conjecture of Dolgachev on 'homaloidal' polynomials in the same context. We generalize these formulas to subschemes of higher codimension in projective space, and describe a simple approach to a theory of 'polynomial Chern classes' for varieties endowed with a morphism to projective space.
48. *A motivic approach to phase transitions in Potts models*, with Matilde Marcolli. Journal of Geom. and Phys., 63 (2013) 6-31.
- We describe an approach to the study of phase transitions in Potts models based on an estimate of the complexity of the locus of real zeros of the partition function, computed in terms of the classes in the Grothendieck ring of the affine algebraic varieties defined by the vanishing of the multivariate Tutte polynomial.
47. *Grothendieck classes and Chern classes of hyperplane arrangements*, Int. Math. Res. Notices (2013) 1873-1900.
- We show that the characteristic polynomial of a hyperplane arrangement can be recovered from the class in the Grothendieck group of varieties of the complement of the arrangement, and from its Chern-Schwartz-MacPherson class. We also give relations with the Segre class of the singularity subscheme of the arrangement, and study the issue of positivity of the Chern-Schwartz-MacPherson class of an arrangement.
46. *Verdier specialization via weak factorization*. Arkiv för Matematik, 51 (2013) 1-28.
- We define a constructible function on arbitrary closed subschemes of nonsingular varieties, recovering and partially extending a construction originally introduced by Verdier. We also introduce a counterpart of the construction in a motivic group, and show that these invariants have a natural 'monodromy' decomposition.
45. *Chern classes of graph hypersurfaces and deletion-contraction relations*. Moscow Math. Jour., 12 (2012) 671-700.
- We establish a weak deletion-contraction relation for Chern classes of graph hypersurfaces, analogous to the relation for the Grothendieck class obtained in previous work, and prove corresponding multiple edge formulas. The main results are obtained as corollary of a formula for the Chern-Schwartz-MacPherson class of a transversal intersection. We also provide an alternative approach, based on Verdier specialization.

44. *Chern classes of free hypersurface arrangements*,
J. Sing. 5 (2012), 19-32.
The Chern class of the sheaf of logarithmic derivations along a simple normal crossing divisor equals the Chern-Schwartz-MacPherson class of the complement of the divisor. We extend this equality to more general divisors, which are locally analytically isomorphic to free hyperplane arrangements.
43. *Graph hypersurfaces and a dichotomy in the Grothendieck ring*, with Matilde Marcolli.
Lett. Math Phys. (2011) 95:223-232
We prove that every graph hypersurface is stably birationally equivalent to a union of points. This implies that Grothendieck classes of graph hypersurfaces span a tiny portion of the Grothendieck ring, while Belkale and Brosnan have proved that they span the whole Grothendieck ring after a suitable localization.
42. *Feynman motives and deletion-contraction relations*, with Matilde Marcolli.
In Topology of Algebraic Varieties and Singularities, Contemporary Mathematics, vol. 538, Amer. Math. Soc., Providence, RI, 2011, pp. 21-64.
We explore analogues of combinatorial deletion-contraction rules for the Feynman rules introduced in our previous work. We prove multiple edge formulas, and formulas for graphs consisting of chains of polygons. We show that the deletion-contraction relation can be lifted to the level of the category of mixed motives.
41. *Algebraic-Geometric Feynman rules*, with Matilde Marcolli.
Int. J. Geom. Methods Mod. Phys. 8 (2011) 203-237
We introduce 'algebraic-geometric' Feynman rules, with values in the Grothendieck groups of varieties (via motivic considerations) and in a polynomial ring (via characteristic classes), and discuss renormalization procedures and some motivic zeta functions.
40. *Parametric Feynman integrals and determinant hypersurfaces*, with Matilde Marcolli.
Adv. Theor. Math. Phys. 14 (2010) 911-963
Modulo the issue of divergences, we express Feynman integrals on a graph as a period of the complement of a determinant hypersurface in a space of matrices with respect to certain loci determined by rank conditions. This provides evidence for a conjectural description of the integrals as periods of mixed Tate motives.
39. *New orientifold weak coupling limits in F-theory*, with Mboyo Esole.
J. High Energy Phys. (2010) no. 2, 1-53.
We produce new explicit examples of weak coupling limits in F-theory, obtaining new brane configurations that would not be admissible in the original framework of the limit constructed by Ashoke Sen. In our approach, limits depend on the choice of a transition from a semistable to an unstable singular fiber of an elliptic fibration. We also classify configurations of smooth D-branes satisfying the tadpole condition.
38. *Chern classes of blow-ups*.
Math Proc. of the Cambridge Phil. Soc. (2010) 148: 227-242.
We generalize a classic formula of Porteous, computing the Chern classes of a blow-up. The classic formula deals with the nonsingular case, while our generalization works whenever the center of the blow-up is regularly embedded.
37. *Limits of PGL-translates of plane curves, II*, with C. Faber.
Journal of Pure and Applied Algebra 214 (2010), 548-564.
We continue the study of degenerations of plane curves, obtaining certain numerical invariants necessary for applications to enumerative geometry.

