The Waisman Laboratory
for Brain Imaging and Behavior

University of Wisconsin
SCHOOL OF MEDICINE
AND PUBLIC HEALTH

Hyperspherical Harmonic (HyperSPHARM) Representation

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October 10, 2017, Florida State University

Abstracts

Existing functional shape models such as the widely used spherical harmonic (SPHARM) representation assume topological invariance, so are unable to simultaneously parameterize multiple disconnected structures. In such a situation, SPHARM has to be applied separately to each individual structure. We present a novel surface parameterization technique using 4D hyperspherical harmonics (HyperSPHARM) in representing multiple disjoint objects as a single analytic form. The underlying idea behind HyperSPHARM is to project an entire collection of disconnected 3D objects onto the 4D hypersphere and simultaneously parameterize them with the 4D hyperspherical harmonics. Hence, HyperSPHARM allows for a holistic treatment of multiple disconnected structures. Although HyperSPHARM may yields similar reconstruction performance as SPHARM, HyperSPHARM can parameterize using much fewer basis functions and projection to 4D dimension obviates SPHARM's burdensome surface flattening. In addition, HyperSPHARM can handle any type of topology. The method is applied in modeling hippocampi and amygdalae of the human brain. The talk is based on paper

Hosseinbor et al., 2015 Medical Image Analysis 22:89-101

Acknowledgements

Pasha Hosseinbor, Nagesh Adluru,
Ross Luo, Houri Voperian,
Seth Pollack, Andrew Alexander,
Hill Goldsmith, Richard Davidson
University of Wisconsin-Madison

NIH funding: EB022856, MH098098, MH061285

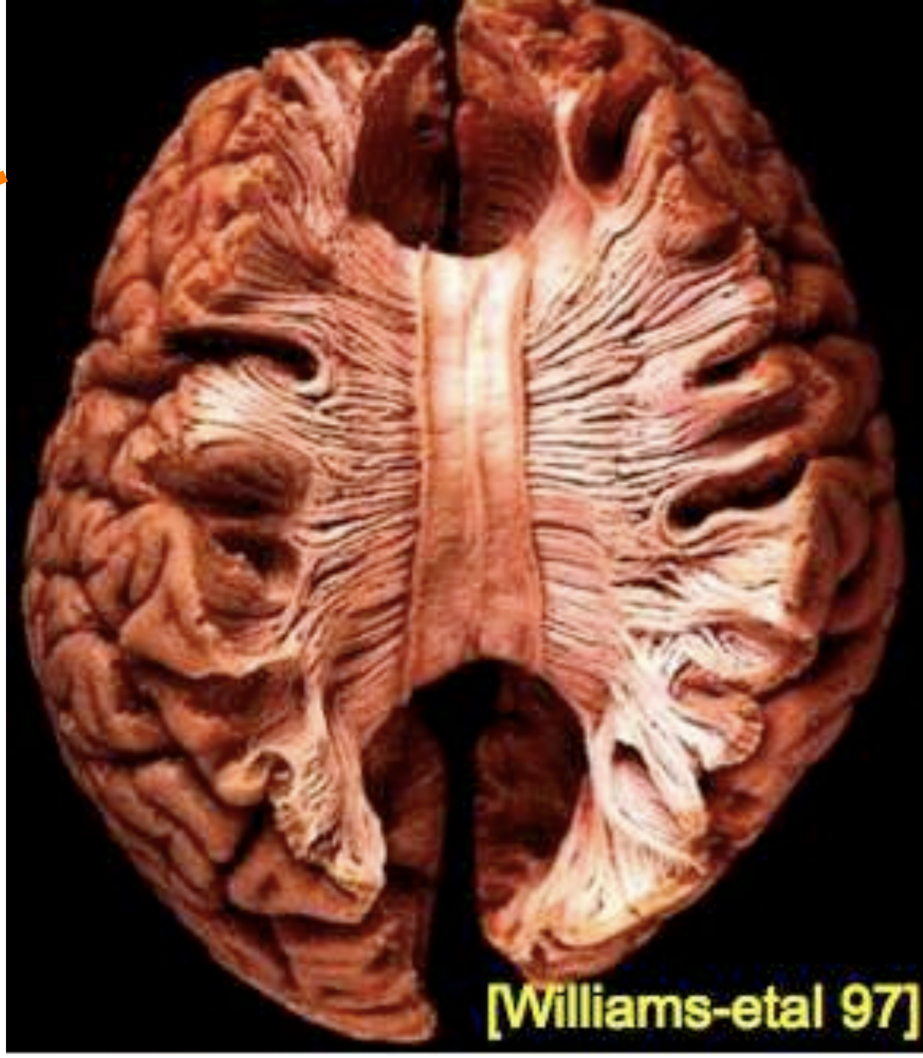
Preliminary

Parametric shape models

Fourier descriptors

Spherical harmonic representation

Laplace-Beltrami eigenfunction expansion

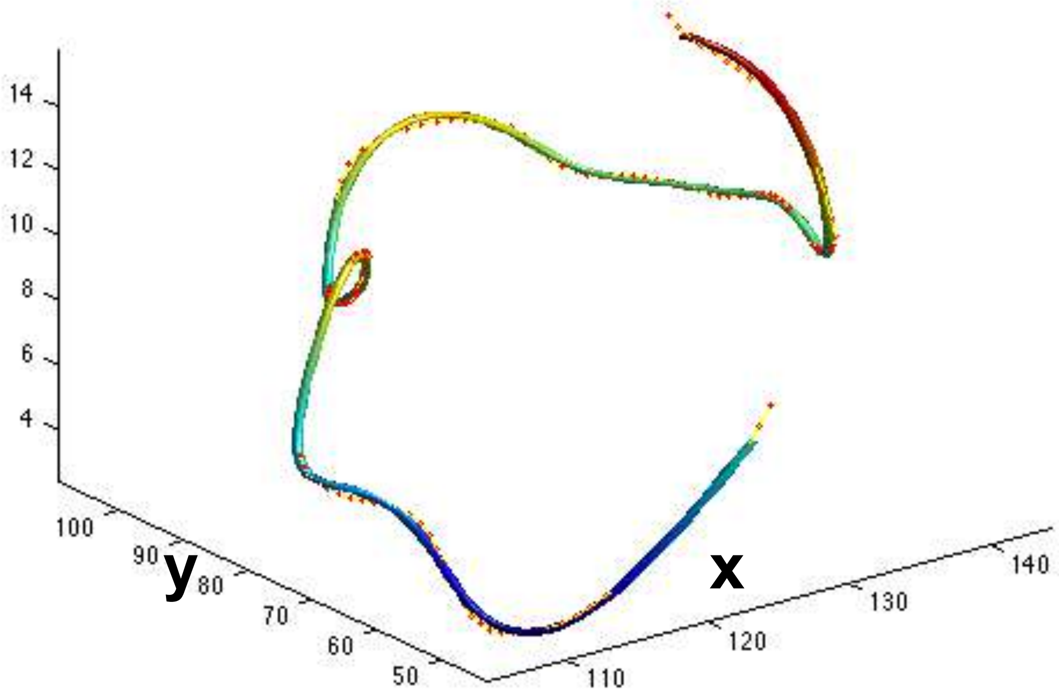


White matter fibers

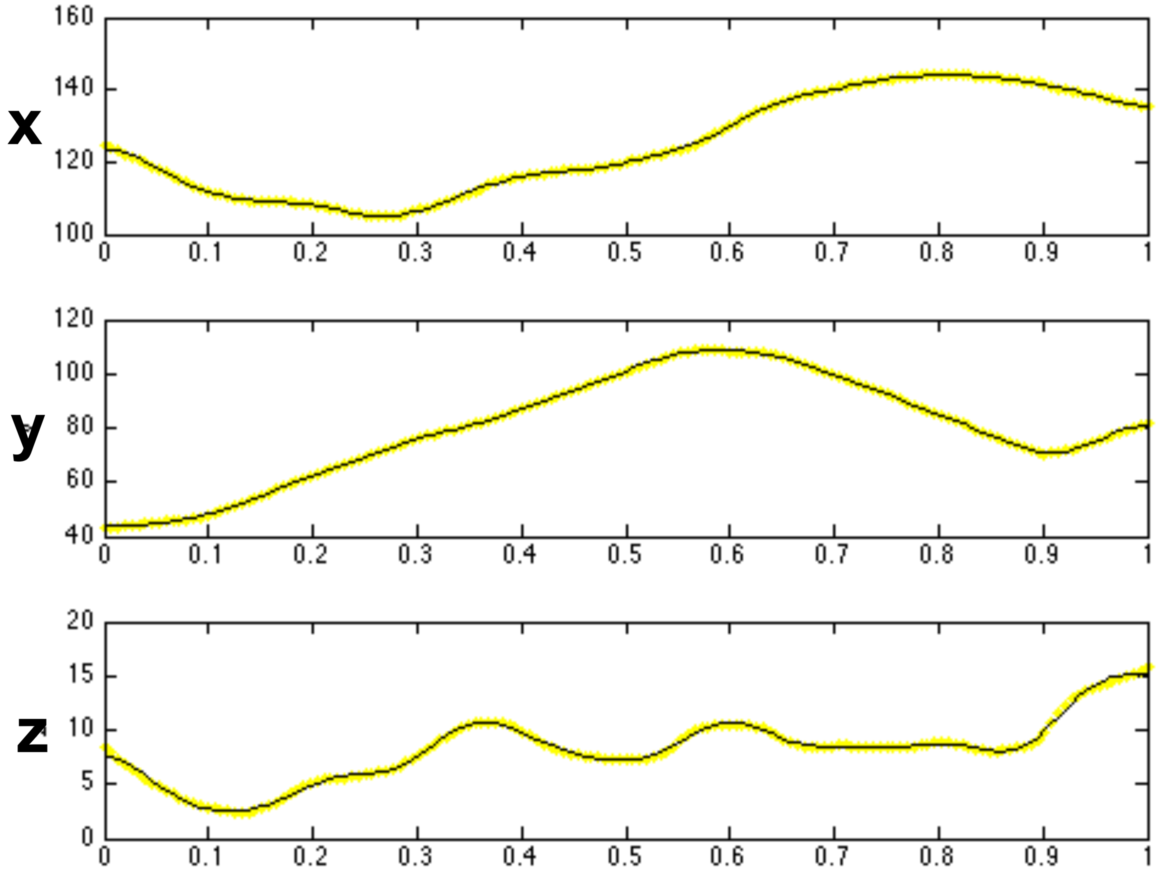
Up to half million tracts

Each tract consists
of about 300 control
points.

Cosine series representation



parameterization



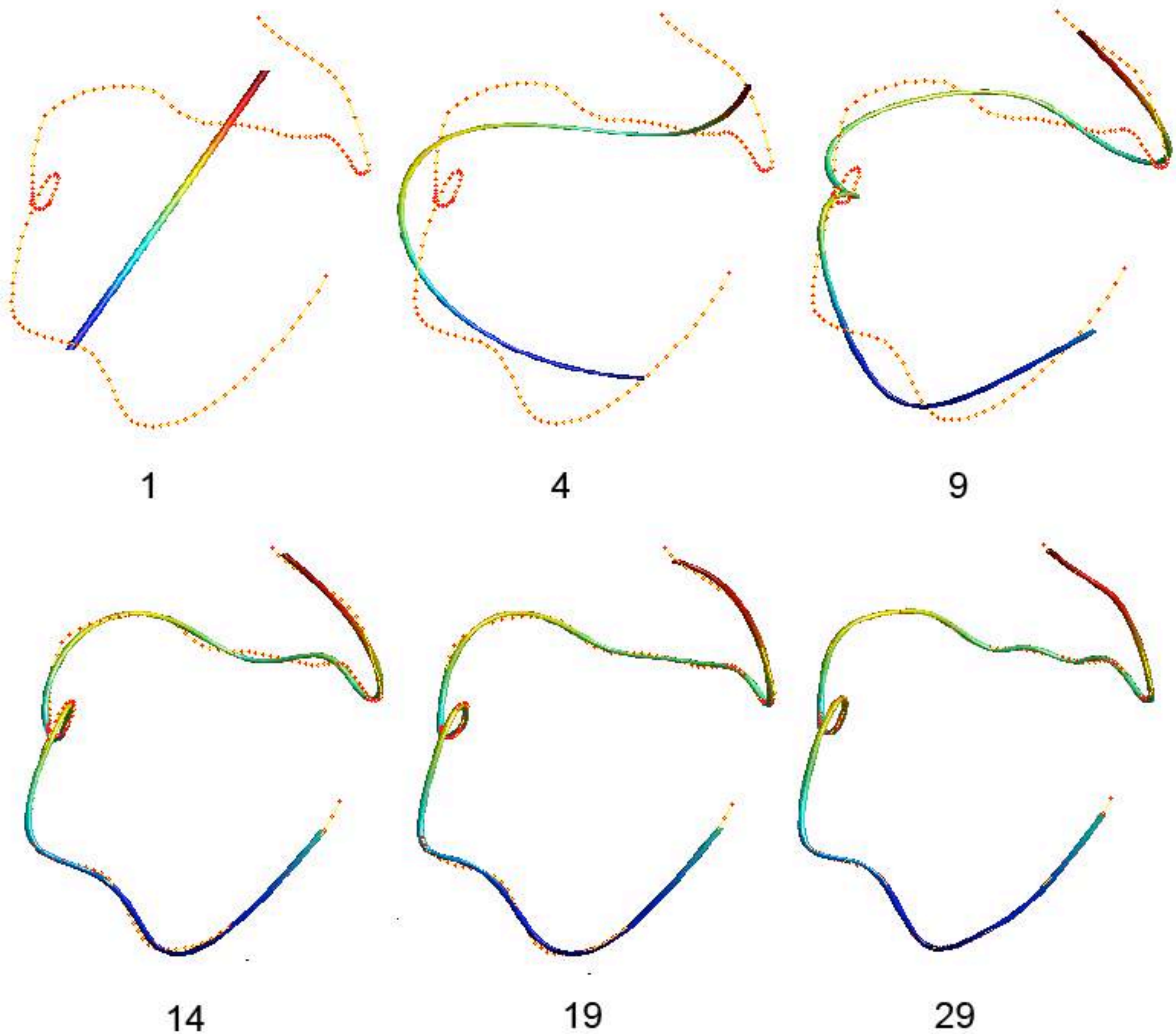
88.1799	56.6336	5.7367
-12.4775	-11.2552	-2.0791
2.4336	-15.4428	-0.4021
4.3956	2.2733	-0.9354
-0.0106	-0.0674	0.6999
2.1773	-2.4194	-0.1176
0.5808	0.8390	1.2942
0.0615	-0.1893	0.1188
-0.2629	0.7524	0.1089
0.7909	-0.7276	-0.1901
0.5458	0.6236	0.6939
0.4295	-0.4337	0.2185
0.2150	0.4157	0.0254
0.1584	-0.1973	0.0762
-0.1557	0.2466	-0.1086
0.0632	-0.0978	-0.0208
0.0389	-0.0143	-0.0284
-0.0014	-0.1193	0.1970
0.0004	0.0129	-0.0198
0.1342	0.0002	0.0260

Any tract can be compactly parameterized with only 60 coefficients.

basis expansion

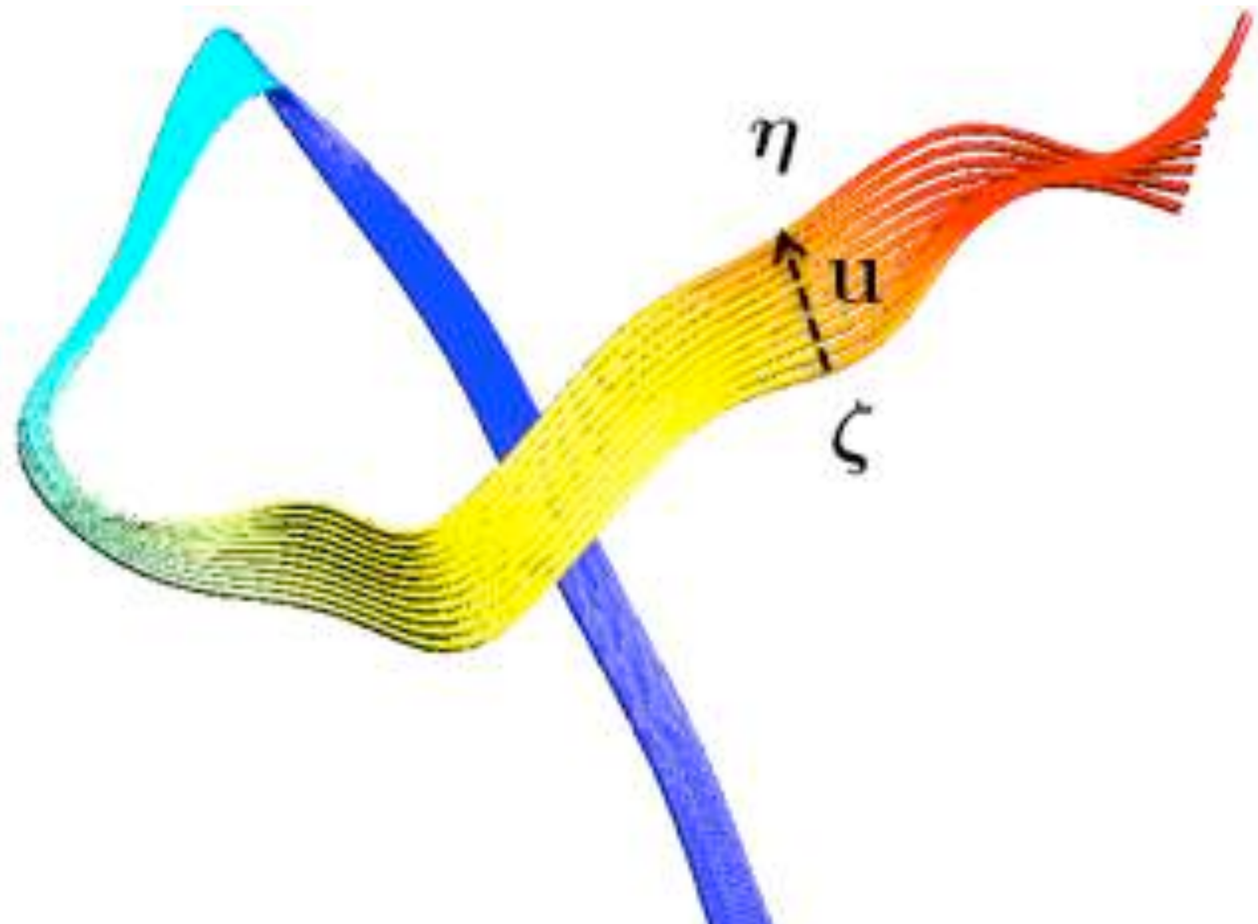
$$(x, y, z)' = \sum_{l=0}^{19} \beta_l \cos(l\pi t)$$

