Chapter 2: Logic

§2.1 Logic Definitions

Definition
A proposition is a declarative sentence that is either
true (denoted either T or 1)
or false (denoted either F or 0).

Notation
most common variables used for propositions are p, q, r

Remark
Not all declarative sentences are propositions

Examples
Give examples from Kirby's notes.

§2.2 Logical Operators.

- Negation of p is not p. It is denoted by $\neg p$.
  Negation is a unary operation

- Conjunction: p and q. It is denoted by $p \wedge q$.
- Disjunction: p or q. ———————— $p \vee q$.
- Exclusive or: exactly one of p or q. ———— p xor q
  $p \oplus q$.

- Implication: if p then q. ———— $p \rightarrow q$.
- Biconditional: p if and only if q ———— $p \leftrightarrow q$.

Truth Tables

Truth Table of Negation

p	$\neg p$
T	F
F	T

Truth Table of Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table of Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Table of XOR

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Truth Table of Implication

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table of Biconditional

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example Construct the truth table of $(p \vee \neg q) \rightarrow (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Example

Let p, q, r be the following statements.

p: You have the flu.
q: You miss the final examination.
r: You pass the corse.

Express the following as English sentences

$p \rightarrow q$: If you have the flu then you miss the final examination.

$p \vee q \vee r$: You have the flu or you miss the final examinat or you pass the course.

$\neg q \leftrightarrow r$: You don't miss the final examination if and only you pass the course.

$(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$: If You have the flu then you don't pass the course or If you miss the final examination, then you don't pass the course.

We can simplify this sentence as follows

If you have the flue or you miss the miss the final examination, then you don't pass the course.

This simplification of the sentence suggests us that we can also simplify the logical expression. to $(p \vee q) \rightarrow \neg r$. How does in general this simplification process work? How can we verify that 2 logical expressions are equivalent? (Coming up soon).

Terminology: For the logical operator implication $p \rightarrow q$, we use the following terminology.

p is called the hypothesis.
q is called conclusion or consequent
$q \rightarrow p$ is the converse
$\neg q \rightarrow \neg p$ is the contrapositive
$\neg p \rightarrow \neg q$ is the inverse

Example
Let p and q be the propositions
p = It is below $0\ C^\circ$
q = It is snowing.
Express the following propositions in English.

$q \rightarrow p$: If it is snowing then it is below $0\ C^\circ$

converse of $(q \rightarrow p)$
inverse of $(q \rightarrow p)$
contrapositive of $(q \rightarrow p)$

Example Construct the truth table of $(p \oplus q) \rightarrow (\neg q \wedge p)$