36. *Limits of PGL-translates of plane curves, I*, with C. Faber.
Journal of Pure and Applied Algebra 214 (2010), 526-547.
We study the degenerations of plane curves under the action of one-parameter families of linear transformations.
35. *Feynman motives of banana graphs*, with Matilde Marcolli.
Comm. in Number Theory and Physics (2009) 1-57.
We study an infinite family of graphs known as 'banana graphs' from an algebro-geometric, intersection-theoretic viewpoint, with the goal of obtaining information about corresponding Feynman integrals.
34. *Chern class identities from tadpole matching in type IIB and F-theory*,
with Mboyo Esole.
J. High Energy Phys. (2009) no. 3, 032, 29 pp.
We prove a family of Chern class identities for elliptic fibrations, conjectured on the basis of considerations in string theory.
33. *Chern classes of Schubert cells and varieties*, with Leonardo Constantin Mihalcea.
J. Algebraic Geom. 18 (2009), 37-61.
We give explicit formulas for the Chern-Schwartz-MacPherson classes of all Schubert varieties in the Grassmannian of d -planes in a vector space, and conjecture that these classes are effective. We prove this is the case for (very) small values of d .
32. *Une nouvelle preuve de la concordance des classes définies par M.-H. Schwartz et par R. MacPherson*, with Jean-Paul Brasselet.
Bull. Soc. math. France 136 (2008) 159-166.
We give a very short proof of the identity between different definitions of Chern classes for singular varieties.
31. *Celestial integration, stringy invariants, and Chern-Schwartz-MacPherson classes*,
in 'Real and Complex Singularities', J.-P. Brasselet, M.A. Soares Ruas Eds., Birkhäuser (2007) 1-14.
We survey the notion of integration on the category of modifications of a variety, that we defined in 28., and several applications.
30. *Limits of Chow groups, and a new construction of Chern-Schwartz-MacPherson classes*.
Pure Appl. Math. Q., Robert MacPherson special issue, part II, 2 (2006), 915-942.
We introduce a new 'proChow functor', and use it to give a new construction of Chern-Schwartz-MacPherson classes, providing a new proof of a conjecture of Deligne and Grothendieck.
29. *Classes de Chern pour variétés singulières, revisitées*,
C. R. Math. Acad. Sci. Paris 342 (2006), no. 6, 405-410.
We summarize a new construction of Chern-Schwartz-MacPherson classes, and use it to give very short proofs of (generalizations of) two known results on these classes.
28. *Modification systems and integration in their Chow groups*,
Selecta Mathematica 11 (2005) 155-202.
We introduce a formal integration on the category of modifications of a given variety. This leads to new invariants, including 'stringy' Chern classes, and is closely related to the theory of Chern-Schwartz-MacPherson classes.
27. *Lectures on characteristic classes of singular varieties*,
in 'Topics in cohomological studies of algebraic varieties', Birkhäuser (2005) 1-32.
Notes for a cycle of five lectures given at the Banach Center, Warsaw, in April 2002.

26. *Chern classes of birational varieties*,
International Mathematics Research Notices 63 (2004), 3367-3377.
We prove that the Chern classes of birational varieties 'with the same canonical class' are images of the same class in a resolution of indeterminacies of any birational map between them. This is an analog of a celebrated theorem of Batyrev, asserting the equality of the Betti numbers of varieties satisfying the same condition.
25. *Shadows of blow-up algebras*,
Tohoku Math. J. (2) 56 (2004), 593-619.
We introduce a new 'blow-up algebra' interpolating between the Rees and Symmetric algebras of an ideal, and use it to give a new construction of the Chern-Schwartz-MacPherson class of a hypersurface in a nonsingular variety.
24. *Inclusion-exclusion and Segre classes, II*,
Contemp. Math. 324 (2003) 51-61.
We pursue the exploration of 'inclusion-exclusion' principles in the theory of Segre classes, by proving such a principle holds for another variation on the notion of Segre class. This is used to provide a simple computation of the classes introduced in our previous work, in certain special (but representative) cases.
23. *Inclusion-exclusion and Segre classes*,
Comm. Algebra 31 (2003) 3619-3630.
We propose a variation of the notion of Segre class, by forcing a naive 'inclusion-exclusion' principle to hold. We establish several general properties of the new class, and obtain an expression for the Milnor class of an arbitrary scheme in terms of this class.
22. *Interpolation of characteristic classes of singular hypersurfaces*, joint with Jean-Paul Brasselet,
Advances in Math 180 (2003) 692-704.
We show that Chern-Schwartz-MacPherson's classes of a hypersurface 'interpolate' two other notions of characteristic classes, provided that a regularity condition on the singular locus is satisfied. We apply this result to obtain a lift of Chern-Schwartz-MacPherson's classes to intersection homology for the same class of hypersurfaces.
21. *Computing characteristic classes for projective schemes*,
Journal of Symbolic Computation 35 (2003) 3-19.
We present an algorithm (implemented in Macaulay2) computing several classes associated to a projective scheme, given its defining homogeneous ideal.
20. *Linear orbits of arbitrary plane curves*, joint with Carel Faber,
Michigan Math. Jour., 48 (2000), 1-37.
We obtain an algorithm computing the degree of the closure of the linear orbit of an arbitrary plane curve, and give explicit formulas for plane curves with irreducible singularities.
19. *Plane curves with small linear orbits II*, joint with Carel Faber,
International Journal of Mathematics, 11 (2000) 591-608.
The completion of the study of plane curves with positive dimensional stabilizer under the action of the group of projective linear transformations of the plane. In this work we analyze curves obtained as unions of lines.