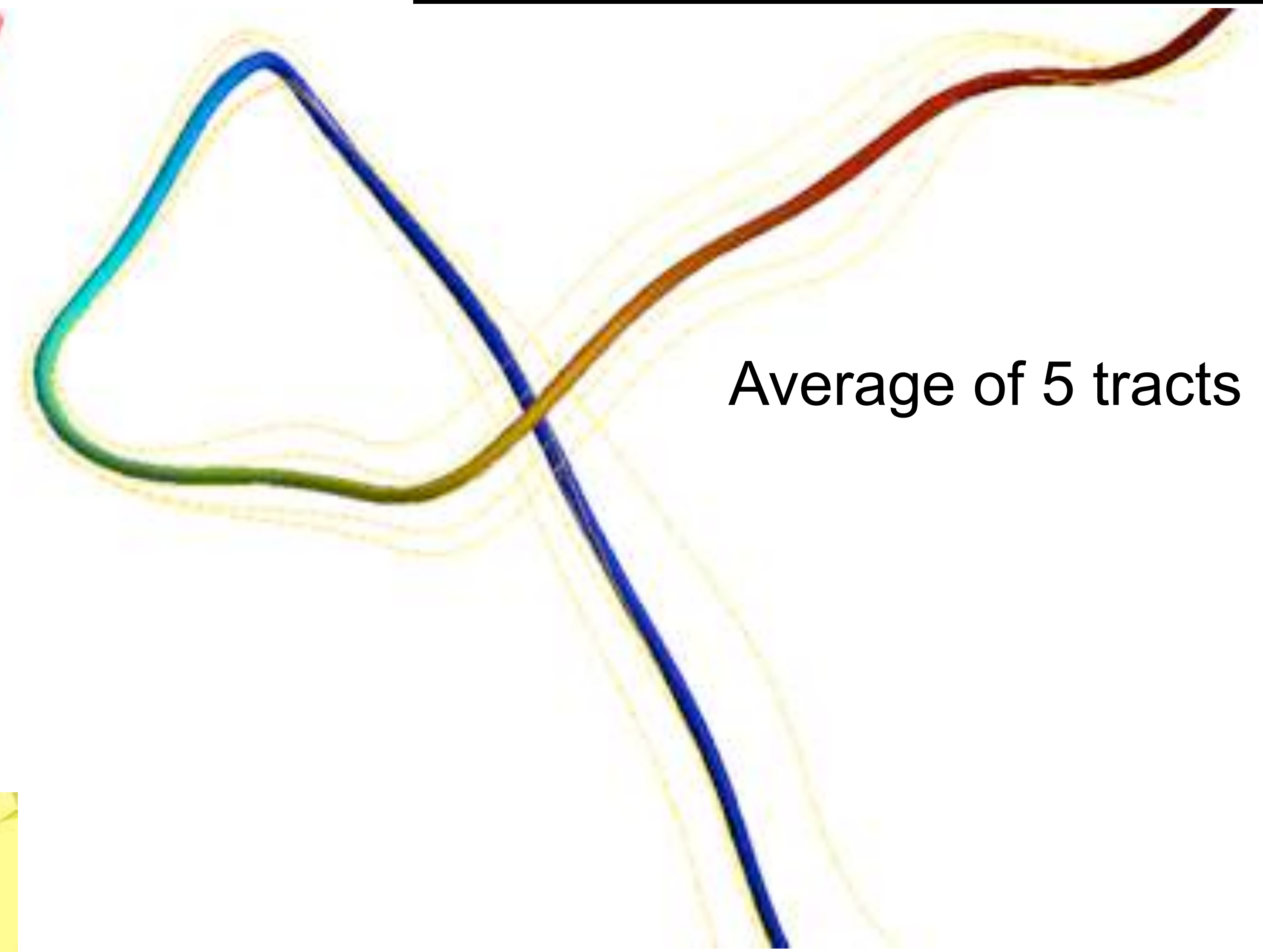
Cosine series representation at various degrees

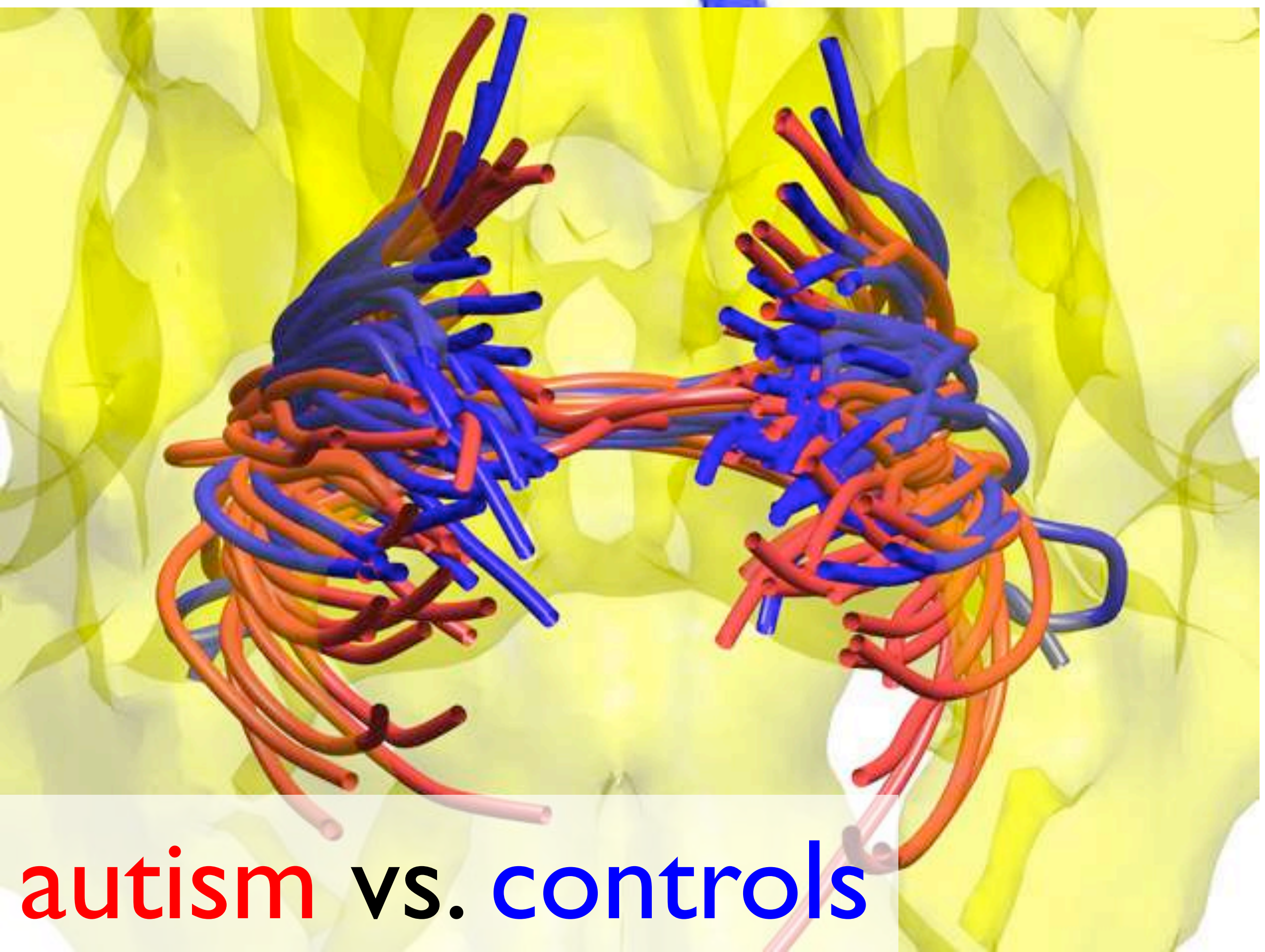


Tract averaging

Tract matching

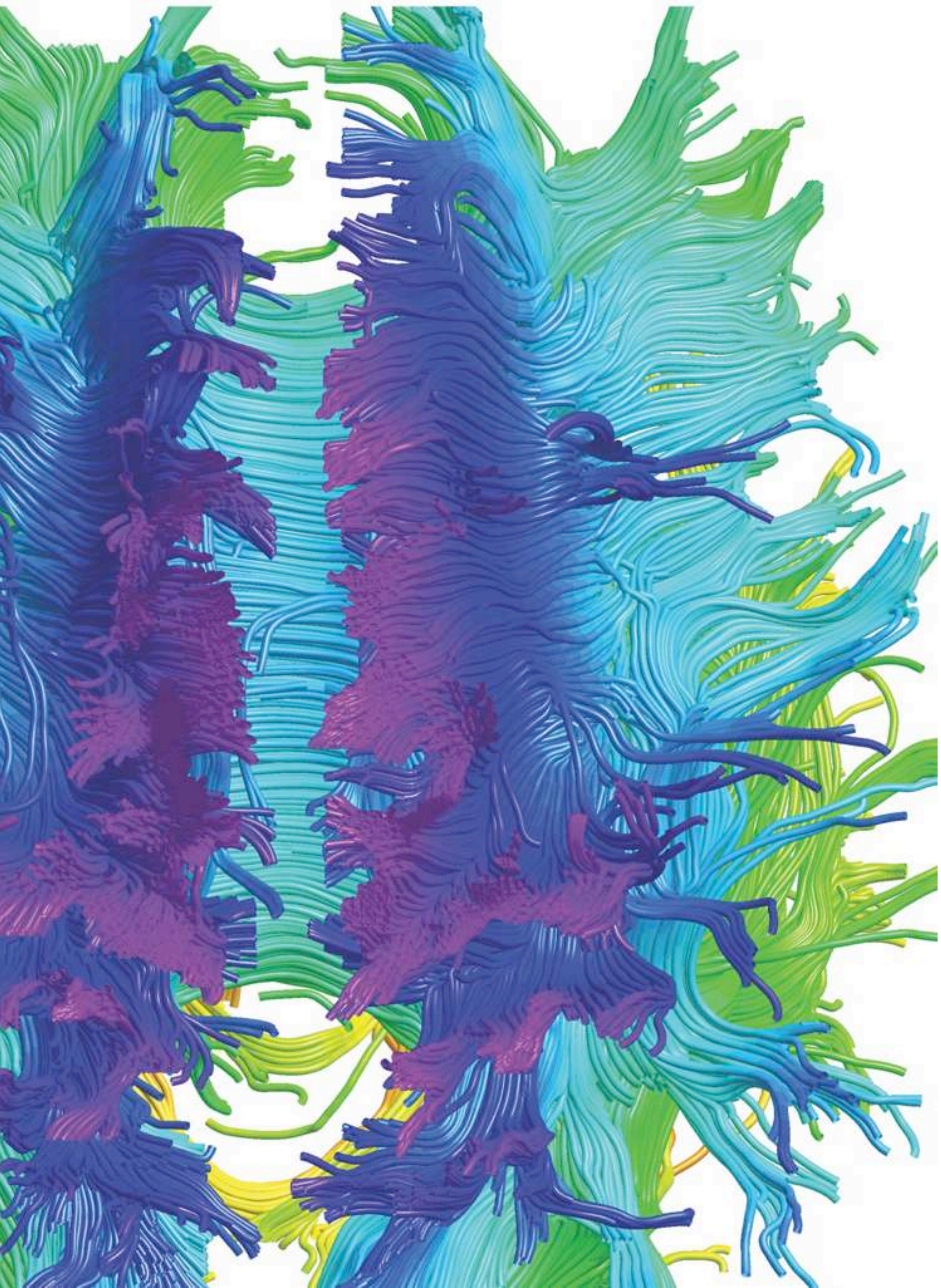






MATLAB:

http://brainimaging.waisman.wisc.edu/~chung/tracts

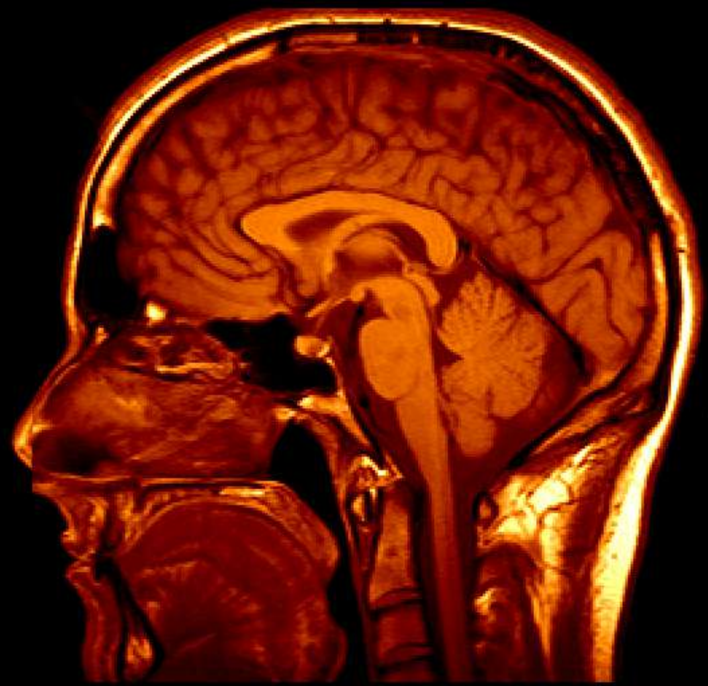


Question:

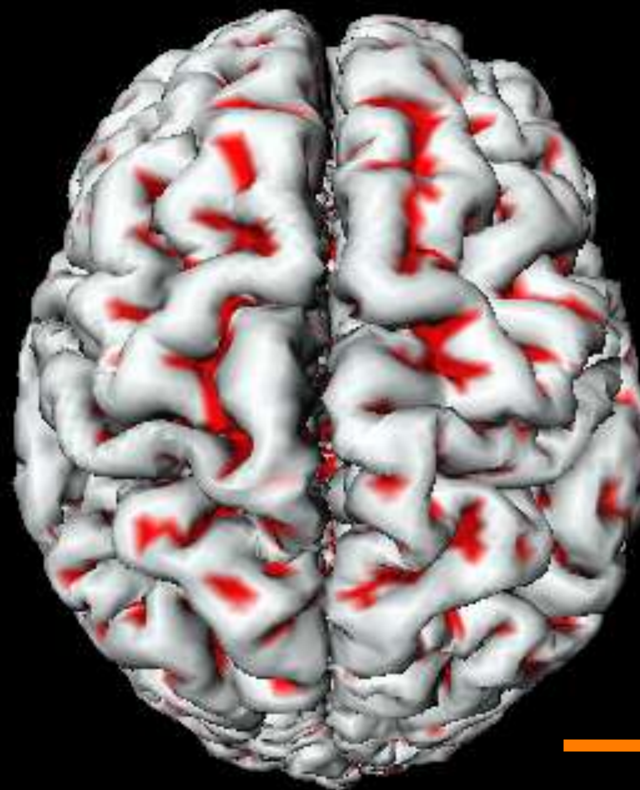
Parameterize the whole white matter fibers using a single parameterization.

