Name:

Instructions: To receive full credit, show all the work. No calculators

Question 1 (10 pts)
If the series \( \sum_{n=1}^{\infty} \frac{3^{n-1} + 6^n}{5^{2n}} \) is convergent, find its exact sum. (Hint: geometric series)
Question 2 (10 pts)
   a) State the comparison test

b) Using comparison test, deduce whether the series \( \sum_{n=1}^{\infty} \frac{1}{2^n + n} \) is convergent or divergent.
Question 3 (10 pts)

a) State the integral test

b) Using the integral test, decide whether the series \( \sum_{n=2}^{\infty} \frac{1}{n \ln n} \) is convergent or divergent
Question 4 (20 pts)

a) State the alternating series test

Given the series \( \sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln(n)} \), answer the following questions

b) Using alternating series test, show that the series is convergent.

c) Use limit comparison test to decide whether the series is absolutely convergent or not?

d) From b) and c), deduce the convergence type of the series?
Question 5 (15 pts)

a) State the ratio test

b) Using the ratio test, decide whether the following series are convergent or divergent

b-1) \[ \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} \]

b-2) \[ \sum_{n=1}^{\infty} \frac{(-10)^n}{n!} \]

b-3) \[ \sum_{n=1}^{\infty} \frac{\pi^n}{ne^n} \]
Question 6 (15 pts)

a) State the root test.

b) Using root test, decide whether the following series are convergent or divergent

b-1) \[ \sum_{n=1}^{\infty} \left( \frac{n^2 + 1}{2n^2 + 1} \right)^n \]

b-2) \[ \sum_{n=1}^{\infty} \frac{n^n}{e^{n^2 + 2n}} \]

b-3) \[ \sum_{n=1}^{\infty} \left( \frac{n}{n + 1} \right)^n \] (Hint: Use the fact that \( \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e \))
Question 7 (10 pts)

Find the radius of convergence and interval of convergence of the series \[ \sum_{n=1}^{\infty} \frac{x^n}{5^n n^5} \]
Question 8 (10 pts)
Find a power series representation for the function \( \ln(x + 1) \) and find its radius of convergence.