18. *Plane curves with small linear orbits I*, joint with Carel Faber, *Annales de l'institut Fourier*, 50 (2000) 151–196.
A study of the enumerative geometry of the orbits of most plane curves with positive dimensional stabilizer under the action of the group of projective linear transformations of the plane.
17. *Differential forms with logarithmic poles and Chern-Schwartz-MacPherson classes of singular varieties*, *Comptes Rendus de l'Académie des Sciences, Série I*, 329 (1999), 619–624.
We prove a formula for Schwartz-MacPherson's chern class of a singular variety in terms of the classes of the sheaf of differential forms with logarithmic poles along the components of a divisor arising in the resolution of singularities of the variety. As an application, we obtain a formula for the class of a singular hypersurface in terms of the Mather-Chern class of a suitable sheaf.
16. *Weighted Chern-Mather classes and Milnor classes of hypersurfaces*, in 'Singularities and Arrangements, Sapporo-Tokyo 1998', *Advanced Studies in Pure Mathematics* 29, p.1–20.
We extend the notion of Chern-Mather class to possibly nonreduced schemes, and show how this new notion can be used to compute the difference between Schwartz-MacPherson's chern class and the class of the virtual tangent bundle of a singular hypersurface of a nonsingular variety.
15. *Chern classes for singular hypersurfaces*, *Trans. Amer. Math. Soc.* 351 (1999) 3989–4026.
A new formula relating Fulton's Chern class and MacPherson's Chern class of a possibly singular hypersurface in a nonsingular variety is proved, and applied to Segre class and Chern class computations.
14. *Characteristic classes of discriminants and enumerative geometry*, *Comm. in Algebra* 26(10) (1998) 3165–3193.
We show how computing characteristic classes of strata of the discriminant of a linear system on a nonsingular variety would lead to enumerative results, and carry out these computations in low codimensions. Also, we compute the classes for all strata of the discriminant of d -tuples of points on the projective line, and of cubic curves in the projective plane.
13. *Singular schemes of hypersurfaces*, *Duke Math. Journal*, 80 (1995) 325–351.
A class is introduced generalizing the notion of Milnor number to possibly nonisolated hypersurface singularities. This is used to recover several results in duality theory, as well as producing new constraints for a scheme to be the jacobian scheme of a hypersurface in a smooth variety.
12. *A blow-up construction and graph coloring*, *Discrete Math.*, 145 (1995) 11–35.
A nonsingular algebraic variety is constructed, encoding the incidence information of a given graph G , and is used to obtain the chromatic polynomial of G in terms of intersection-theoretic information.
11. *A remark on the Chern class of a tensor product*, with Carel Faber, *Manu. Math.* 88 (1995) 85–86.
A short note showing that the $(r + 1)$ -st Chern class of a rank- r element α in the Grothendieck group of vector bundles over a scheme does not change if α is tensored by a line bundle.

