1. Section 9.2 The Parabola

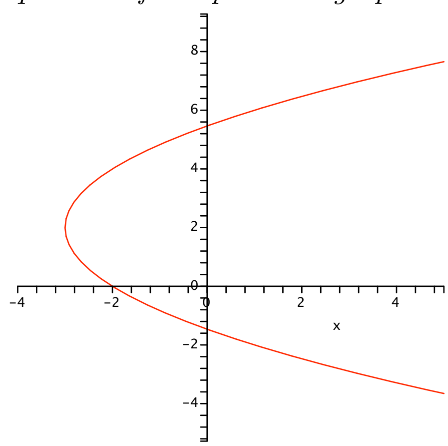
Definition 1.1. *A* **parabola** *is the set of all points, $P = (x, y)$, that are equidistant from a fixed point called the* **focus** *and a fixed line called the* **directrix**.

2. Equations, $a > 0$

Equation	Vertex	Focus	Directrix	Sketch
$(y-k)^2 = 4a(x-h)$				
$(y-k)^2 = -4a(x-h)$				
$(x-h)^2 = 4a(y-k)$				
$(x-h)^2 = -4a(y-k)$				

Example 2.1. *Sketch the graph of* $x^2 = -cy$ *if* $c < 0$.

Example 2.2. *Find the equation of the parabola graphed below.*



Example 2.3. *Find the equation of the parabola with focus* $(1, -2)$ *and vertex at* $(1, 1)$.

Example 2.4. *Find the equation of the parabola with focus* $(-1, -2)$ *and vertex at* $(1, -2)$.

Example 2.5. *Find the equation of the parabola with vertex* $(-1,-2)$ *and directrix* $x = 3$.

Example 2.6. *Find the equation of the parabola with focus* $(-1,-2)$ *and directrix* $y = 3$.

Example 2.7. *Find the focus, directrix and vertex of the parabola with the equation* $x^2 = 6y$

Example 2.8. *Find the focus, directrix and vertex of the parabola with the equation* $2(y+2)^2 = x+3$

Example 2.9. *Find the focus, directrix and vertex of the parabola with the equation* $(y+2)^2 = -2x+3$

3. 9.3 The Ellipse

Definition 3.1. *An* **ellipse** *is the set of all points, $P = (x, y)$, such that the sum of the distances between two points, called the* **foci** *of the ellipse, is constant.*

The **major axis** *is the axis through the foci.*

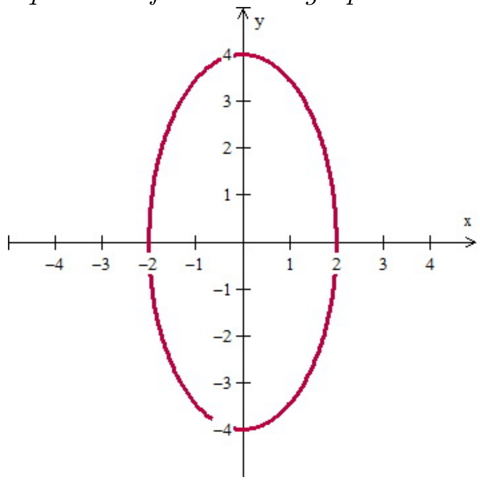
The **minor axis** *is the axis through the center perpendicular to the major axis.*

The **vertices** *of he ellipse are the points where the ellipse intersects the major axis.*