Surface parameterization

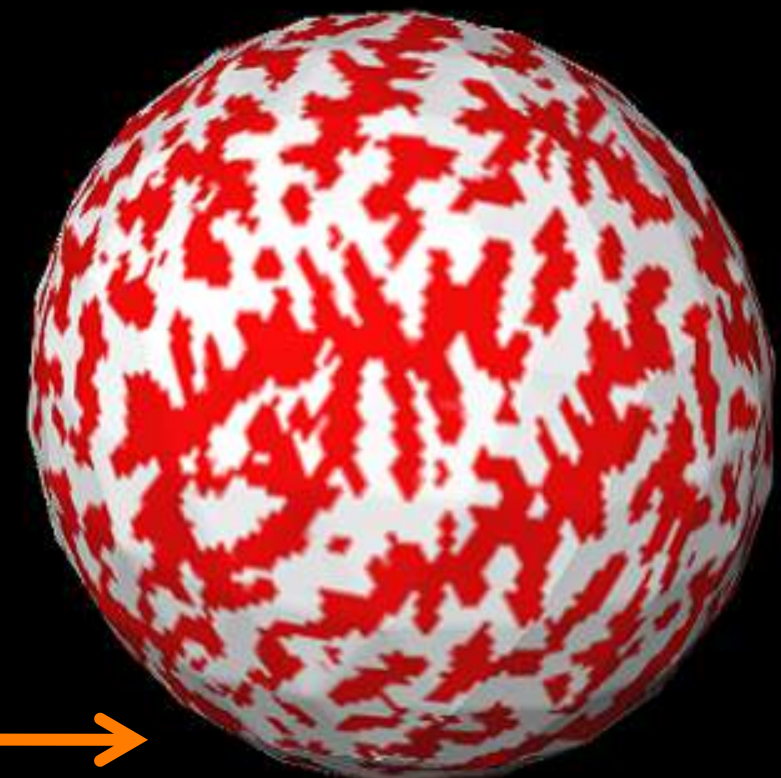
3T MRI

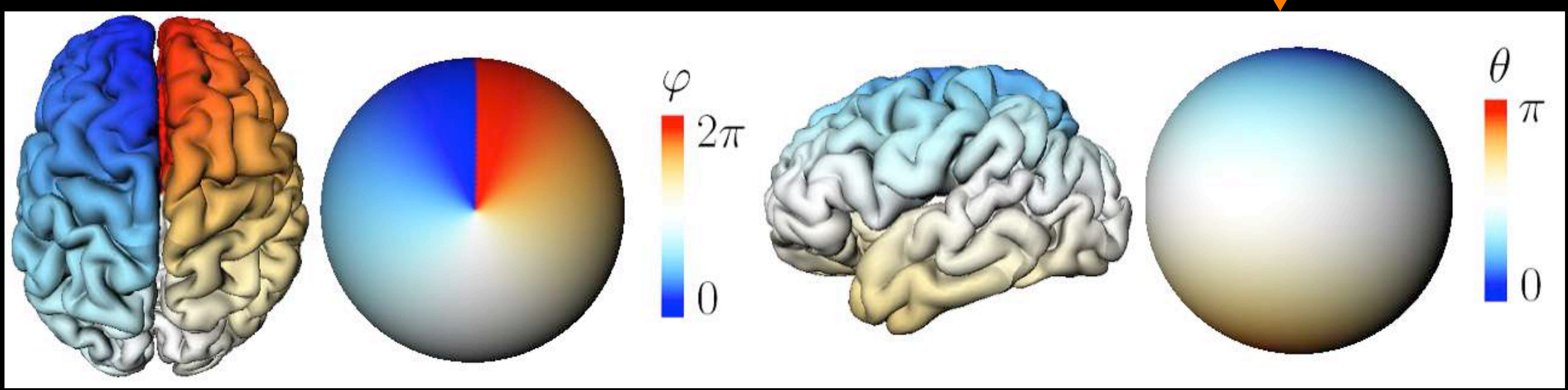


Surface segmentation



Surface flattening

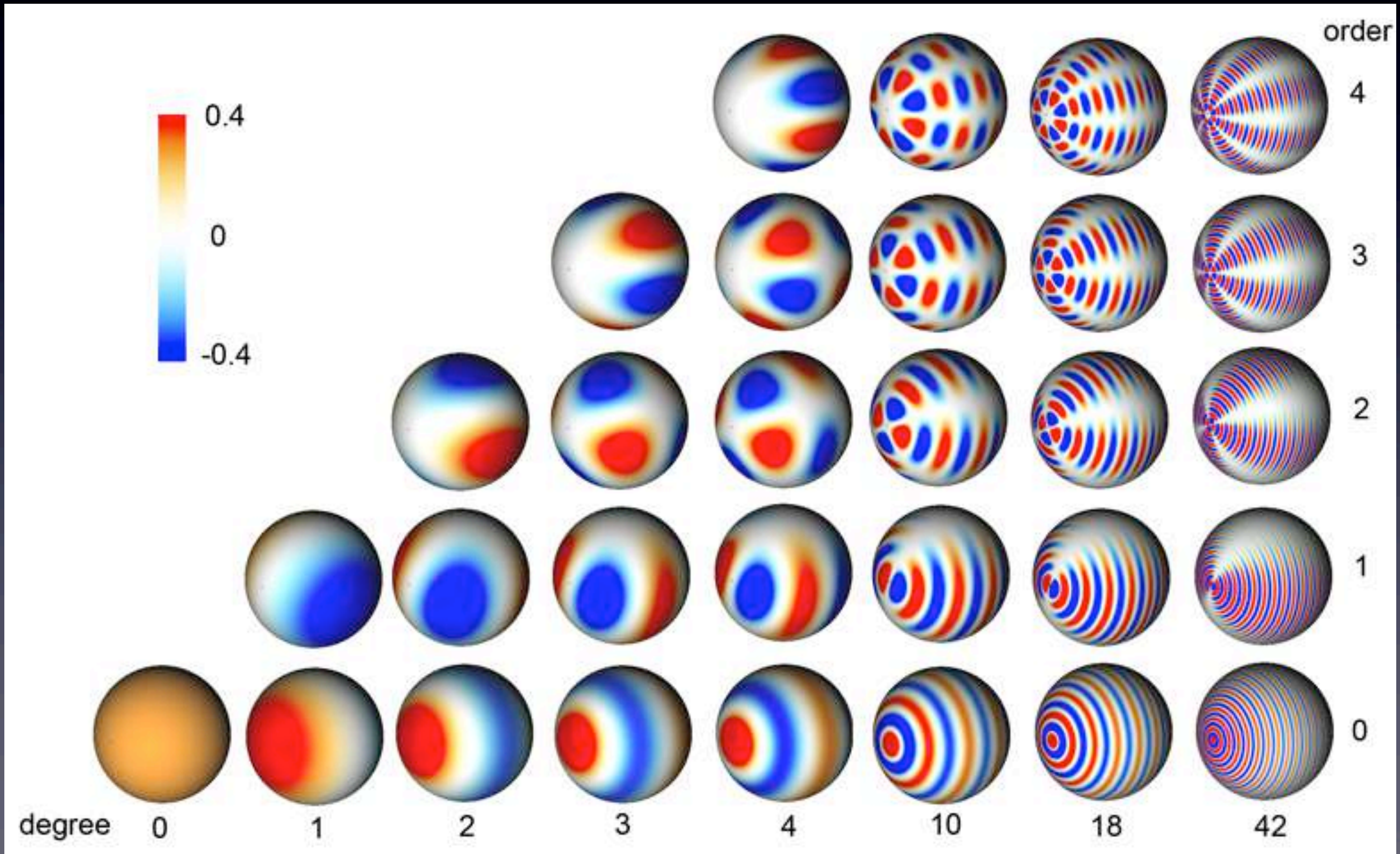




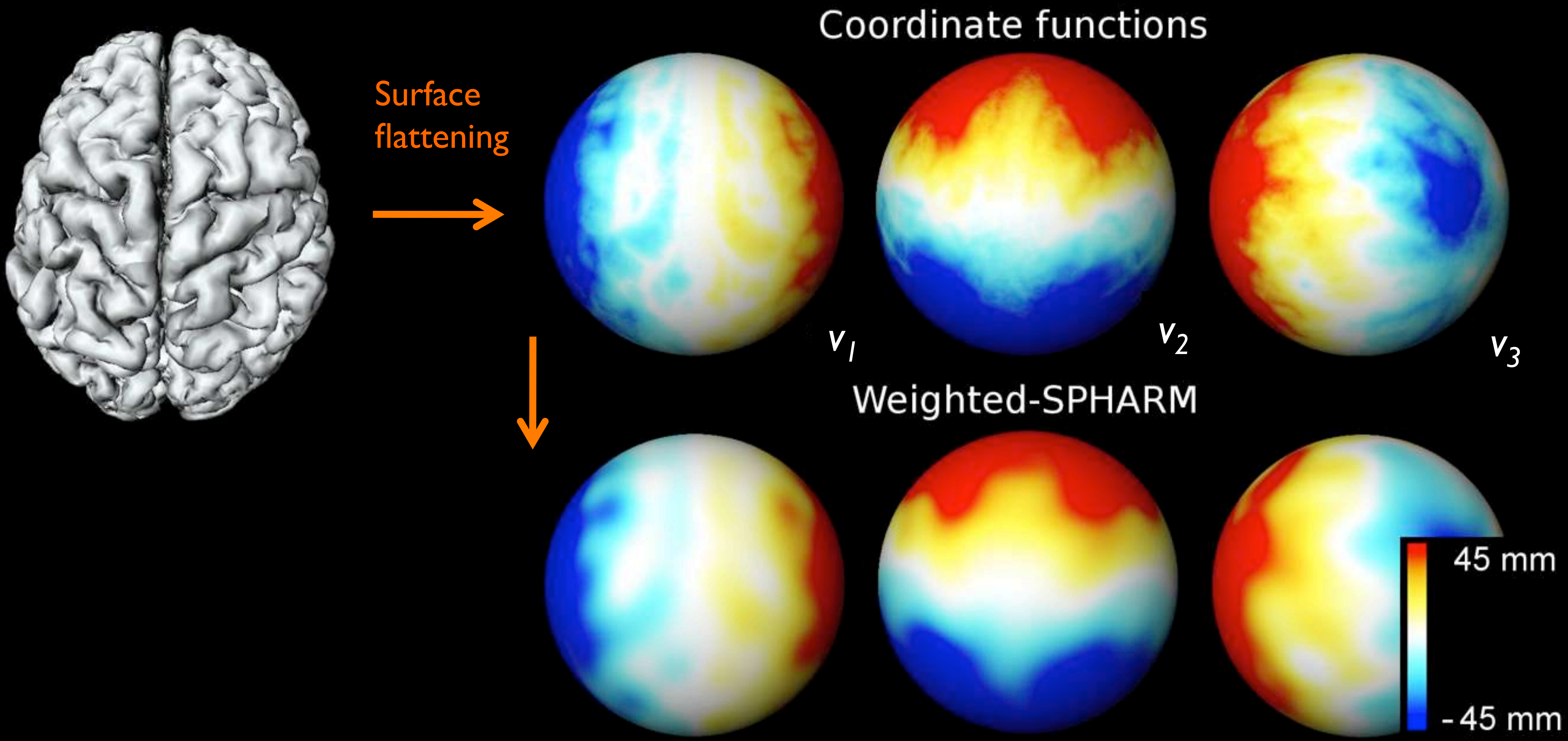
Spherical angle based coordinate system

Spherical harmonic of degree *l* and order *m*

$$Y_{lm} = \begin{cases} c_{lm} P_l^{|m|}(\cos\theta)\sin(|m|\varphi), & -l \le m \le -1, \\ \frac{c_{lm}}{\sqrt{2}} P_l^0(\cos\theta), & m = 0, \\ c_{lm} P_l^{|m|}(\cos\theta)\cos(|m|\varphi), & 1 \le m \le l, \end{cases}$$

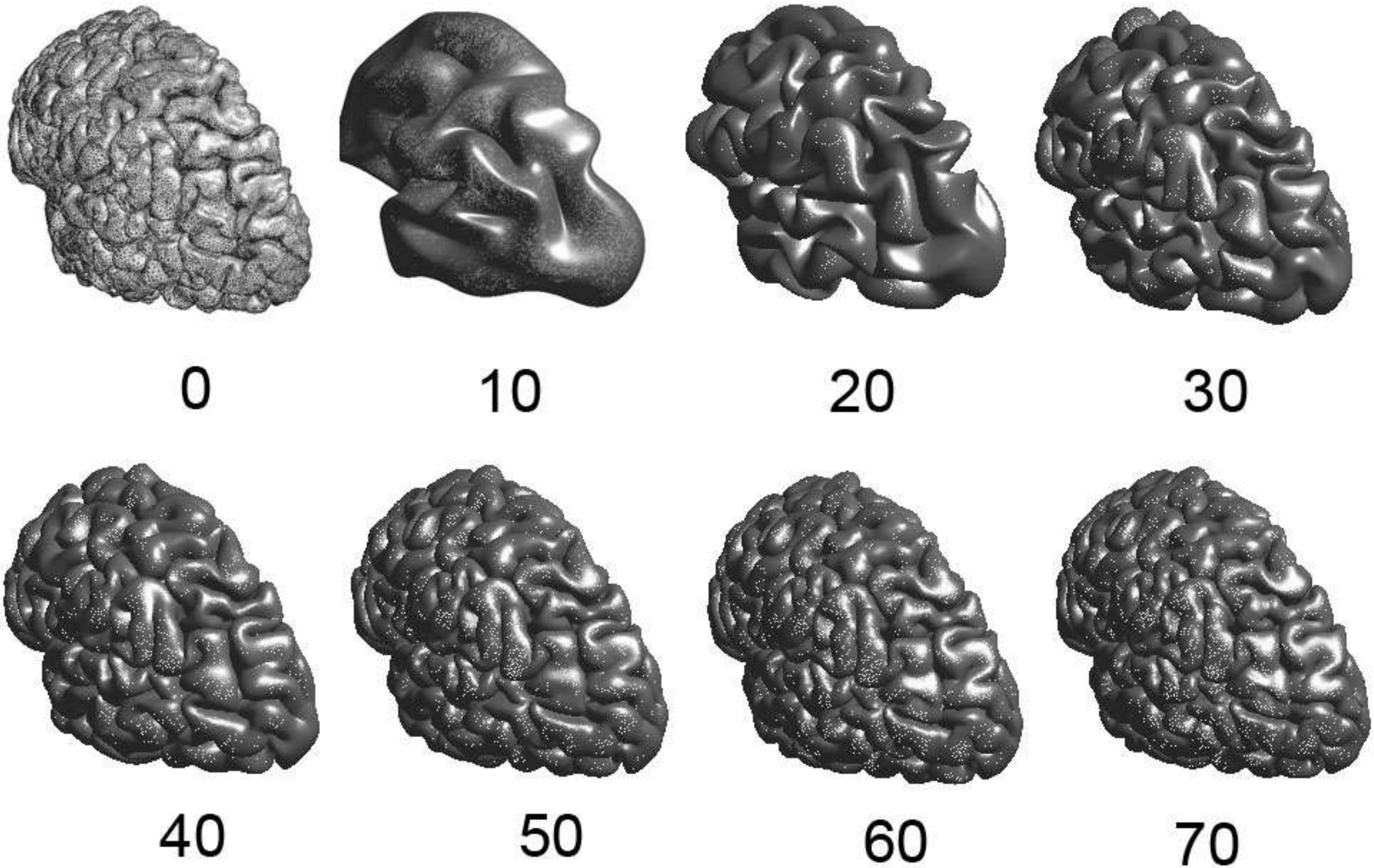


Weighted-Spherical harmonics (SPHARM)



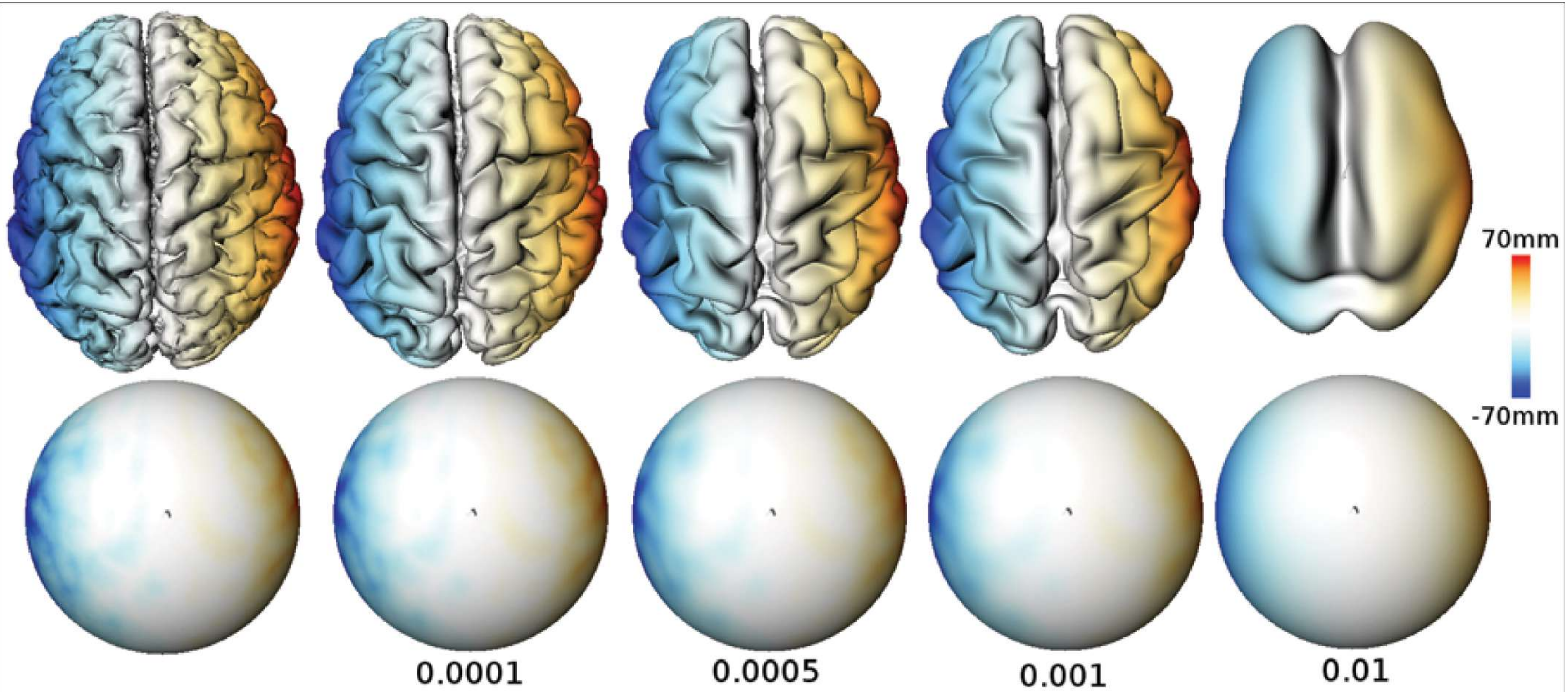
$$v_i(\theta, \varphi) = \sum_{l=0}^{k} \sum_{m=-l}^{l} e^{-l(l+1)\sigma} f_{lm}^i Y_{lm}(\theta, \varphi)$$