10. *MacPherson's and Fulton's classes of hypersurfaces*,
International Mathematics Research Notices (1994), 455–465.
It is shown that MacPherson's Chern class of a hypersurface in a smooth variety agrees, at least numerically, with a construction inspired by Fulton's canonical class of a scheme.
9. *Linear orbits of d -tuples of points in P^1* ,
with Carel Faber, Jour. für die R. und Ang. Math. 444 (1993), 205–220.
Orbit closures of sets of points of the projective line under the action of the automorphism group of the latter are studied in terms of their degree and multiplicity along their boundary.
8. *Multiplicities of discriminants*, with Fernando Cukierman,
Manu. Math. 78 (1993), 245–258.
The multiplicity of the discriminant of a line bundle over a nonsingular variety at a section is computed in terms of the Segre class of the jacobian scheme of the section.
7. *Linear orbits of smooth plane curves*,
with Carel Faber, Journal of Alg. Geom. 2 (1993) 155–184.
A desingularization of the orbit closures of plane curves under the action of the automorphism group of the plane is constructed and used to study such orbits.
6. *Two characteristic numbers for smooth plane curves of any degree*,
Trans. of the Amer. Math. Soc., vol. 329 (1992), 73–96.
A sequence of blow-ups over projective spaces parametrizing plane curves is used to obtain enumerative results concerning smooth plane curves of arbitrary degree.
5. *Some characteristic numbers for nodal and cuspidal plane curves of any degree*,
Manu. Math. 72 (1991), 425–444.
A compactification of the space of reduced plane curves is used to compute characteristic numbers for singular plane curves of arbitrary degree.
4. *The enumerative geometry of plane cubics II: nodal and cuspidal cubics*,
Math. Annalen 289 (1991), 543–572.
A variety of complete cubics is used to study the enumerative geometry of several families of irreducible singular plane cubics.
3. *How many smooth plane cubics with given j -invariant are tangent to 8 lines in general position?*,
Contemporary Mathematics 123 (1991), 15–29.
Formulas are obtained for the characteristic numbers of families parametrized by hypersurfaces in the projective space of plane cubics.
2. *The enumerative geometry of plane cubics I: smooth cubics*,
Trans. of the Amer. Math. Soc., vol. 317 (1990), 501–539.
A variety of complete plane cubics is constructed and employed to obtain a thorough analysis of the enumerative geometry of smooth cubics.
1. *The characteristic numbers for smooth plane cubics*,
Proceedings of 'Algebraic Geometry, Sundance 1986', Springer Lecture Notes 1311, 1–8.
The characteristic numbers for the family of smooth plane cubics are computed, verifying results of Maillard and Zeuthen.

Ph.D. students

- **Deborah Jones, Ph.D. 2003**
Dissertation: *Intersection numbers of divisors in graph varieties*
 - **Dimitre Tzigantchev, Ph.D. 2006**
Dissertation: *Predegree polynomials of plane configurations in projective space*
 - **Judson Stryker, Ph.D. 2011**
Dissertation: *Chern-Schwartz-MacPherson classes of graph hypersurfaces and Schubert varieties*
 - **James Fullwood, Ph.D. 2012**
Dissertation: *On elliptic fibrations and F-theory compactifications of string vacua*
 - **Xia Liao, Ph.D. 2013**
Dissertation: *Chern classes of sheaves of logarithmic vector fields for free divisors*
 - **William Adams, Ph.D. 2015**
Dissertation: *Lagrangian Specialization via Log Resolutions and Schwartz-MacPherson Chern Classes*
 - **Corey Harris, Ph.D. 2017**
Dissertation: *Effective methods in intersection theory and combinatorial algebraic geometry*
 - **Xiping Zhang, Ph.D. 2018**
Dissertation: *Characteristic Classes and Local Invariants of Determinantal Varieties and a Formula for Equivariant Chern-Schwartz-MacPherson Classes of Hypersurfaces*
- CURRENT DOCTORAL STUDENT:
- **Grayson Jorgenson**, candidacy 2017, expected Ph.D. 2020
- DOCTORAL STUDENTS CO-DIRECTED:
- **Dan Li, Ph.D. 2012**. Major professor: Prof. Matilde Marcolli (Caltech).
Dissertation: *Periods and motives: applications in mathematical physics*
 - **Yunyi Shen, Ph.D. 2017**. Major professor: Prof. Matilde Marcolli (Caltech).
Dissertation: *Arithmetic aspects of noncommutative geometry: Motives of noncommutative tori and phase transitions on $GL(N)$ and Shimura varieties systems*

Selected invited talks (last 5 years)

- 2018 **Simons Foundation, New York, NY**, MPS Conference on Singularities: Geometric, Topological, and Analytic Aspects—in honor of S. Cappell, (M. Banagl, G. Friedman, S. Weinberger, R. Young).
Chern Classes of Embeddable Schemes
- 2018 **Boston, MA**, AMS meeting #1139, Special session on Singularities of Spaces and Maps, (T. Gaffney, D. Massey).
Segre zeta functions
- 2018 **Worldwide center of Mathematics, Cambridge, MA**, (D. Massey).
Segre classes and other intersection-theoretic invariants. 3 lectures.
- 2018 **Nashville, TN**, AMS meeting #1138, Special session on Algebraic Geometry, Representation Theory, and Applications, (S. Kumar, J.M. Landsberg, L. Oeding).
Degrees of projections of rank loci
- 2018 **University of North Carolina, Chapel Hill, NC**, Algebra seminar, (R. Rimanyi).
Segre zeta functions