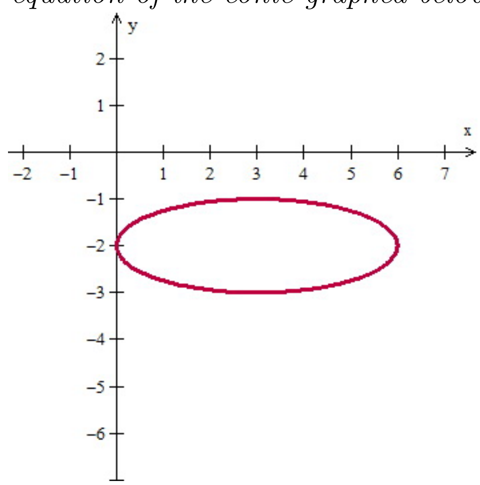
4. Equations

	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
$a > b$, relation between a, b, and c	$a^2 = b^2 + c^2$	$a^2 = b^2 + c^2$
Center		
Foci		
Vertices		
Intercepts minor axis		
Sketch		

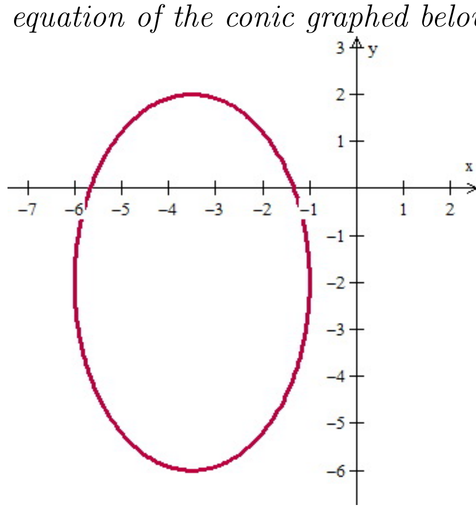
Example 4.1. *Find the equation of the conic graphed below.*



Example 4.2. *Find the equation of the conic graphed below.*



Example 4.3. *Find the equation of the conic graphed below.*



Example 4.4. *Write (using lower case x) the formula for y^2 in the ellipse with center $(0,0)$, focus at $(0,4)$ and vertex at $(0,-6)$.*

Example 4.5. *Find the equation of the ellipse with foci at $(-1,2)$ and $(3,2)$ and vertex at $(4,2)$.*

Example 4.6. *Plot the graph and find the center, foci, vertices, major axis, and minor axis for the ellipse*

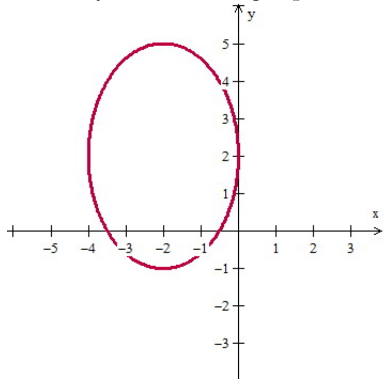
$$\frac{x^2}{8} + \frac{y^2}{16} = 1.$$

Example 4.7. *Plot the graph and find the center, foci, vertices, major axis, and minor axis for the ellipse*

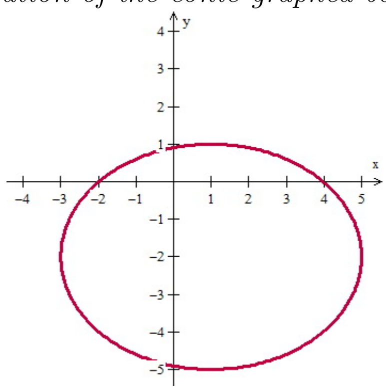
$$2(x+1)^2 + (y-2)^2 = 16.$$

5. Practice

Exercise 5.1. *Find the equation of the conic graphed below.*



Exercise 5.2. *Find the equation of the conic graphed below.*



Exercise 5.3. *Find the equation of the ellipse with foci at* $(-1, 2)$ *and* $(-1, -4)$ *and vertex at* $(-1, -6)$.