SPHARM with different degrees



Chung et al., 2007 IEEE Transactions on Medical Imaging 26:566-581

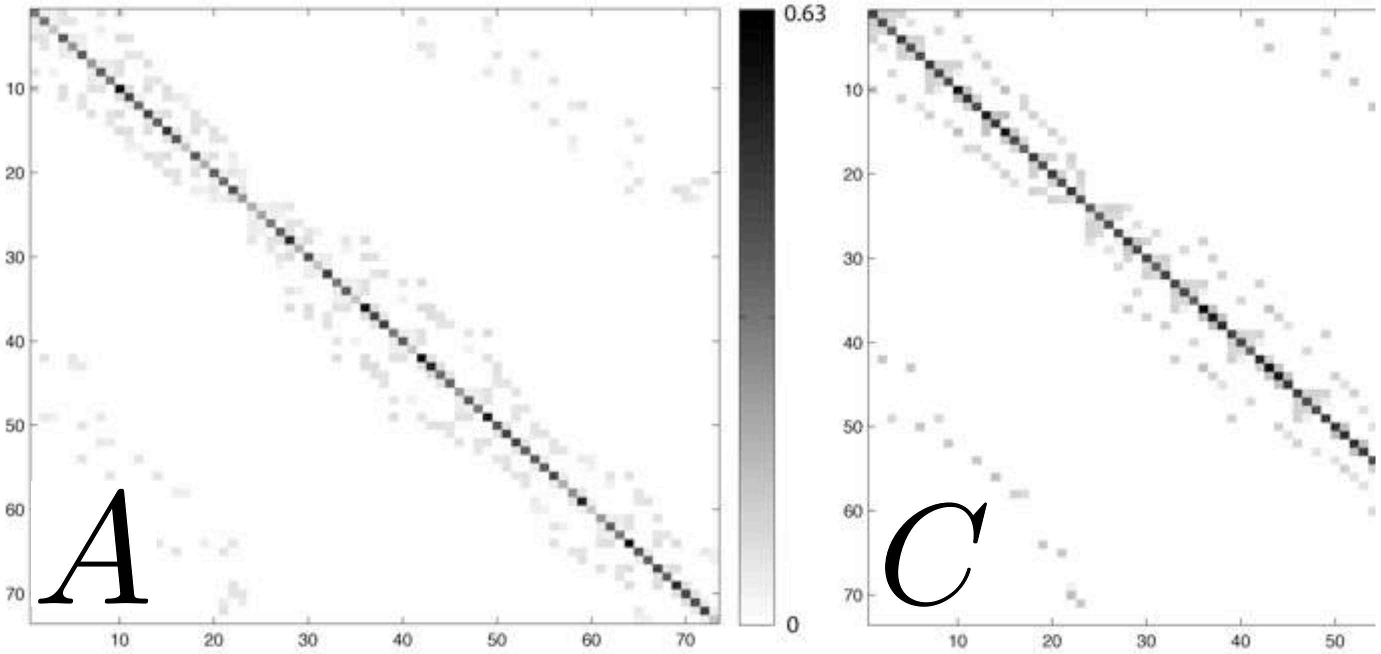
Weighted-SPHARM

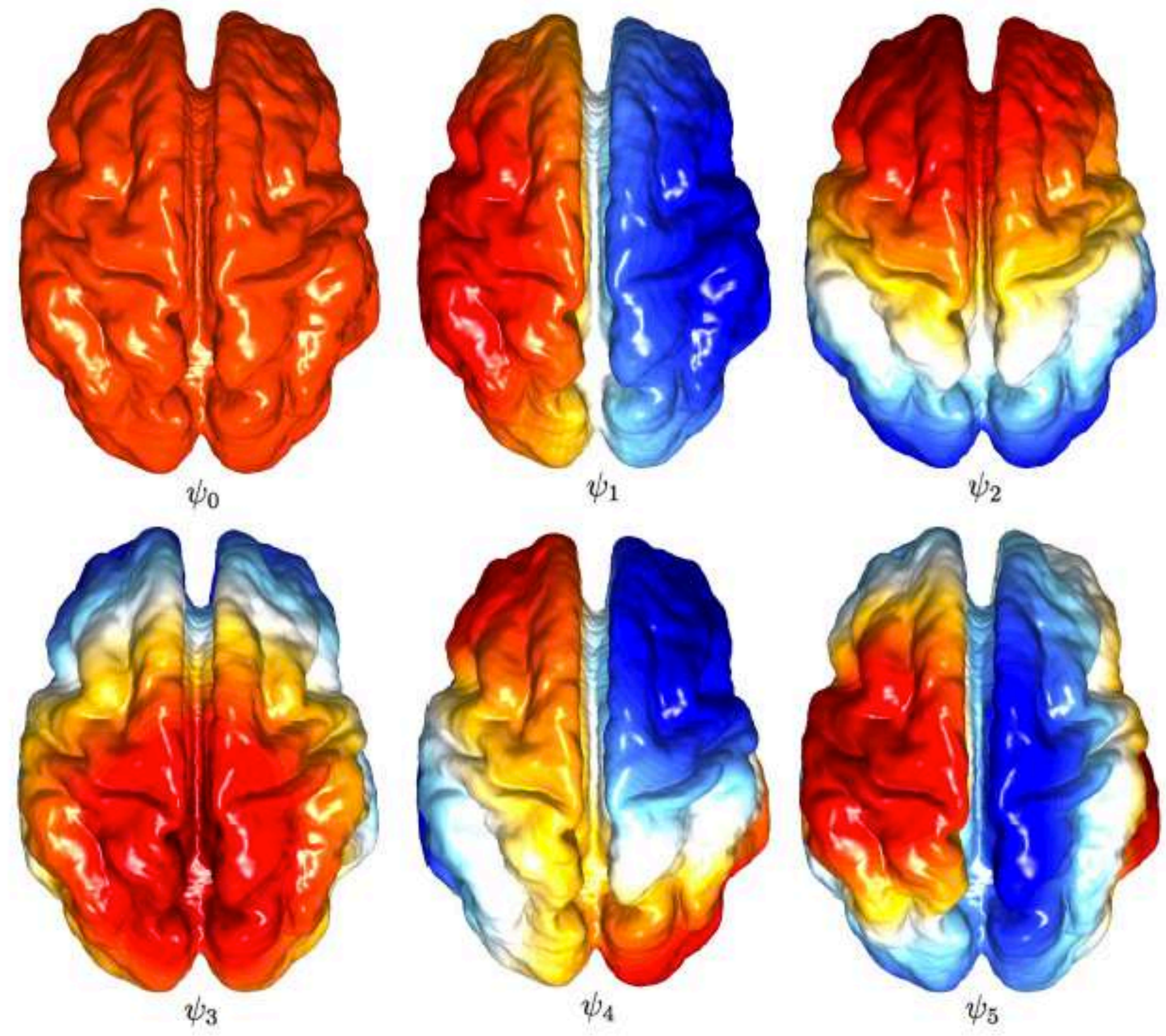


heat kernel bandwidth, diffusion time

Matlab: http://www.stat.wisc.edu/~mchung/softwares/weighted-SPHARM/weighted-SPHARM.html

Laplace Beltrami eigenfunction expansion



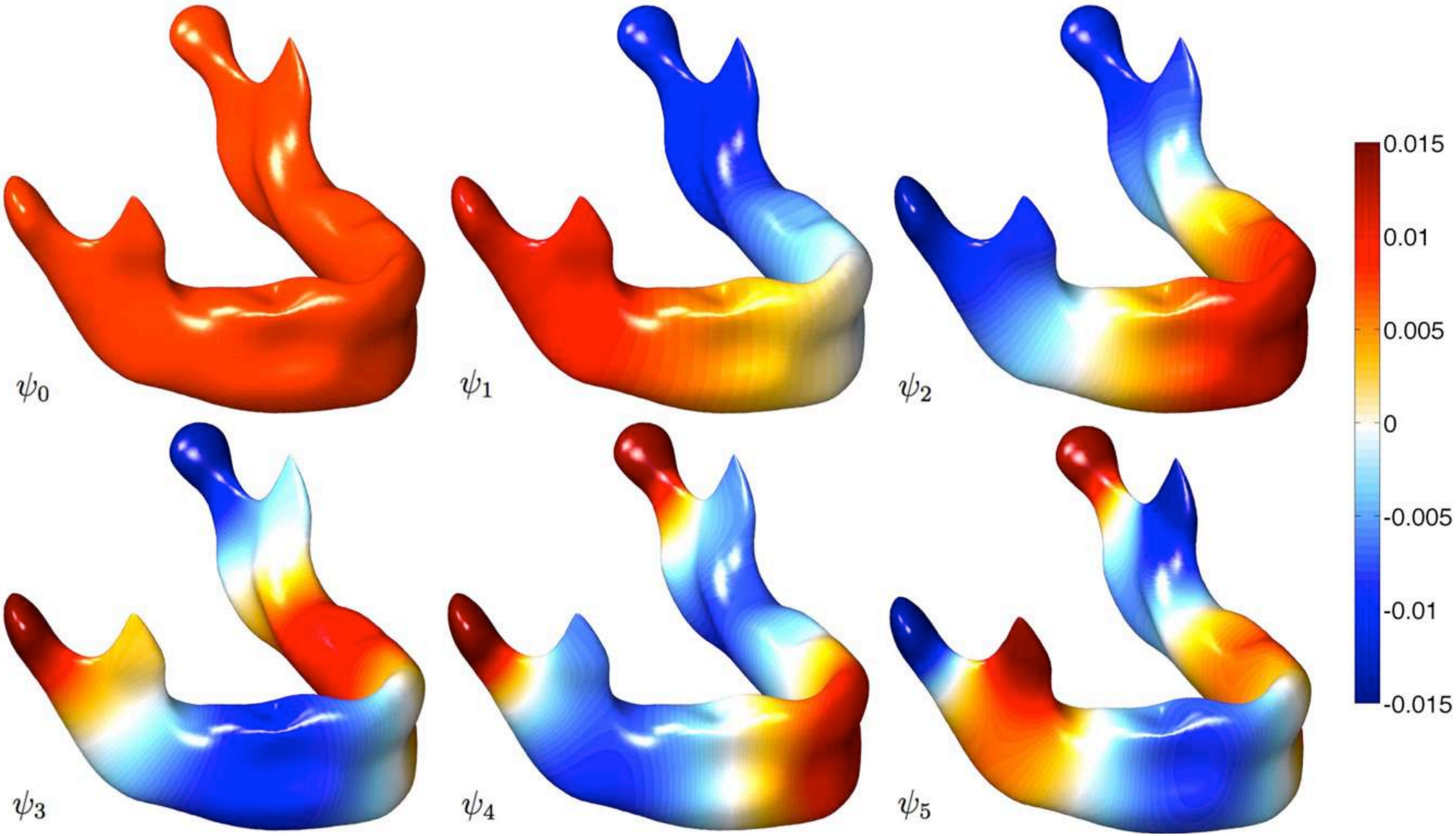


$$\Delta f = \lambda f \quad \dashrightarrow \quad C\psi = \lambda A\psi$$

MATLAB:

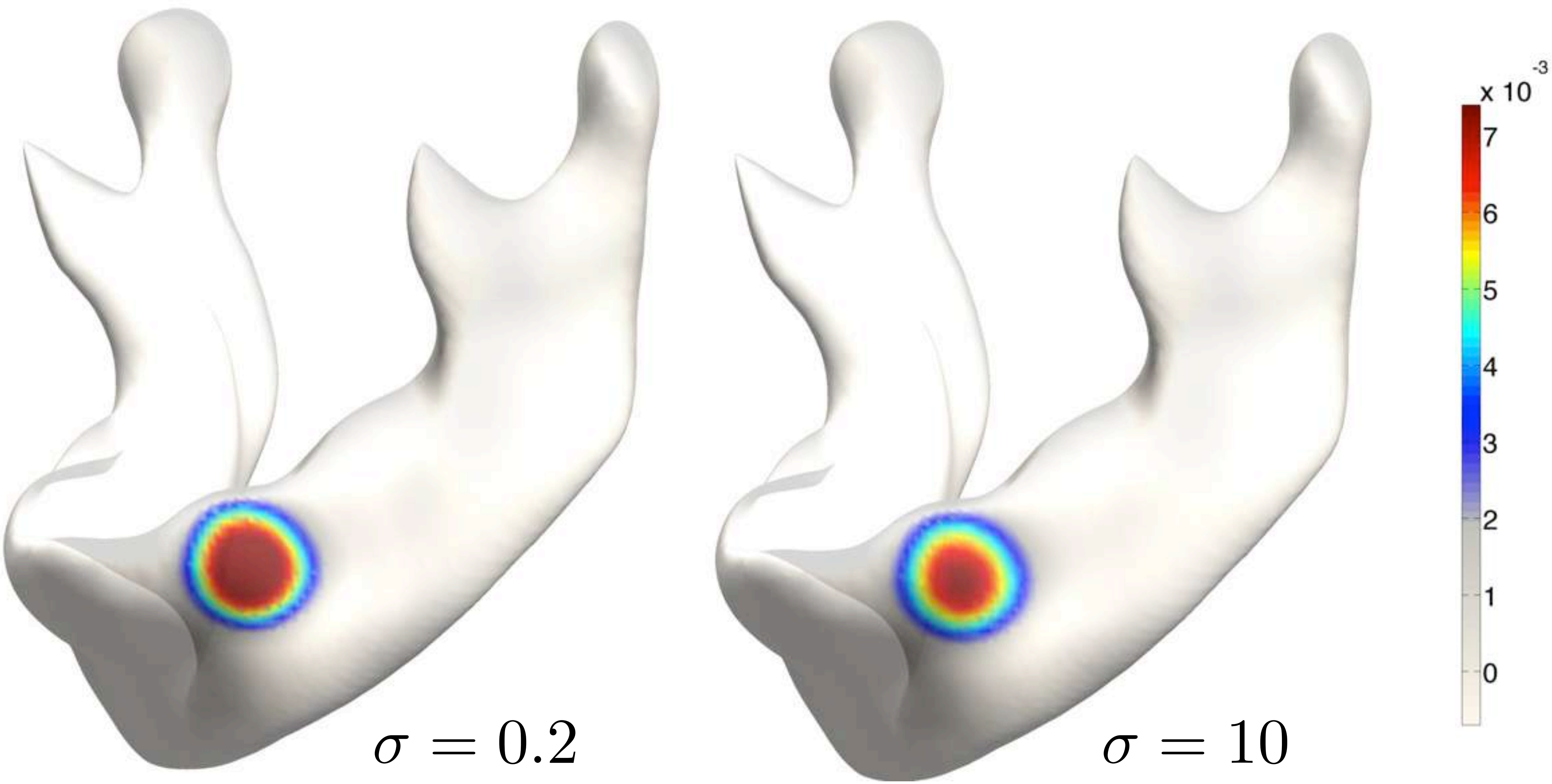
http://brainimaging.waisman.wisc.edu/~chung/lb

Laplace-Beltrami eigenfunctions on mandible



$$\Delta\psi_j = \lambda_j\psi_j$$

Heat kernel = probability distribution on manifold



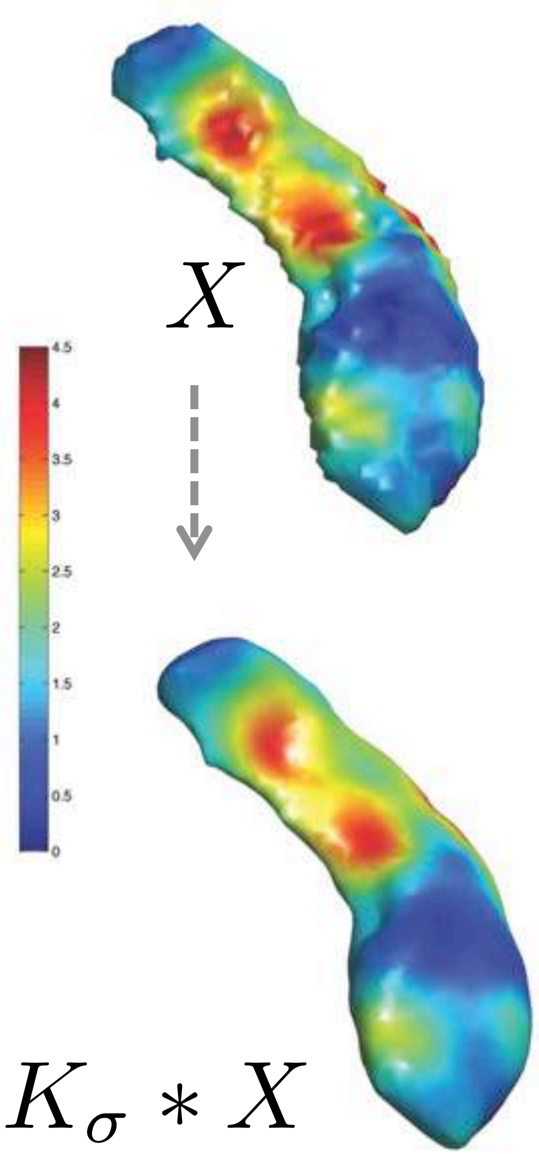
$\sigma = 0.2$ $\sigma = 10$

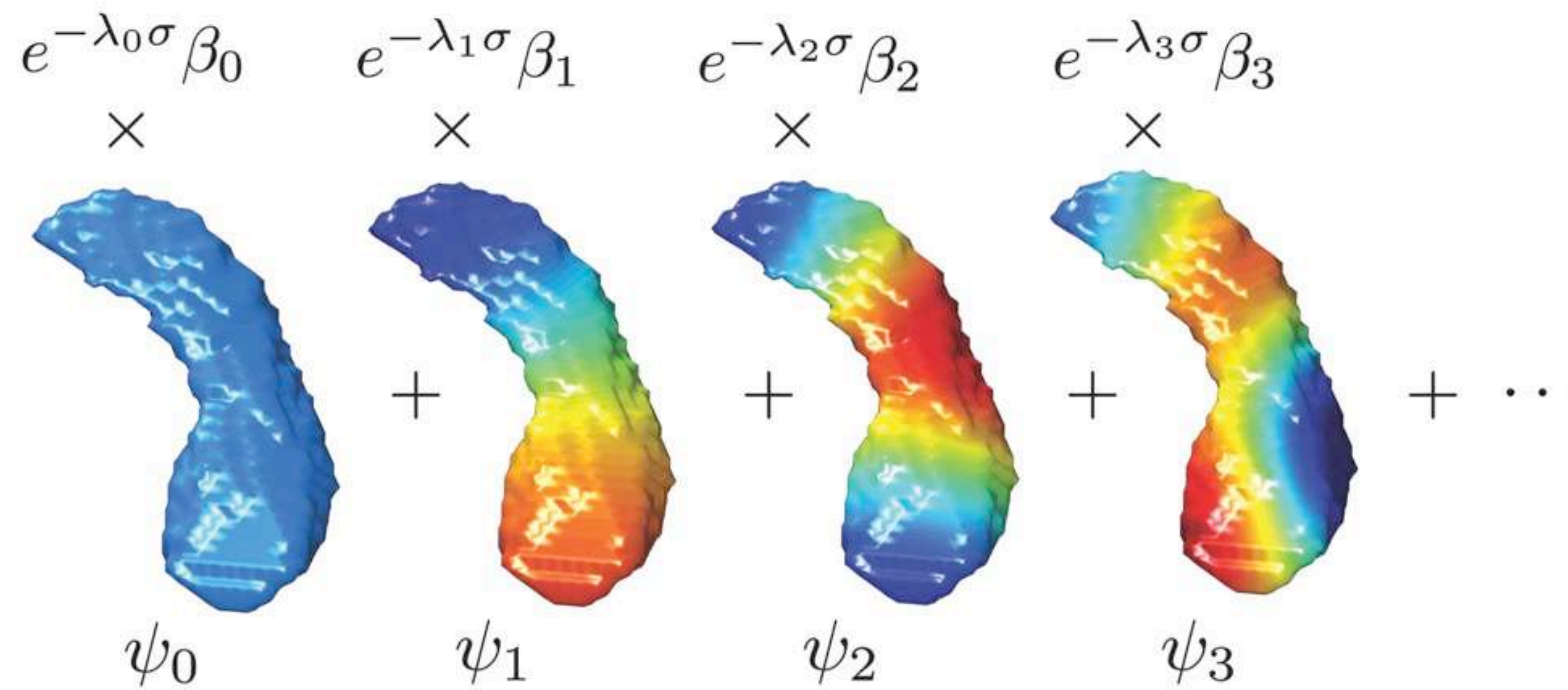
$$K_\sigma(p,q) = \sum_{j=0}^{\infty} e^{-\lambda_j \sigma} \psi_j(p)\psi_j(q)$$