- 2017 **University of Oslo**, *Oslo, Norway*, Polar geometry — A conference celebrating the work of Ragni Piene on the occasion of her 70th birthday, (John Christian Ottem, Kristian Ranestad, Jorgen Vold Rennemo).
Chern-Schwartz-MacPherson classes of Schubert varieties
- 2016 **Fields Institute**, *Toronto, ON, Canada*, Workshop on Stratified Spaces: Perspectives from Analysis, Geometry and Topology, (M. Banagl, E. Bierstone, S. Cappell, L. Maxim, T. Weigand).
Chern-Schwartz-MacPherson classes of Schubert varieties
- 2016 **University of Washington**, *Seattle, WA*, Algebra seminar, (S. Billey, J. Pevtsova).
Chern-Schwartz-MacPherson classes of Schubert varieties
- 2016 **UCLA**, *Los Angeles, CA*, Algebraic geometry seminar, (B. Totaro).
Chern-Schwartz-MacPherson classes of Schubert varieties
- 2016 **Caltech**, *Pasadena, CA*, Algebraic geometry seminar, (P. Solis, T. Graber).
Segre classes of monomial schemes and a Segre zeta function
- 2016 **University of California, Berkeley**, Applied algebra seminar, (M. Helmer).
The number of equations needed to define a Segre class
- 2016 **University of Hawaii**, *Honolulu, HI*, Geometry seminar, (A. Hadari, L. Wilson).
The number of equations needed to define a Segre class
- 2016 **Università di Torino**, *Turin, Italy*, Algebraic Geometry in Torino—A conference on the occasion of Alberto Collino's retirement, (A. Albano, C. Casagrande).
Segre classes of monomial schemes and a Segre zeta function
- 2016 **Politecnico di Torino**, *Turin, Italy*, Algebraic Geometry seminar, (A. Albano).
Projective duality and a Chern-Mather involution
- 2015 **Virginia Tech**, *Blacksburg, Va*, Algebraic Geometry seminar, (L. Mihalcea).
General linear sections and characteristic classes
- 2015 **ICERM**, *Providence, RI*, Computational Geometric Topology in Arrangement Theory, (N. Budur, G. Denham, D. Matei, L. Maxim, H. Schenck, M. Wakefield).
Characteristic classes and arrangements (two lectures)
- 2015 **University of Wisconsin**, *Madison, WI*, Stratified spaces in geometric and computational topology and physics, (S. Cappell, L. Maxim, M. Nicolau, K. Orr).
Degrees of projections of determinantal varieties
- 2015 **Centre International de Rencontres Mathématiques**, *Luminy, France*, Invariants locaux et globaux des singularités, (Nicolas Dutertre, Anne Pichon).
Segre classes of monomial schemes and Segre zeta functions
- 2014 **Greensboro, NC**, AMS meeting #1105, Special session on Geometry and Combinatorics on Homogeneous Spaces, (L. Mihalcea, R. Rimanyi).
Degrees of projections of rank loci
- 2014 **AGNES workshop Penn 2014**, *Philadelphia, PA*, (R. Donagi, A. Grassi, T. Pantev).
Segre classes of monomial schemes
- 2014 **SACNAS**, *Los Angeles, CA*, Session on combinatorial commutative algebra, (D. Karp, M. Vazquez, R. Wilson).
Monomial schemes and polygamma functions
- 2014 **Texas A&M**, *College Station, TX*, Algebraic geometry seminar, (J. M. Landsberg).
Segre classes of monomial schemes
- 2014 **University of Hawaii**, *Honolulu, HI*, Colloquium, (L. Wilson).
Segre classes of monomial ideals

2014 **University of Trento**, *Levico Terme, Italy*, Winter school on Curves and Equations, (C. Fontanari, E. Sernesi).
Linear orbits of plane curves (5 lectures)

Visits (≥ 1 month)