Exercise 5.4. *Plot the graph and find the center, foci, vertices, major axis, and minor axis for the ellipse*

$$3(x-2)^2 + (y+1)^2 = 12.$$

6. Section 9.4 The Hyperbola

Definition 6.1. *A* **hyperbola** *is the set of all points, $P = (x, y)$, such that the difference of the distances between two points, called the* **foci** *of the ellipse, is constant.*

The **center** *is the point halfway between the two foci.*

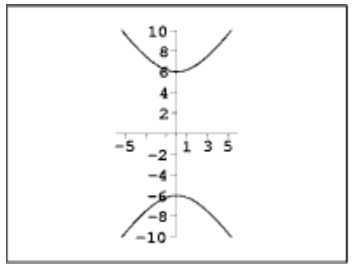
The **transverse axis** *is the axis through the foci. The* **conjugate axis** *is the axis through the center perpendicular to the transverse axis.*

The **vertices** *of he hyperbola are the points where the hyperbola intersects the transverse axis.*

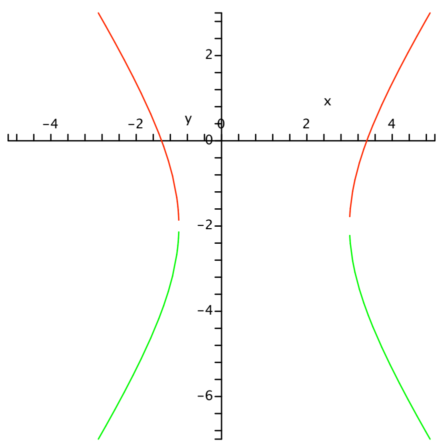
7. Equations

	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-h)^2}{a^2} - \frac{(x-k)^2}{b^2} = 1$
relation between a, b, and c		
Center		
Foci		
Vertices		
Asymptotes		
Sketch		

Example 7.1. *Find the equation of the conic graphed below.*



Example 7.2. *Find the equation of the conic graphed below.*



Example 7.3. *Write (using lower case x) the formula for y^2 in the hyperbola with center $(0,0)$, focus at $(0,-6)$ and vertex at $(0,4)$.*

Example 7.4. *Find the equation of the hyperola with foci at $(-2,2)$ and $(4,2)$ and vertex at $(3,2)$.*

Example 7.5. *Find the center, foci, vertices, transverse axis, conjugate axis, and the asymptotes for the conic*

$$\frac{x^2}{8} - \frac{y^2}{16} = 1.$$

Example 7.6. *Find the center, foci, vertices, transverse axis, conjugate axis, and the asymptotes for the conic*

$$2(x+1)^2 - (y-2)^2 = 16.$$

Example 7.7. *Find the center, foci, vertices, transverse axis, conjugate axis, and the asymptotes for the conic*

$$2(y+1)^2 - (x-2)^2 = 16.$$

8. Discriminant

The equation of a conic (parabola, ellipse, or hyperbola) may be written in the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

where A, B, C, D, E, and F are real numbers. We may determine which conic the above formula is for by examining the **discriminant**

Discriminant = ______________________________

	type of equation (or degenerate)
Discriminant $= 0$	
Discriminant < 0	
Discriminant > 0	

9. Completing the Square

To change an equation of a conic from the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

into an equation of the forms discussed earlier we must **complete the square**.

Steps:

(1) Group the terms with x together, the terms with y together, and move the constant to the other side.

(2) Factor coefficient of x^2 and coefficient of y^2 out of each group.

(3) To complete the square of $(x^2 + Mx)$ we add $(M/2)^2$ since

$$x^2 + Mx + (M/2)^2 = (x + M/2)^2.$$

(4) Keep the equation balanced by adding equivalent values to the other side. Keep in mind the value factored out in step 2.

Example 9.1. *Describe the graph of*

$$4x^2 + 9y^2 - 16x - 18y = 11$$

That is, find the type of graph and where applicable vertices, foci, directrix, asymptotes, etc.

Example 9.2. *Describe the graph of*

$$4x^2 - y^2 + 8x + 4y = 4$$

Example 9.3. *Describe the graph of*

$$0 = y^2 - 8x - 4y + 12$$