Heat kernel smoothing

$$K_\sigma * X(p) = \sum_{j=0}^{\infty} e^{-\lambda_j \sigma} X_j \psi_j(p)$$

$$\beta_j = \int X(p)\psi_j(p)\, d\mu(p)$$





Chung, Qiu et al. 2015 Medical Image Analysis 22:63-76

Limitations

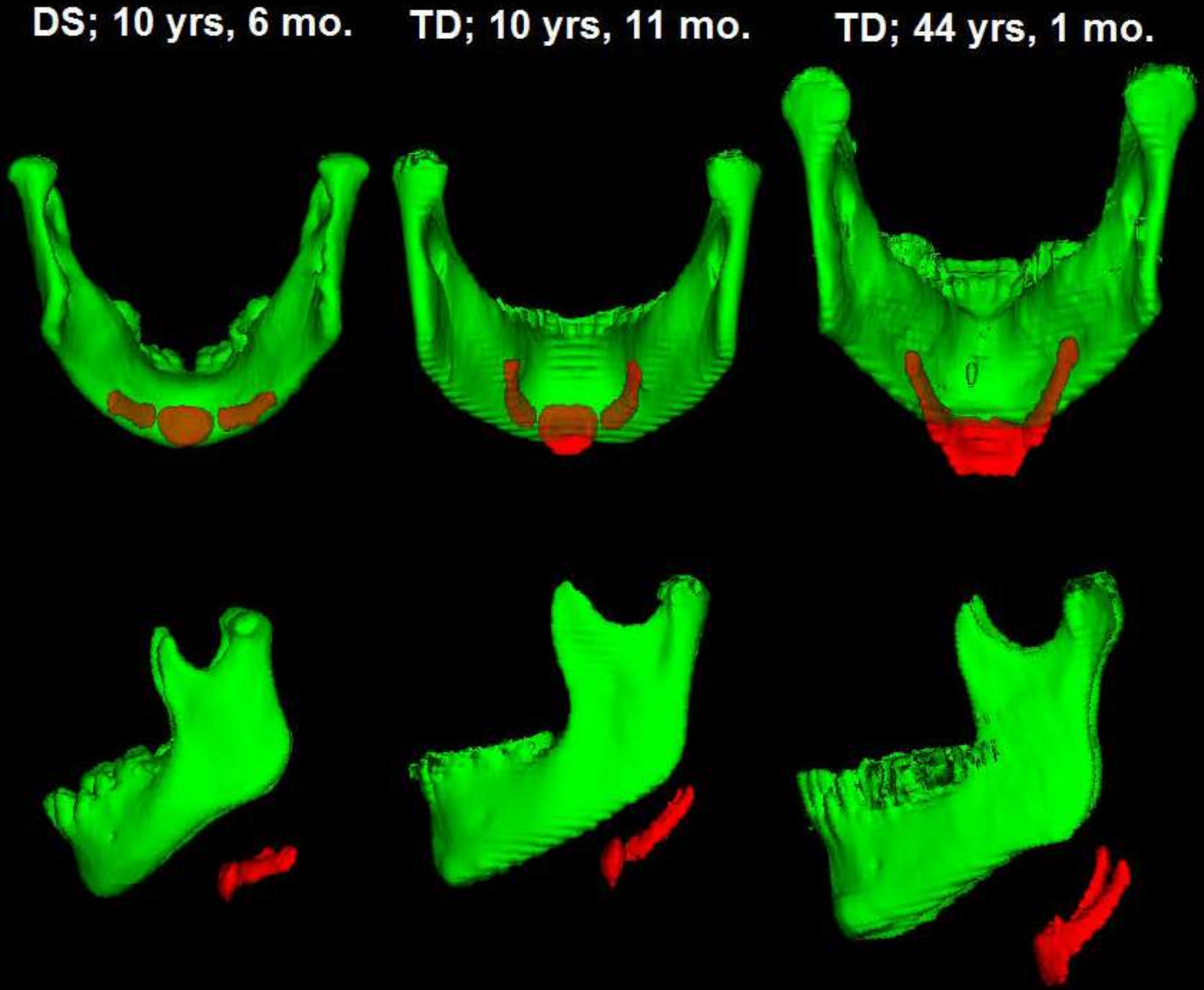
Existing parametric shape representations do *not* work for different topology

Cancer growth

Stroke lesions in brain

Bone fusion

Hyoid bone fusion

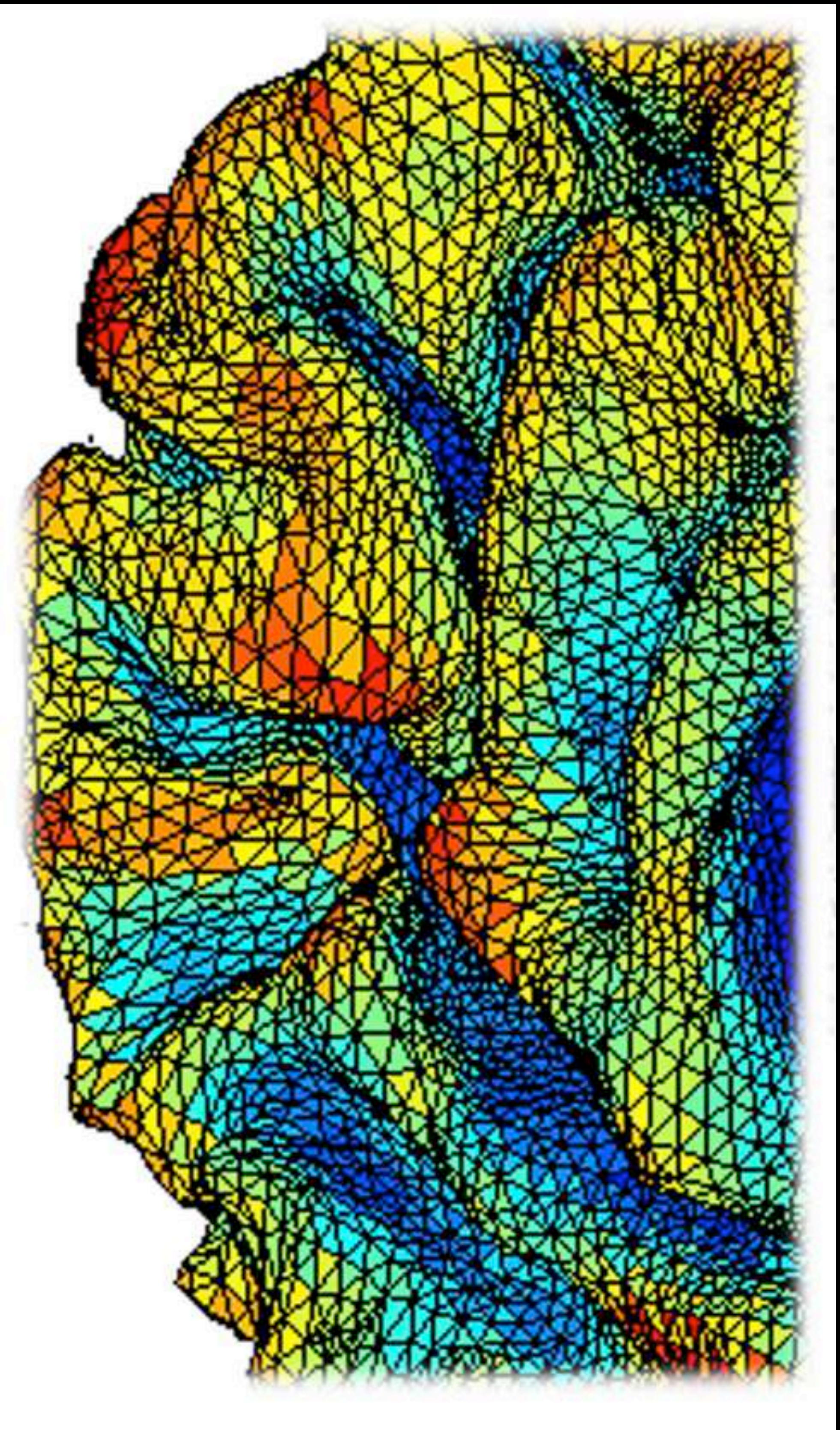


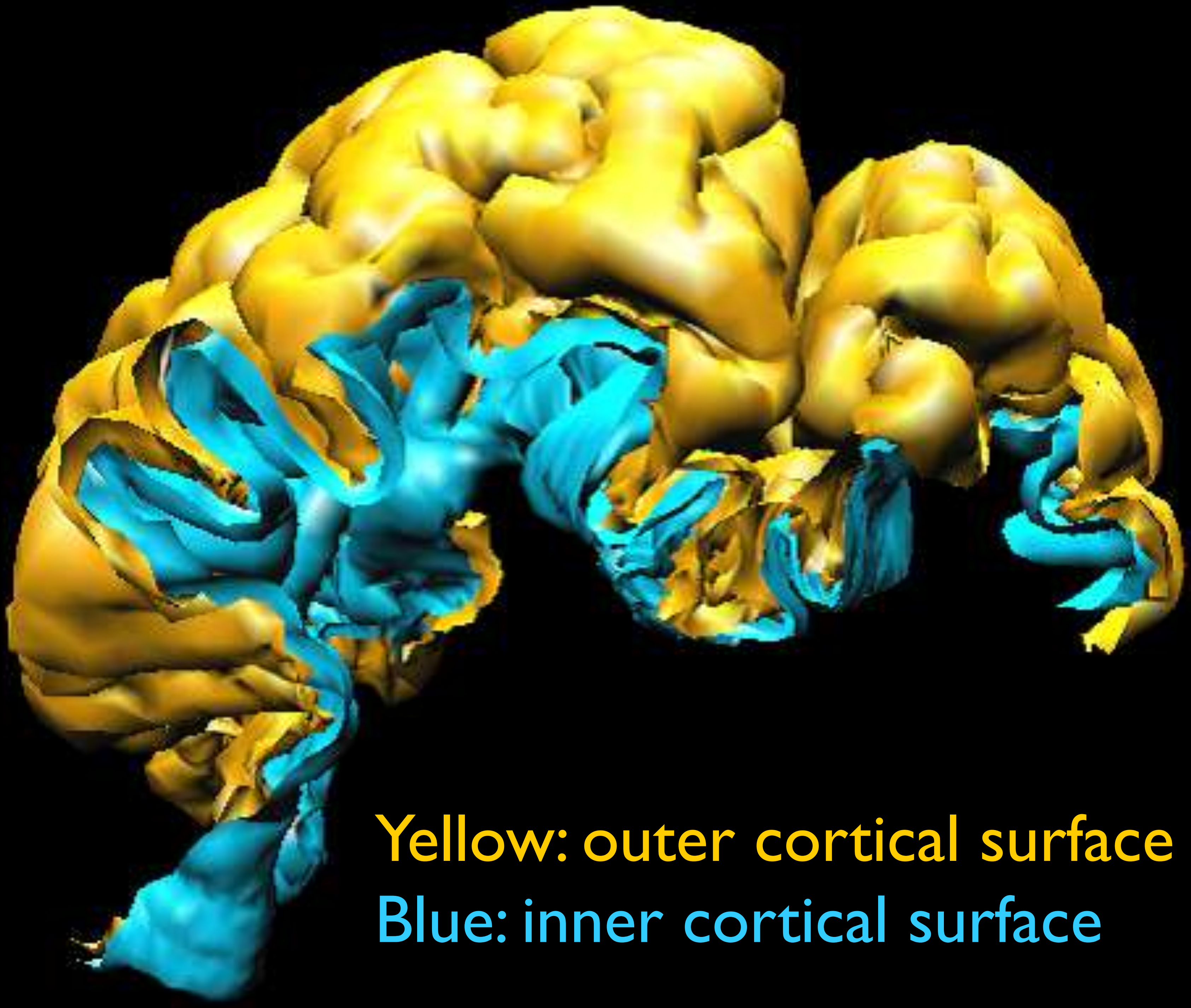
DS: down syndrome

TD: typically developing

Bessel Fourier Reconstruction (BFOR)