Jan.-June 2018 **University of Toronto**.
May-June 2017 **California Institute of Technology**.
May'15-July'16 **California Institute of Technology**.
May-July 2014 **California Institute of Technology**.
June-July 2013 **California Institute of Technology**.
Jan.-July 2012 **California Institute of Technology**.
Summer 2011 **California Institute of Technology**.
Summer 2010 **California Institute of Technology**.
July 2009 **Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
Jan.-April 2009 **Mathematical Sciences Research Institute**, *Berkeley*.
Sep.-Dec. 2008 **California Institute of Technology**.
May-July 2008 **Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
June-July 2007 **Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
April-May 2007 **Mittag-Leffler Institut**, *Stockholm*, Sweden.
Jan.-Mar. 2007 **Institut de Mathématiques de Luminy**, *France*.
June-July 2006 **Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
June-July 2005 **Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
May 2005 **Institut de Mathématiques de Luminy**, *France*.
May-July 2004 **Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
May-July 2003 **Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
Sep. 01-July 02 **Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
May-July 2001 **Institut de Mathématiques de Luminy**, *France*.
July 2000 **Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
Sep.-Dec. 1999 **Harvard University**.
Sep. 96-Apr. 97 **Mittag-Leffler Institut**, *Stockholm*, Sweden.
June-July 1996 **Mathematisches Forschungsinstitut**, *Oberwolfach*, Germany.
May 1996 **University of Chicago**.
May 1995 **University of Chicago**.
May 1994 **University of Chicago**.
Jan.-June 1993 **Mathematical Sciences Research Institute**, *Berkeley*.
Jan.-June 1992 **Max-Planck-Institut für Mathematik**, *Bonn*, Germany.
May-July 1990 **Mathematisches Institut**, *Erlangen*, Germany.

Service (to the profession)

- Managing Editor, *Journal of Singularities*, 2014—present (Duties shared with Javier Fernández de Bobadilla and Victor Goryunov)
- Associate Editor, *Journal of Singularities*, 2009–14 (David Massey, Editor-in-Chief)
- Editor, *ISRN Geometry*, 2011–12

- Refereeing: *Advances in Mathematics*; *Advances in Applied Mathematics*; *Annales scientifiques de l'École normale supérieure* *Annali dell'Università di Ferrara* *Arkiv for Matematik*; *Aspects of Mathematics*; *Asian Journal of Mathematics*; *American Journal of Mathematics* (2); *Canadian Journal of Mathematics*; *Canadian Mathematical Bulletin* (2); *Central European Journal of Mathematics*; *Communications in Algebra* (3); *Compositio Mathematica* (3); *Comptes Rendus*; *Contemporary Mathematics* (4); *Duke Math Journal*; *Geometriae Dedicata*; *Geometry and Topology* (4); *Impanga Lecture Notes*; *IMRN* (4); *Inventiones Mathematicae*; *Journal of Algebra*; *Journal of Algebraic Geometry* (3); *Journal of Complexity*; *Journal of Geometry*; *Journal of Geometry and Physics* (2); *Journal of Mathematics and Music*; *Journal of Pure and Applied Algebra* (2); *Journal of Singularities* (2) *Journal of Software for Algebra and Geometry* (3); *Journal of the Australian Math Society*; *Journal of Topology*; *Mathematica Scandinavica*; *MEGA 2013* (3); *Pure and Applied Math Quarterly* (2); *K-theory*; *Mathematics of Computer Science*; *Mathematische Zeitschrift* (2); *Proceedings of the American Mathematical Society* (3); *Rendiconti del circolo matematico di Palermo*; *Springer Lecture Notes*; *Topology and its applications*; *Transactions of the American Mathematical Society* (5); *Transformation groups*.
- Grant reviewing: N.S.A. (4), N.S.F. (25); Swedish Research Council (8); Dutch NWO 'Veni' and 'Vidi' programs.
- NSF panelist, (2003).
- Co-organizer (with Ettore Aldrovandi and Eriko Hironaka) of the Special Session on Algebraic Geometry and Topology at the Sectional Meeting #994 of the AMS, Tallahassee, March 12-13, 2004.
- *Mathematical Reviews* (63 reviews)
- Outside Ph.D. thesis examiner for Dmitry Kerner, Tel-Aviv University, 2007 (Thesis: 'Enumeration of singular hypersurfaces'; E. Shustin, Ph.D. advisor)
- Outside evaluator of the mathematics program at Bethune-Cookman University, May 2009
- Co-organizer (with Mboyo Esole) of the Special Session on Singularities and Physics at the Sectional Meeting #1097 of the AMS, Knoxville, TN, March 21-23, 2014.
- Scientific committee, INPANGA '15, Będlewo, Poland.
- Evaluator, IMPAN International Fellowships, 2015.
- Co-organizer (with Lara Anderson, Mboyo Esole, Shing-Tung Yau) of a three-workshop series on the mathematics and physics of F-theory: Virginia Tech (2016), Harvard (2018), Florida State (2019).
- NSF panelist, (2017).

Service (departmental/university)

- DIRECTOR OF PURE MATHEMATICS: 2005-07 (through Fall '07)
Duties included: revision of graduate curriculum; organization of graduate and undergraduate courses; graduate student recruitment and advising; membership in the executive, faculty evaluation, and hiring committees.
- Co-director of pure mathematics: Spring 2015.
- COMMITTEES:
 - Brennan Professor selection committee: 2014, 2015, 2018
 - Chair nomination (mathematics): 2004-05 (chair); 2011-12
 - Chair nomination (statistics): 2007-08

- COFRS evaluation: 2011, 2014, 2015
- Colloquium: 1998-99
- Curriculum: 1993-96
- Doctoral preliminary examination: 1999-2001; 2002-03; 2003-05 (chair); 2009-10; 2014-15; 2017-18
- Election: 2002-04
- Executive: 1999-2000; 2005-07 (through fall '07); 2012-18
- Facilities and technology (self-study): 1992-93
- Faculty evaluation: 2000-01; 2002 (fall); 2004 (spring); 2004-07 (through fall '07); spring 2013-15; fall 2017-present
- FSU math society: 1995-96
- Graduate: 2006-07, 2010, 2015
- Graduate student recruitment and financial aid: 1997-2001; 2002-05
- GPC committee, department member, 2013
- Hiring: 1999-2000; 2004-05; 2017-8
- Library: 1994-96, 1997-2001
- Preliminary examination policy (chair): 1999-2000
- Pure mathematics: 2004-05 (chair)
- Quality Enhancement Review: 2012-13 (chair)
- Retirement: 2003 (spring)
- Science Area Promotion & Tenure Committee: 2017
- Study Abroad Scholarship Committee: 2016-present
- SPP peer evaluation: 2011
- TIP peer evaluation: 1994-95, 1998-99
- Unit assessment survey ('QER'): 2003-04
- University Senate: 2000-01; 2006-08
- Undergraduate major recruitment and retention: Fall 2004 (chair)
- Visibility: 2000-present

— STUDENT COMMITTEES:

- *Advanced topics examination/Ph.D. candidacy:*
W. Adams (Apr. 98, chair); D. Jones (Dec. 98, chair); B. Goforth (Apr. 99);
C. Stockwell (Apr. 99); D. Tzigantchev (Jan. 03, chair); A. Novocin (Feb. 06);
R. Todd (Oct. 06); G. Levy (Nov. 06); E. Tatar (Dec. 06); Y. Lebedev (Nov. 07);
Q. Yuan (Nov. 07); Y. Cha (Jan. 08); J. Stryker (Apr. 08, chair); D. Li (Aug. 08);
T. Fang (Nov. 09); X. Liao (Nov. 09, chair); I. Kunwar (Mar. 10); I. Dungan
(Apr. 10); J. Fullwood (Apr. 10, chair); Y. Shen (Aug. 11); S. Aksoy (Oct. 12);
M. Aktas (Dec. 13); C. Harris (Dec. 13, chair); E. Imamoglu (Jan. 14); L. Sparaco
(Apr. 14); X. Zhang (Dec. 14, chair); Y. Valdes (Oct. 15); G. Jorgenson (Oct. 17,
chair); S. Aldossari (Jan. 18).
- *Master's committee:*
J. de Marco (2004, chair).
- *Honors thesis committee:*
R. Orlando, 'A Study of Physics Beyond the Standard Model' (H. Prosper, chair;
2018).