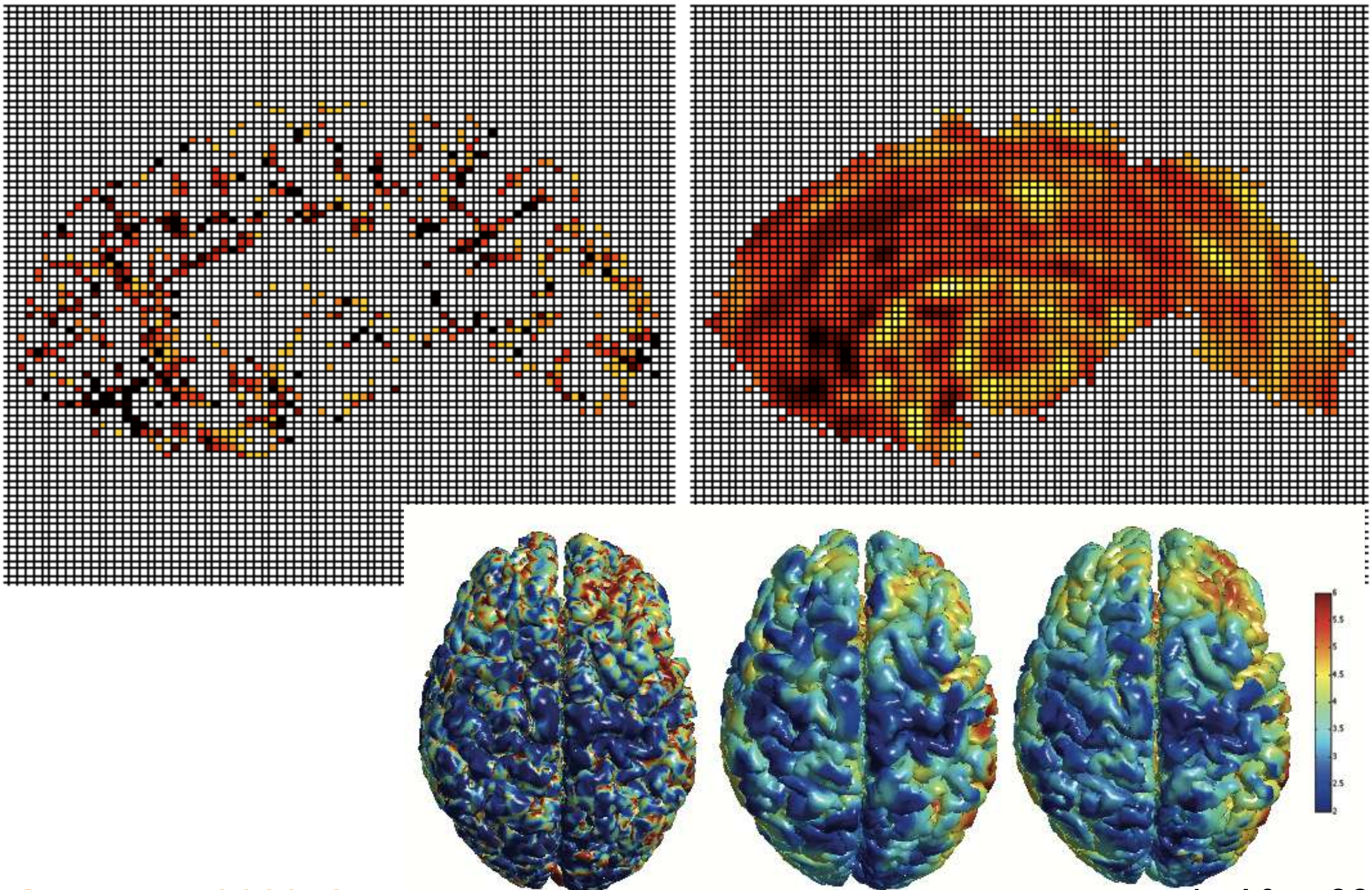
2D cortical thickness





Chung et al. 2003 NeuroImage 18:198-213

Bessel Fourier reconstruction (BFOR) on cortical thickness

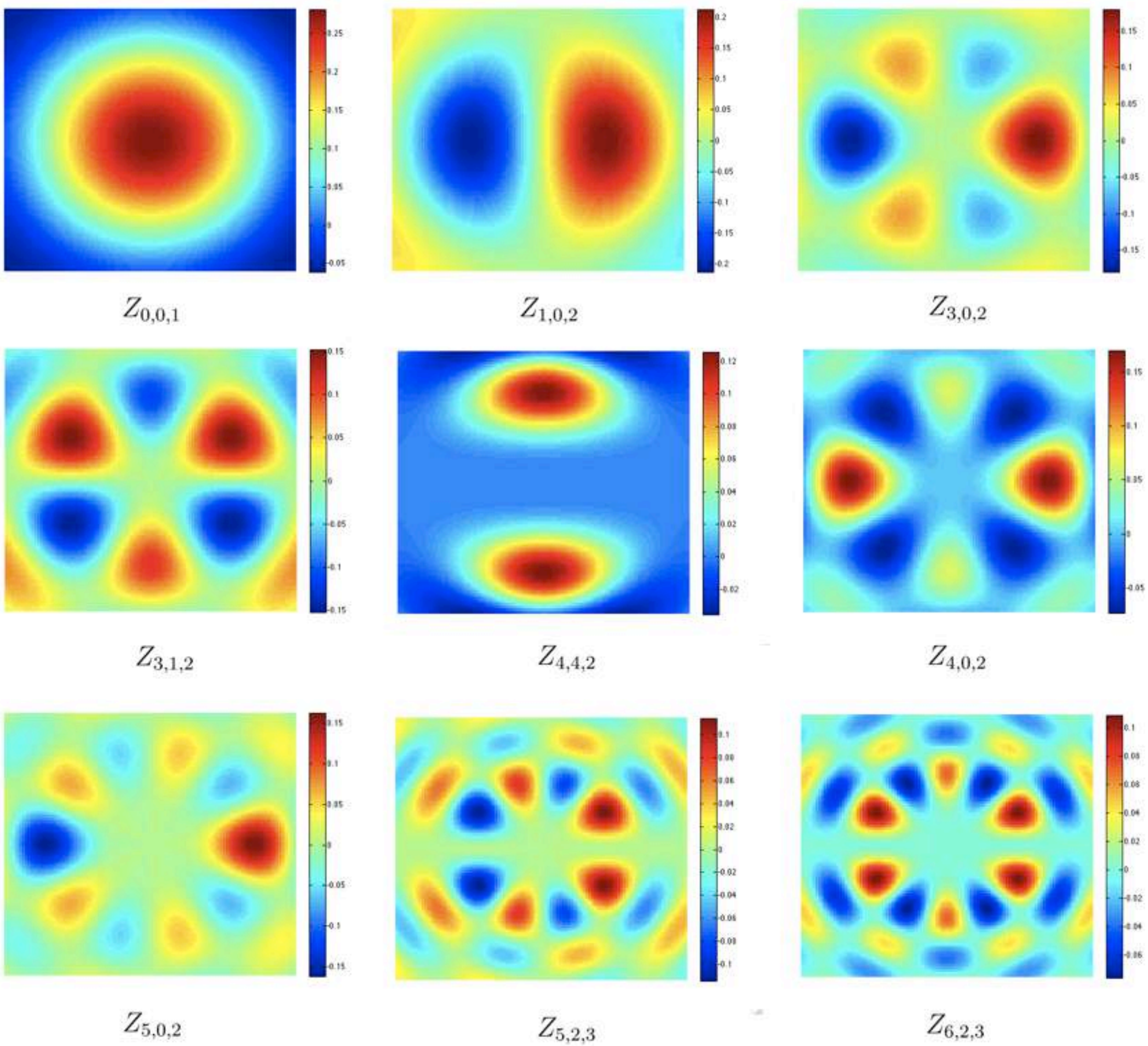


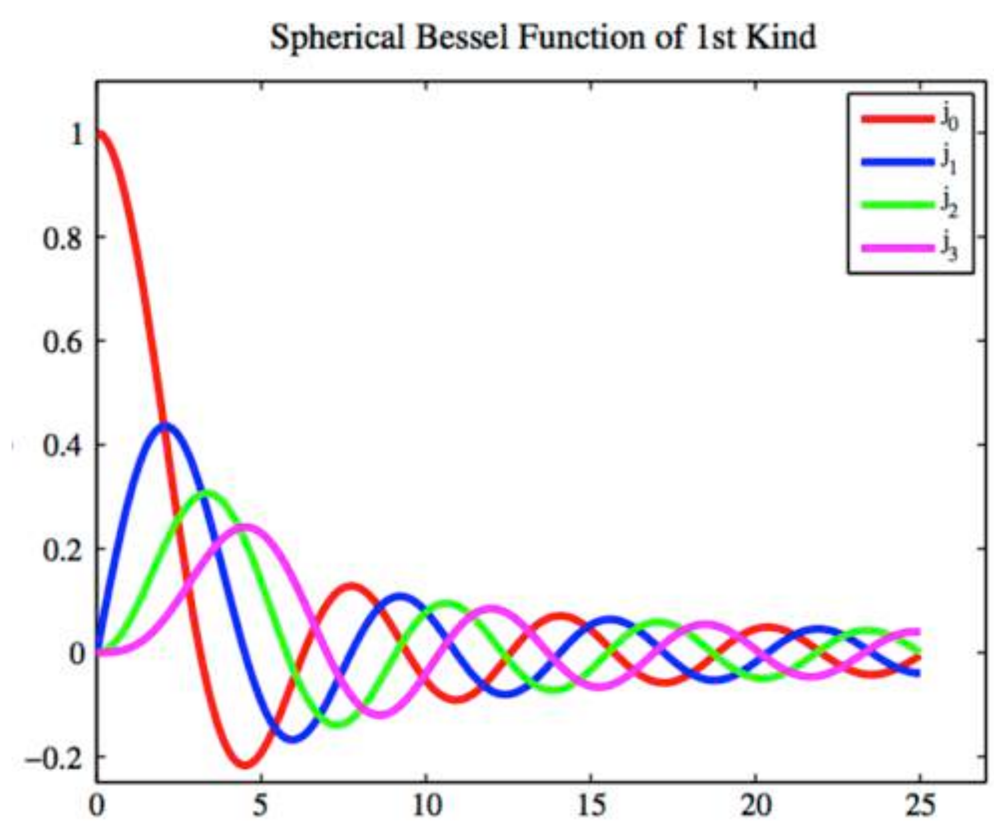
Chung et al. 2009 ISBI

$$f(r,\theta,\varphi) \approx \sum_{l=0}^{k} \sum_{m=-l}^{l} \sum_{n=1}^{j} \beta_{lmn} Z_{lmn}(r,\theta,\varphi)$$

$$\begin{aligned} & Z_{lmn}(r,\theta,\varphi) \\ = \quad & S_l(\sqrt{\lambda_{ln}}r) Y_{lm}(\theta,\varphi) \end{aligned}$$

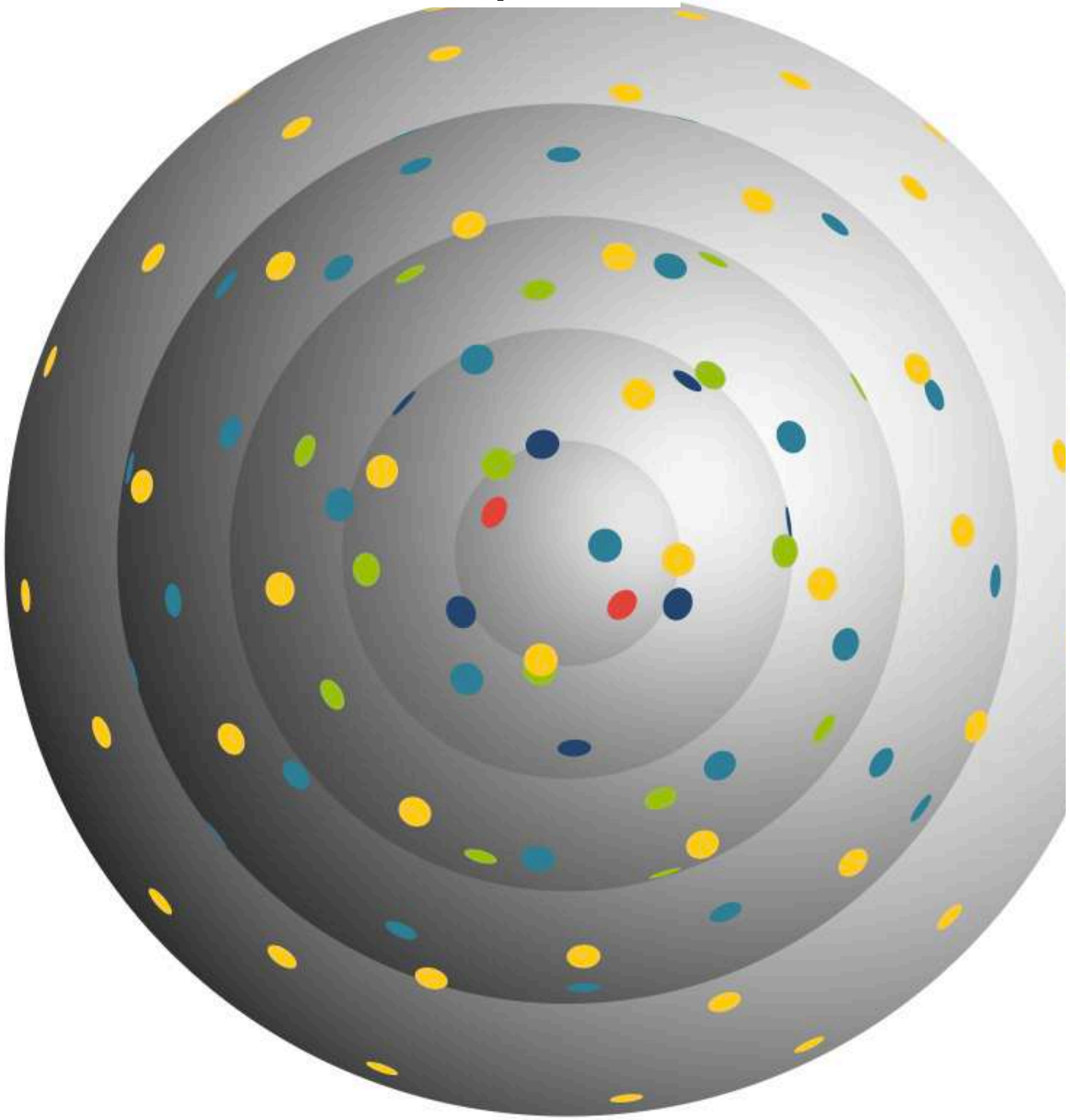
$$S_l(x) = \sqrt{\frac{\pi}{2x}} J_{l+1/2}(x)$$