- *Ph.D. committees (mathematics)*:
 - B. Goforth (April 2005, dissertation: 'Description and analysis of a two-variable version of the NTRU crypto system'. Sam Huckaba, major professor).
 - A. Novocin (April 2008, dissertation: 'Factoring Univariate Polynomials over the Rationals'. Mark van Hoeij, major professor).
 - Y. Lebedev (November 2008, dissertation: 'Openmath library for computing on Riemann surfaces'. Mika Seppälä, major professor).
 - G. Levy (December 2009, dissertation: 'Solutions of Second Order Recurrence Relations'. Mark van Hoeij, major professor).
 - E. Tatar (May 2010, dissertation: 'Picard 2-Stacks and length 3 complexes of abelian sheaves'. Ettore Aldrovandi, major professor).
 - Y. Cha (December 2010, dissertation: 'Closed form solutions of linear difference equations'. Mark van Hoeij, major professor).
 - Q. Yuan (March 2012, dissertation: 'Finding all Bessel type solutions for linear differential equations with rational function coefficients'. Mark van Hoeij, major professor).
 - T. Fang (October 2012, dissertation: 'Solving linear differential equations in terms of hypergeometric functions by 2-descent'. Mark van Hoeij, major professor).
 - I. Dungan (April 2014, dissertation: ' N -butterflies: modeling weak morphisms of strict N -groups'. Ettore Aldrovandi, major professor).
 - M. Aktas (April 2017, dissertation: 'Topology of N -Gonal Curve Complements'. Eriko Hironaka and Washington Mio, major professors).
 - E. Imamoglu (May 2017, dissertation: 'Algorithms for solving linear differential equations with rational function coefficients'. Mark van Hoeij, major professor).
 - W. Xu (November 2017, dissertation: 'Third order A -hypergeometric functions'. Mark van Hoeij, major professor).
 - Y. Valdes (April 2018, dissertation: 'The 1-type of algebraic K-Theory as a multifunctor'. Ettore Aldrovandi, major professor).
- *Ph.D. committees (university representative)*:
 - Fernando Febres Cordero (Physics, 2004 prospectus; 2007 Ph.D. dissertation: 'Next-to-leading-order corrections to weak boson production with a massive quark jet pair at hadron colliders'. Laura Reina, major professor)
 - Farrooh Fattoyev (Physics, 2009 prospectus; 2011 Ph.D. dissertation: 'Sensitivity of neutron star properties to the equation of state'. Jorge Pieckarevycz, major professor)
 - Paulo Rottmann (Physics, 2011 Ph.D. dissertation: 'Z-sum approach to loop integrals'. Laura Reina, major professor)
 - Benjamin Thayer (Physics, 2012 Ph.D. dissertation: 'On-shell methods applied to exotic Higgs production at hadronic colliders'. Laura Reina, major professor)
 - Heribertus Hartanto (Physics, 2010 prospectus; 2013 Ph.D. dissertation: 'Top-Quark associated production with one hard photon at hadron colliders'. Laura Reina, major professor)
 - Steve Honeywell (Physics, 2014 prospectus; 2017 Ph.D. dissertation: 'Automated one-loop QCD and electroweak calculations with NLOX'. Laura Reina, major professor)
 - Lewis Jeter (Music, 2016 preliminary exam. Cliff Callender and Nancy Rogers, major professors)

Teaching

— FLORIDA STATE UNIVERSITY (1991-PRESENT):

Graduate:

- *Abstract algebra I*: Fall 1995; Fall 2003
- *Abstract algebra II*: Spring 1996
- *Groups, Rings, Vector Spaces I*: Fall 1994; Fall 2002; Fall 2004; Fall 2005; Fall 2009, Fall 2013, Fall 2016
- *Groups, Rings, Vector Spaces II*: Spring 1995; Spring 2003; Spring 2005; Spring 2006; Spring 2010; Spring 2014; Spring 2017
- *Groups, Rings, Vector Spaces III*: Fall 2005; Fall 2006; Fall 2010; Fall 2014; Fall 2017
- *Algebraic geometry (topics course)*: Fall 1993; Spring 1994; Fall 1997; Spring 1998; Fall 2000; Spring 2006; Spring 2014; Spring 2015
- *Homological and commutative algebra (topics course)*: Spring 2011
- *Euler calculus (topics course)*: Fall 2012
- *Algebra seminar*: Fall 1992; Spring 2000; Fall 2000
- *Algebraic geometry seminar*: Fall 1998; Spring 1999; Spring 2006; Fall 2009; Spring 2010; Fall 2010, Spring 2011

Undergraduate:

- *Applied Linear Algebra I*: Fall 2007
- *Calculus I*: Fall 1991, 2 sections
- *Calculus II*: Spring 1995
- *Calculus III*: Fall 1992, 2 sections; Fall 1994; Spring 1996; Fall 1998; Spring 2000; Fall 2000; Fall 2002; Fall 2003
- *College geometry*: Fall 1998
- *Complex variables*: Fall 2016
- *Discrete mathematics I*: Fall 1993; Fall 1995
- *Discrete mathematics II*: Spring 1994
- *Introduction to abstract algebra I*: Fall 1997; Fall 2000; Fall 2003; Fall 2004; Fall 2006; Fall 2007; Fall 2014; Fall 2017
- *Introduction to abstract algebra II*: Spring 1998; Spring 2005; Spring 2008; Spring 2015
- *Introduction to advanced mathematics*: Spring 1999; Spring 2010; Fall 2010; Fall 2011; Fall 2012; Fall 2013
- *Introduction to real analysis*: Fall 2009
- *Mathematical logic*: Spring 2011

— BROWN UNIVERSITY (1984–87):

- Elementary Calculus I, II; Intermediate Calculus I, II.

— THE UNIVERSITY OF CHICAGO (1987-89):

- Honors Calculus, I, II, III; Abstract Algebra I, II, III; Introduction to Algebraic Geometry.

— OKLAHOMA STATE UNIVERSITY (1989-91):

- Business Calculus, Calculus II, Differential Equations, Linear Algebra, Combinatorics, Complex Analysis.

— KTH, STOCKHOLM, SWEDEN (JANUARY-APRIL 1997):

- Intersection theory and enumerative geometry.

— CALIFORNIA INSTITUTE OF TECHNOLOGY (FALL 2008):

- Homological algebra.