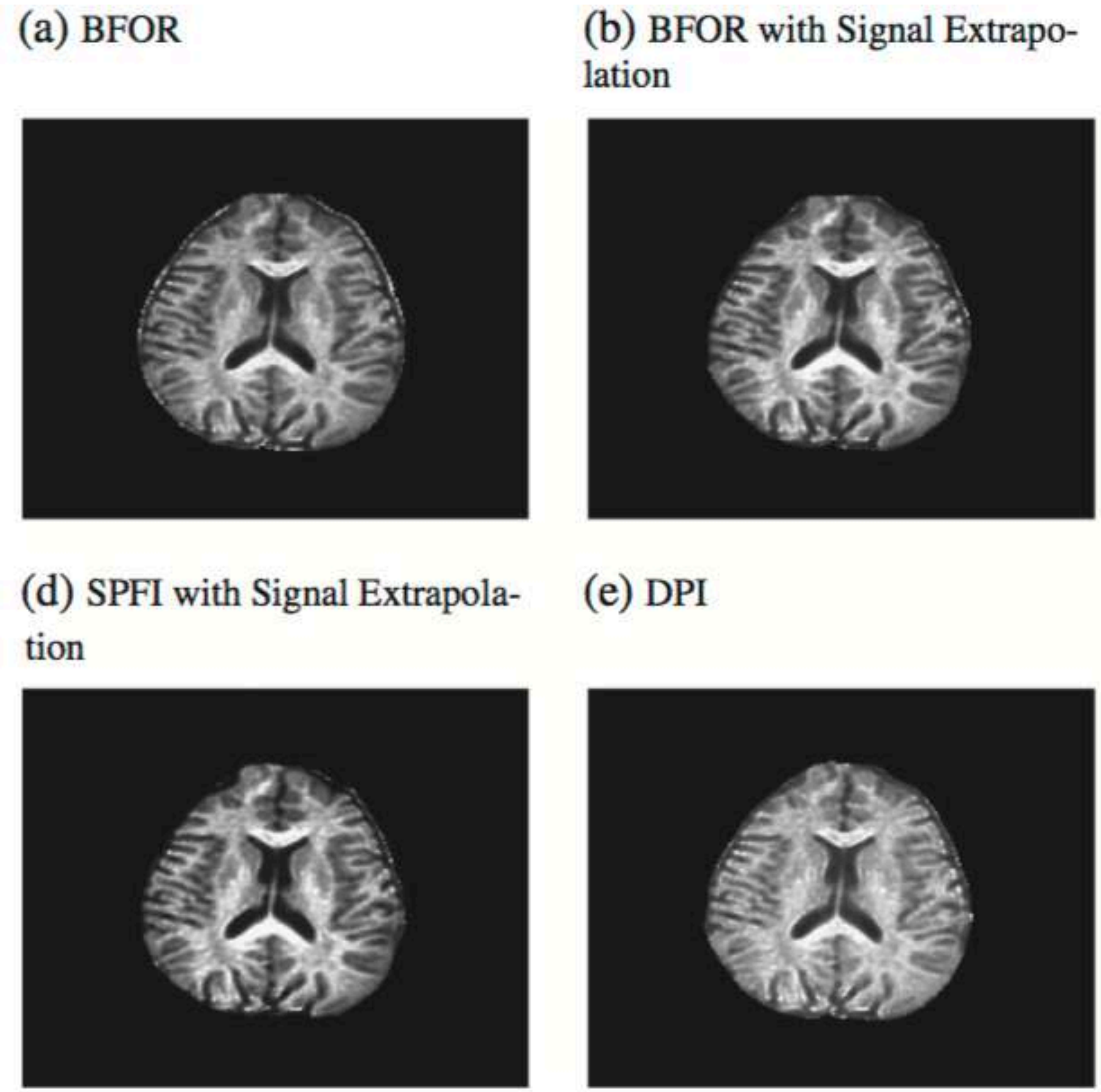




Multi-shell reconstruction in diffusion weighted imaging

5 shells, 126 data points



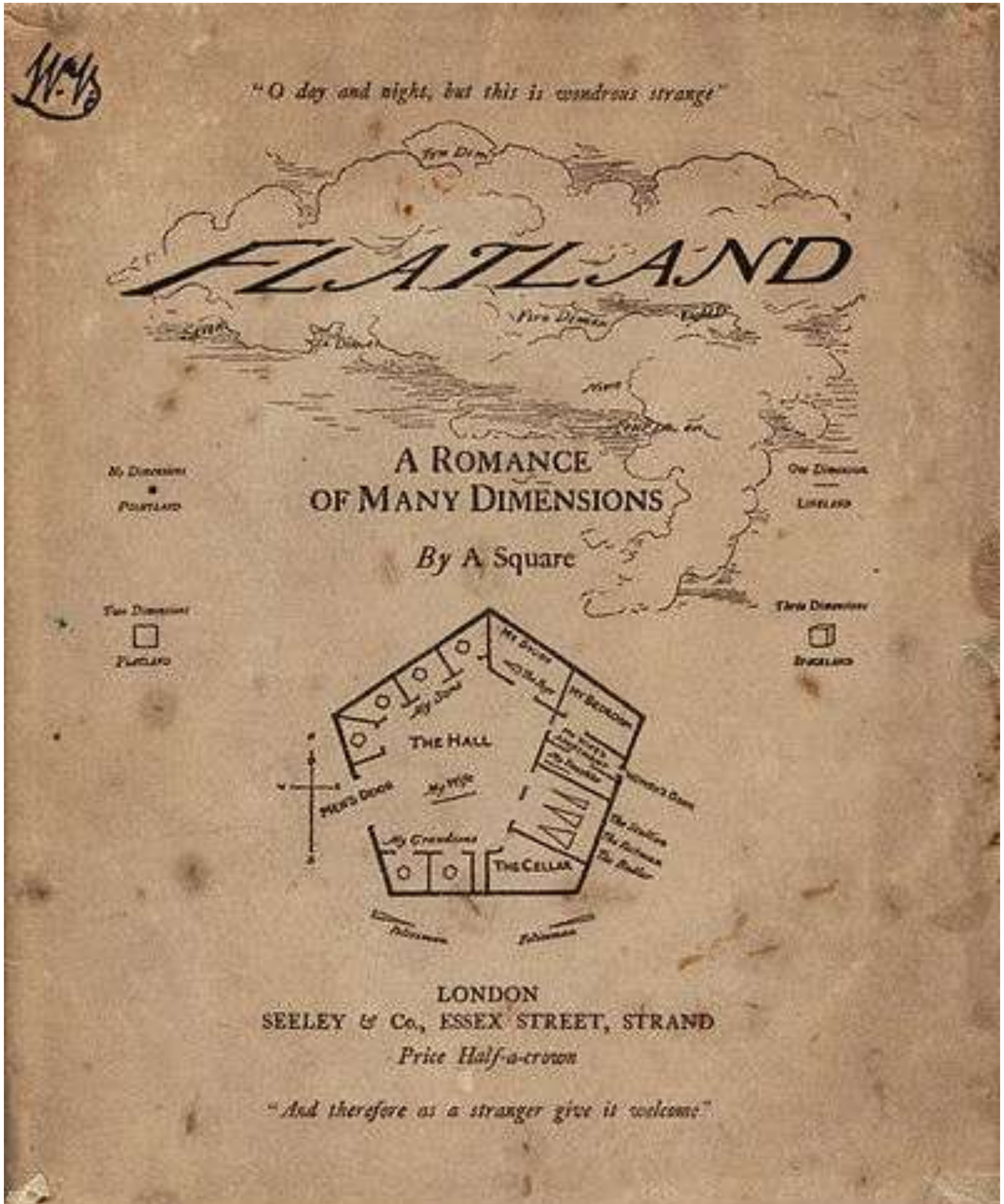


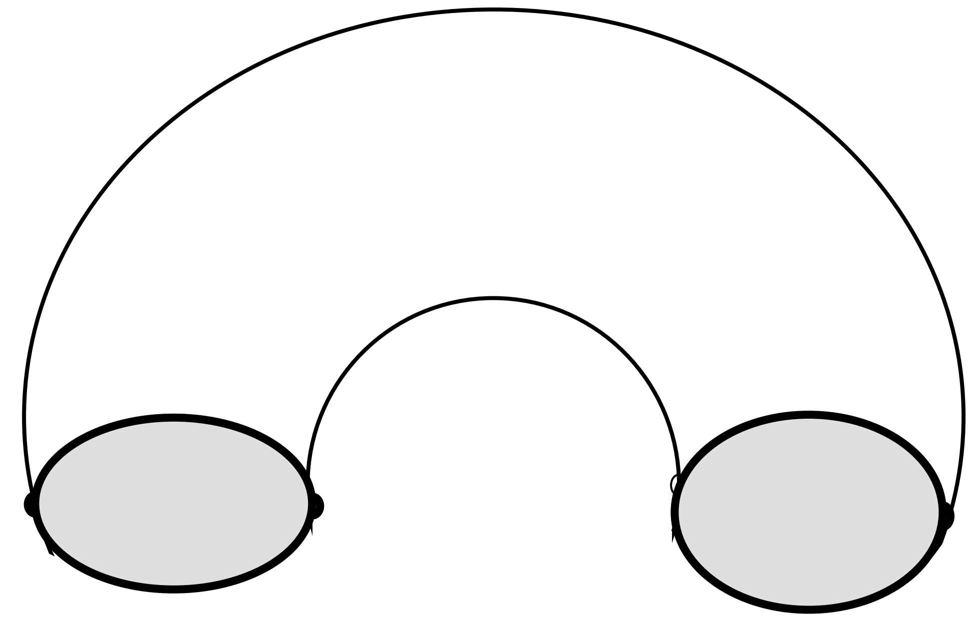
P_0 image

Hosseinbor et al. 2013 NeuoImage 64:650-670

Hyper Spherical Harmonic (SPHARM) Representation

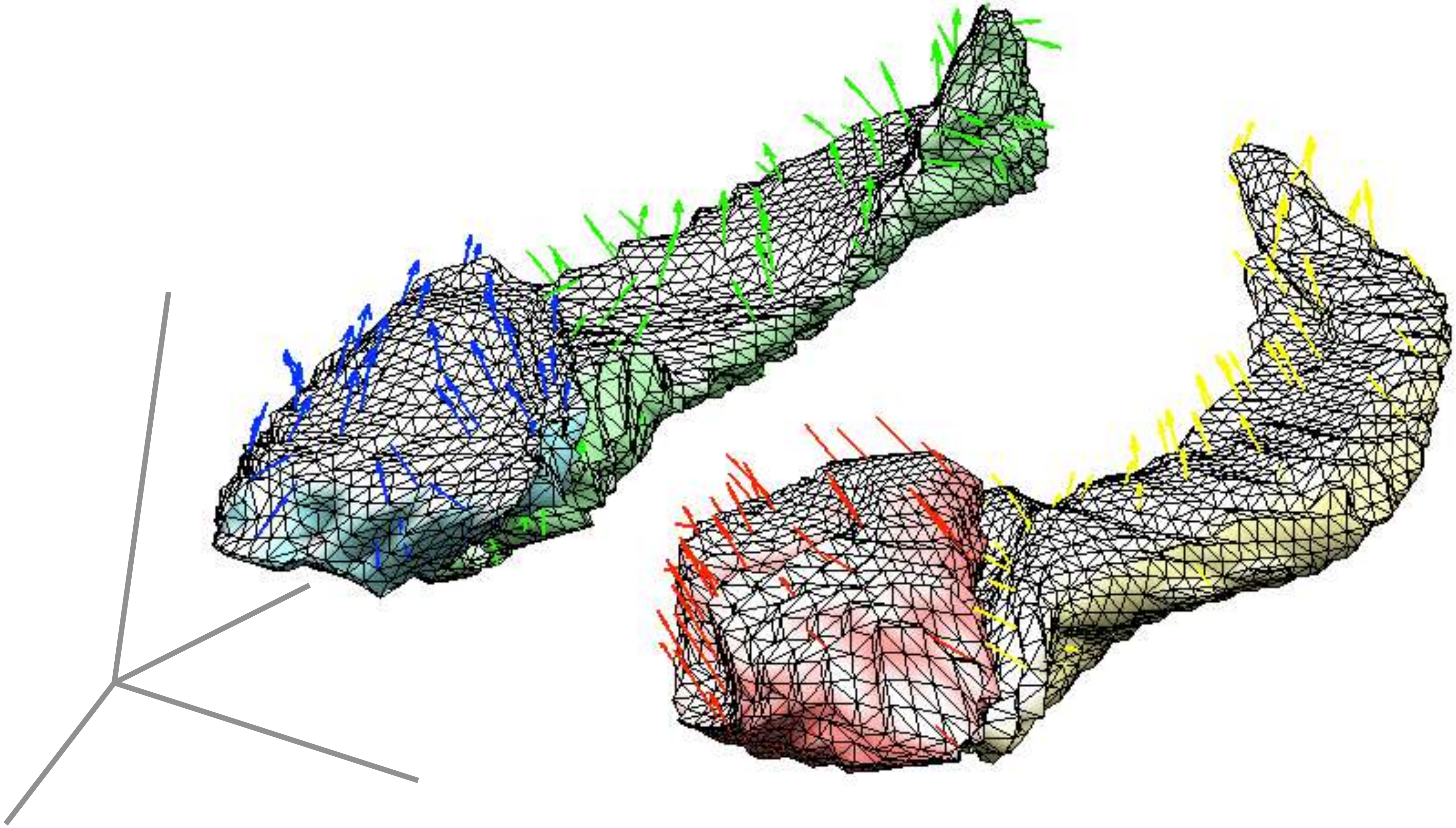
Flatland by Edwin A. Abbott, 1884





Connected in 3D

Question: Connect disconnected structures

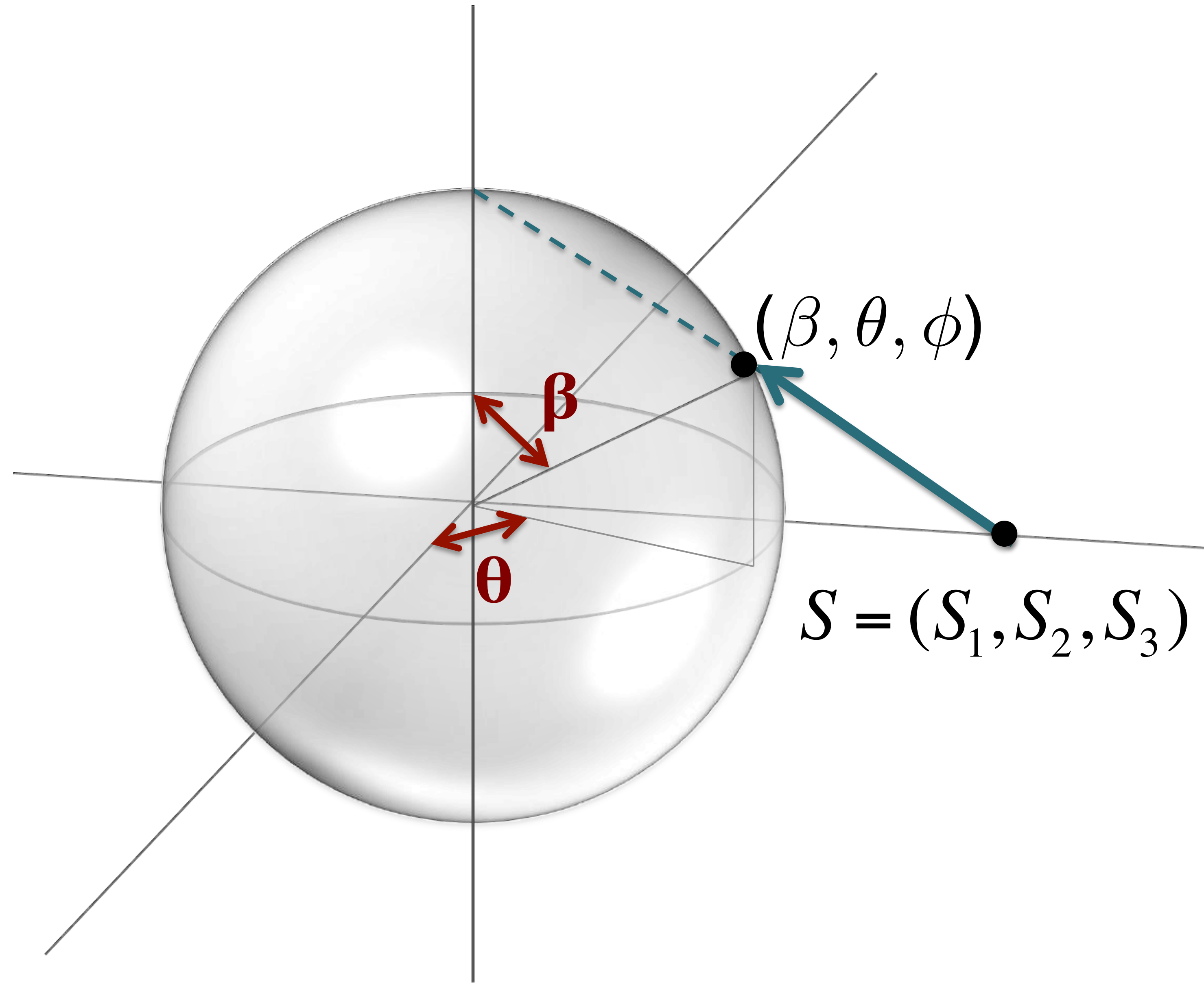


Connected in 4D

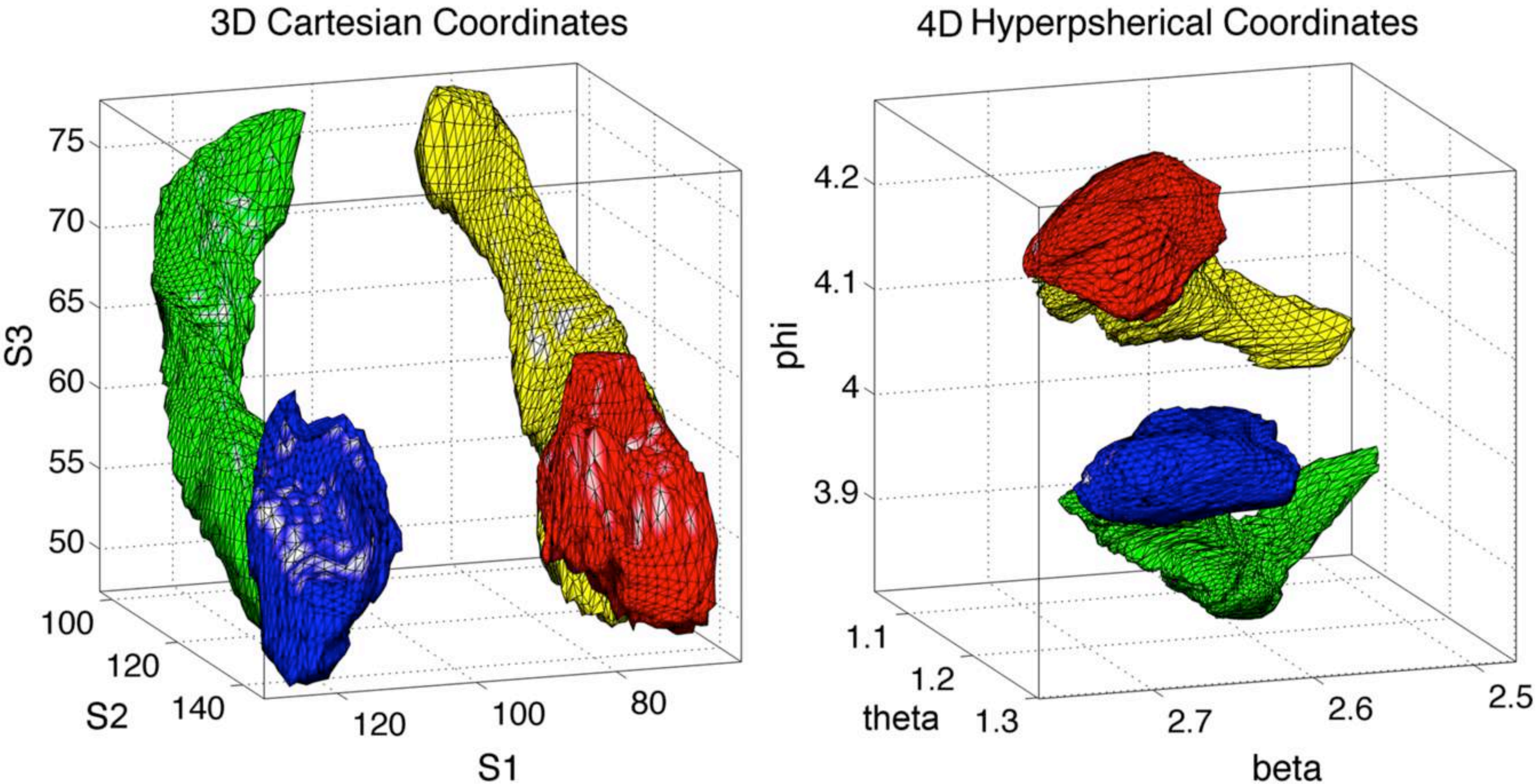
Question: Connect disconnected structures

3D stereographic projection

4D stereographic projection



4D stereographic projection



Hyper Spherical harmonic representation

3D coordinates $S = (S_1, S_2, S_3)$

$$S_i = \sum_{n=0}^{N} \sum_{l=0}^{n} \sum_{m=-l}^{l} C_{nlm}^{i} Z_{nl}^{m}(\beta, \theta, \phi)$$

Spherical angles of a hypersphere

$$Z_{nl}^{m}(\beta,\theta,\phi) = 2^{l+1/2}\sqrt{\frac{(n+1)\Gamma(n-l+1)}{\pi\Gamma(n+l+2)}}\Gamma(l+1)\sin^{l}\beta\; C_{n-l}^{l+1}(\cos\beta)\; Y_{l}^{m}(\theta,\phi)$$

Gegenbauer polyonomials

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\pi} Z_{nl}^{m}(\Omega) Z_{n'l'}^{m'*}(\Omega)\sin^2\beta\sin\theta\, d\beta\, d\theta\, d\phi = \delta_{nn'}\delta_{ll'}\delta_{mm'}$$

Hosseinbor et al., 2015 Medical Image Analysis 22:89-101

SPHARM mean squared error. *1764 parameters*

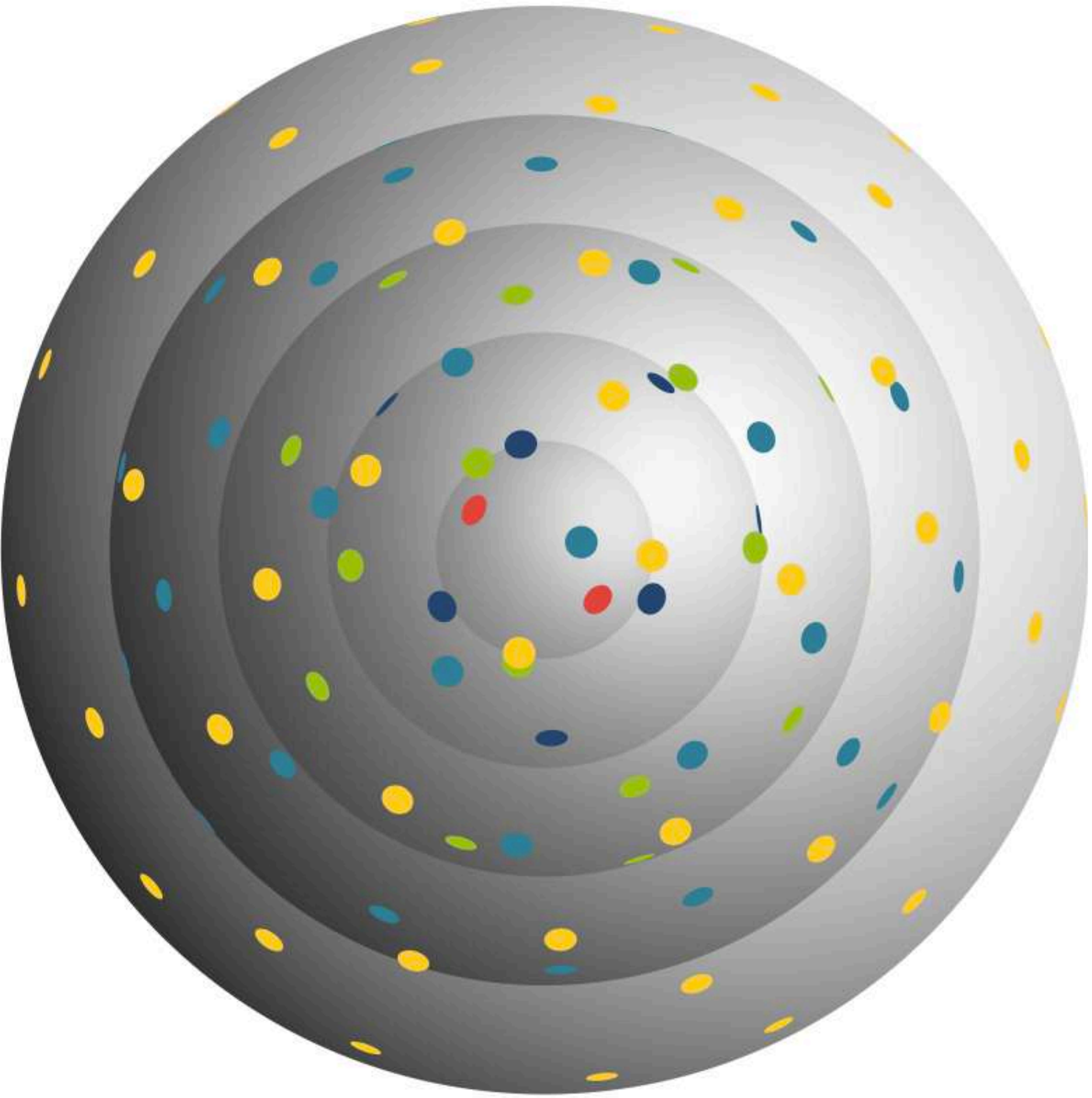
	MSE_{SPHARM}
Left Amygdala	0.0843 ± 0.0183
Right Amygdala	0.0941 ± 0.0165
Left Hippocampus	0.364 ± 0.732
Right Hippocampus	0.192 ± 0.314

HyperSPHARM mean squared error. *140 parameters*

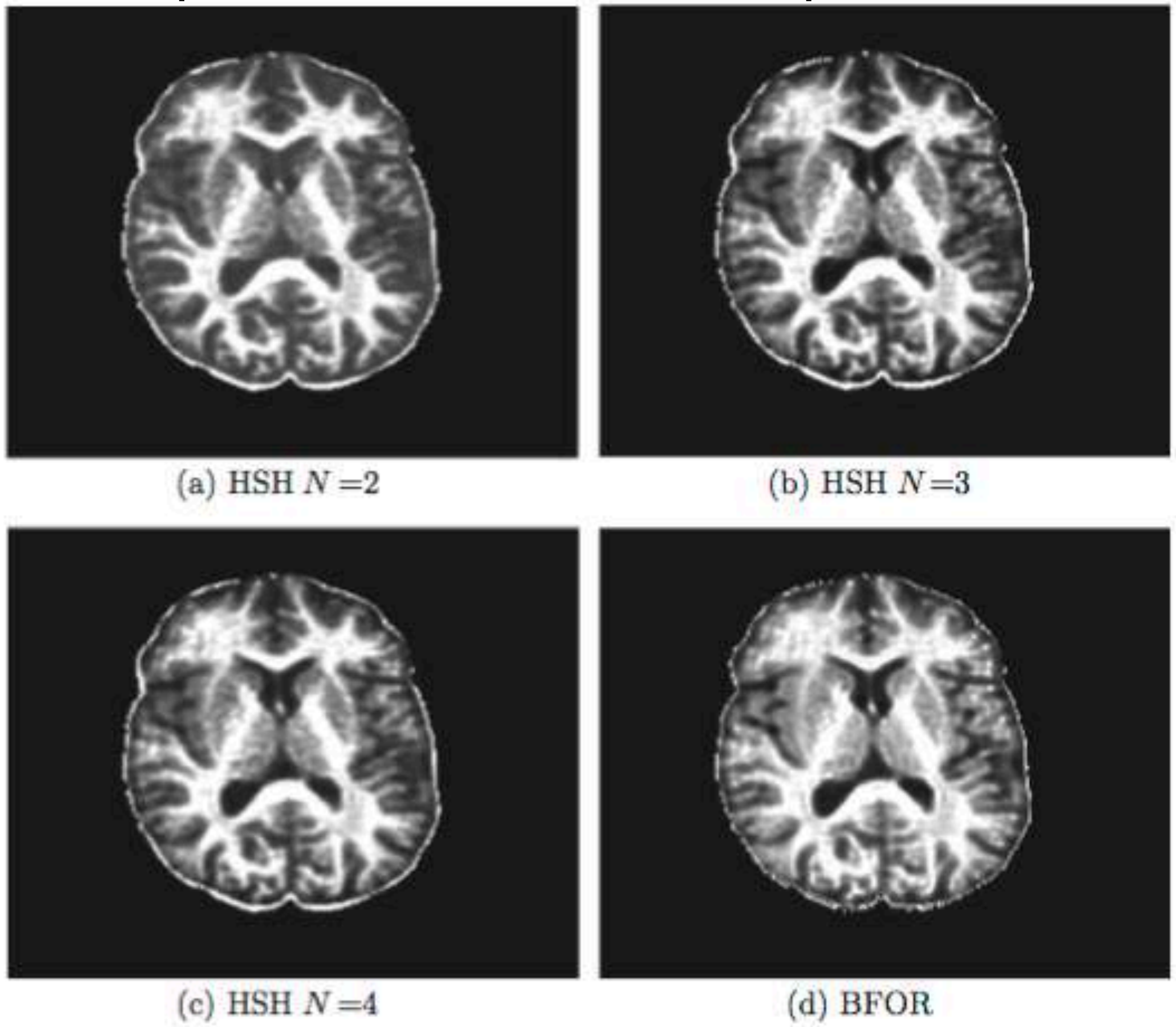
	MSE_{HSH}
Left Amygdala	0.147 ± 0.609
Right Amygdala	0.148 ± 0.632
Left Hippocampus	0.129 ± 0.511
Right Hippocampus	0.127 ± 0.504

Multi-shell reconstruction in diffusion weighted imaging

5 shells, 126 data points



14 parameters | 30 parameters

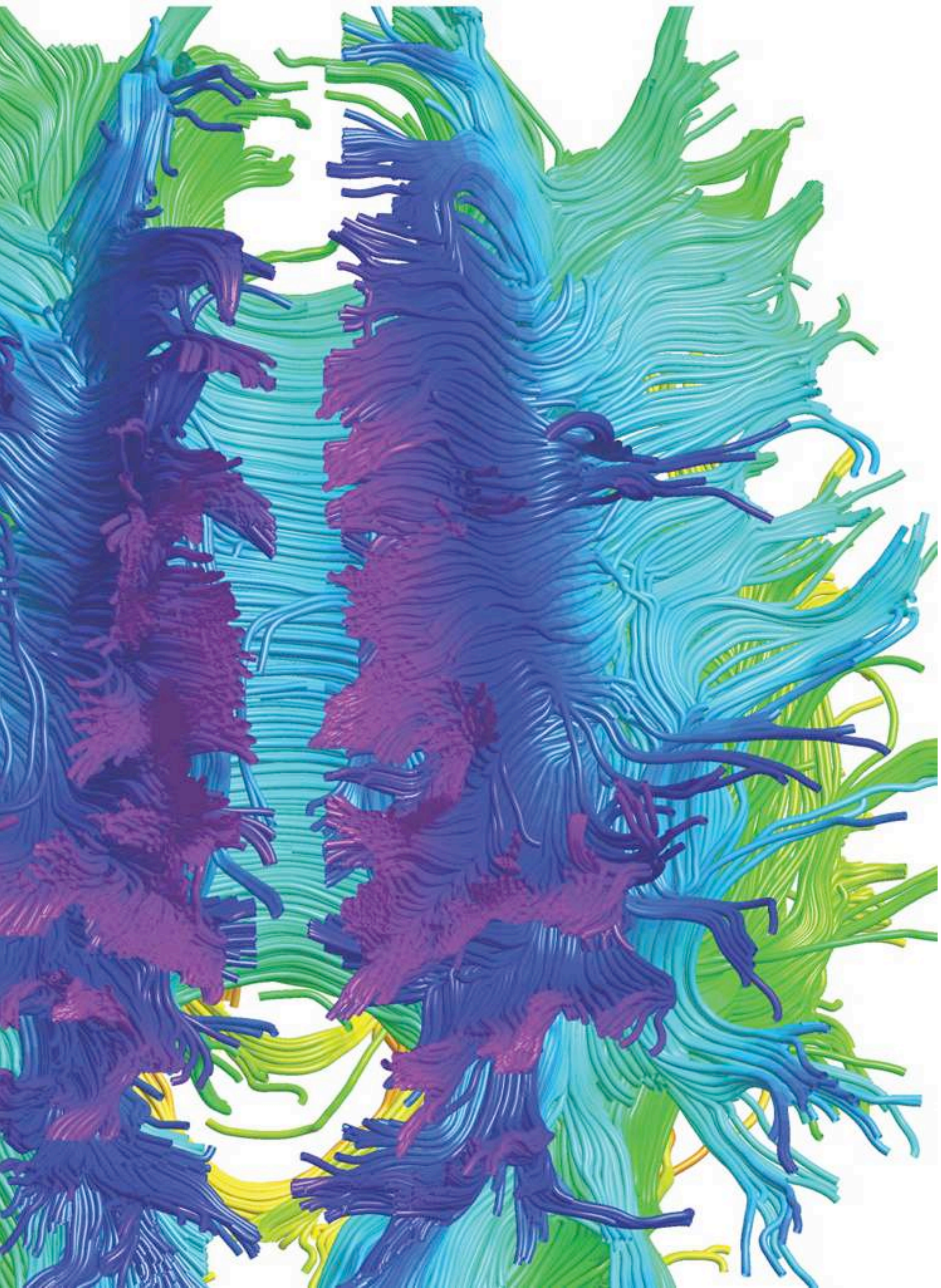


P_0 image

Hosseinbor et al., 2015 Medical Image Analysis 21:15-28

What Next?

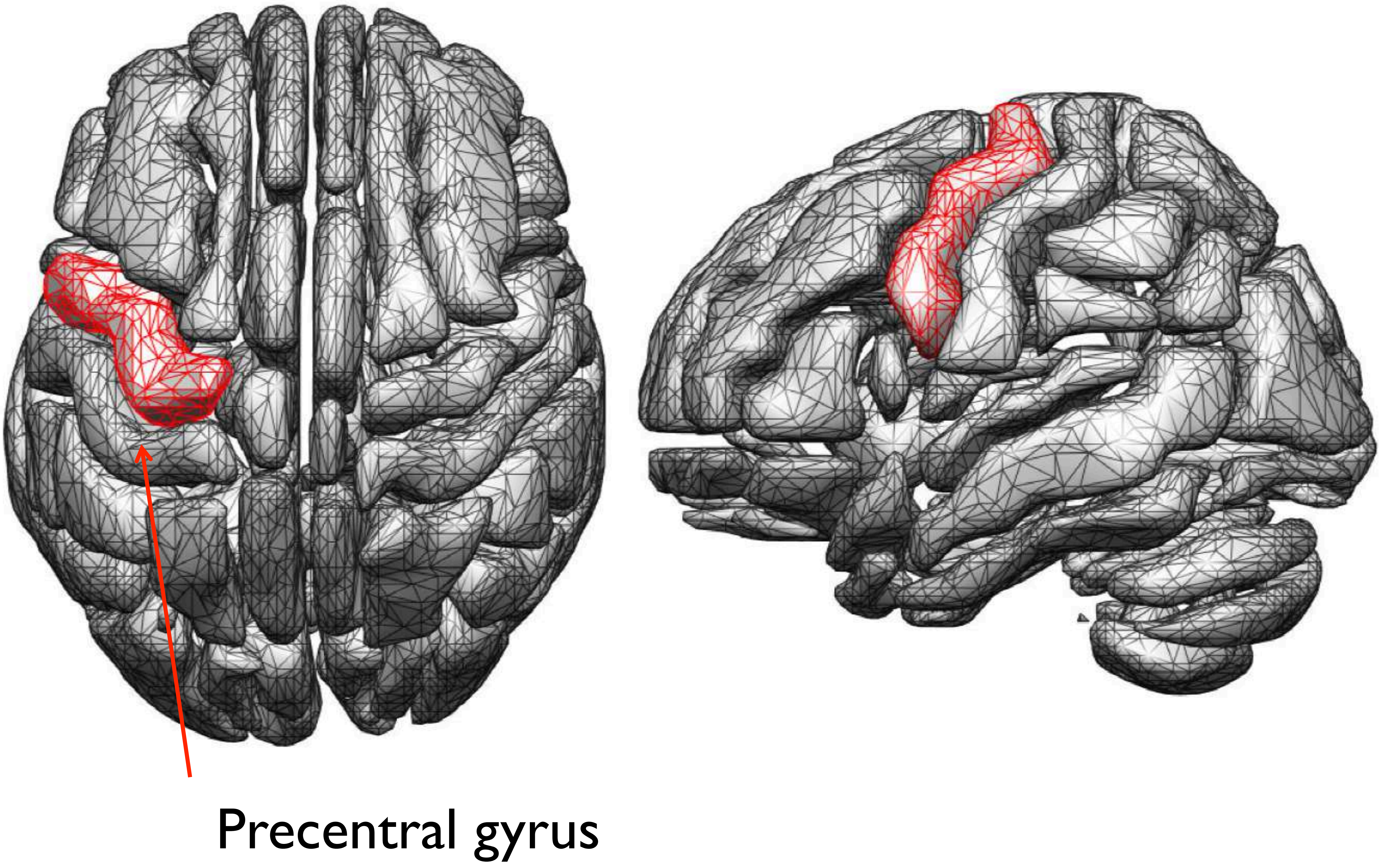
Extremely complex multiple disconnected anatomical structures



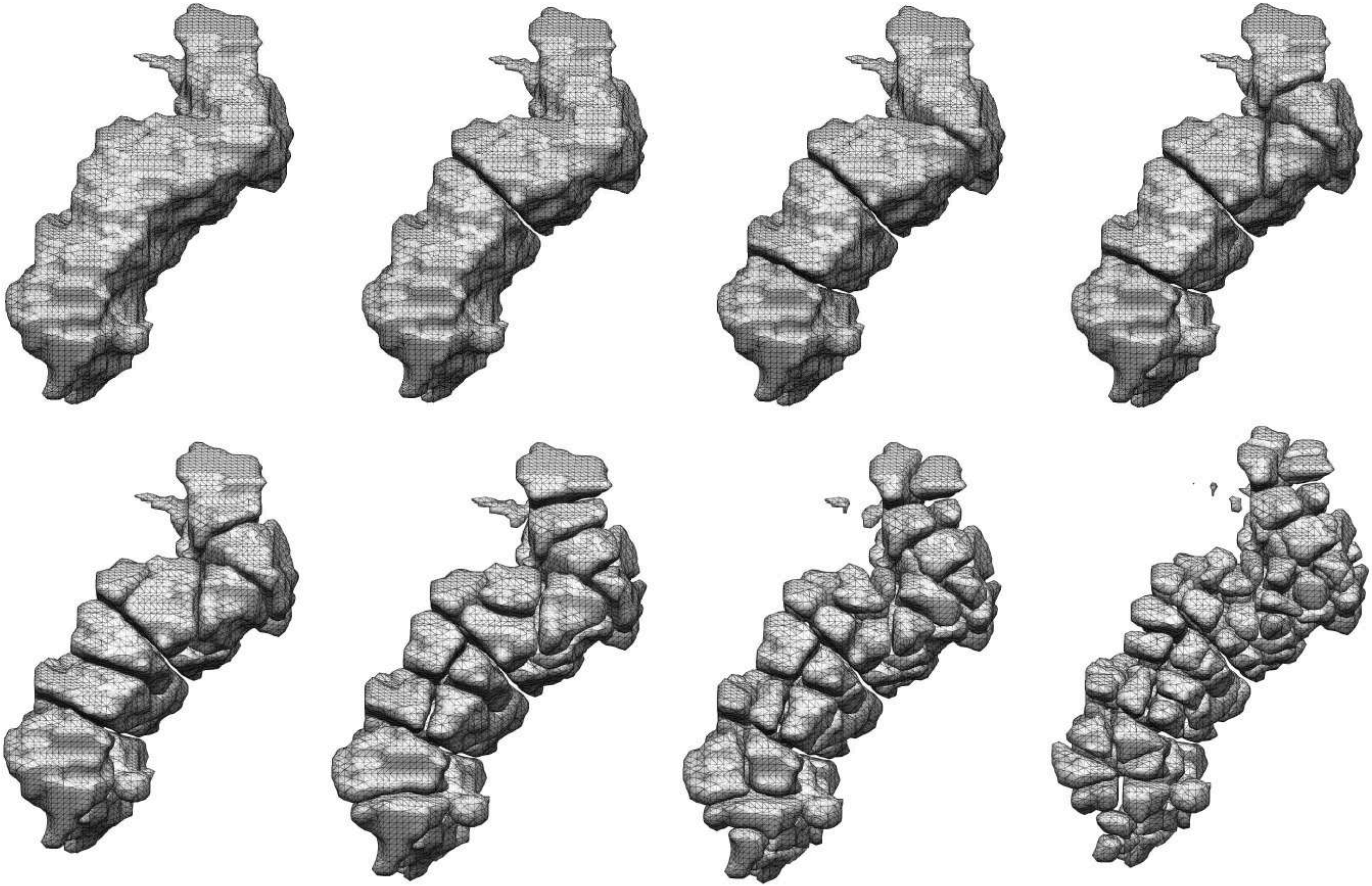
Challenge:

Parameterize the whole white matter fibers using HyperSPHARM.

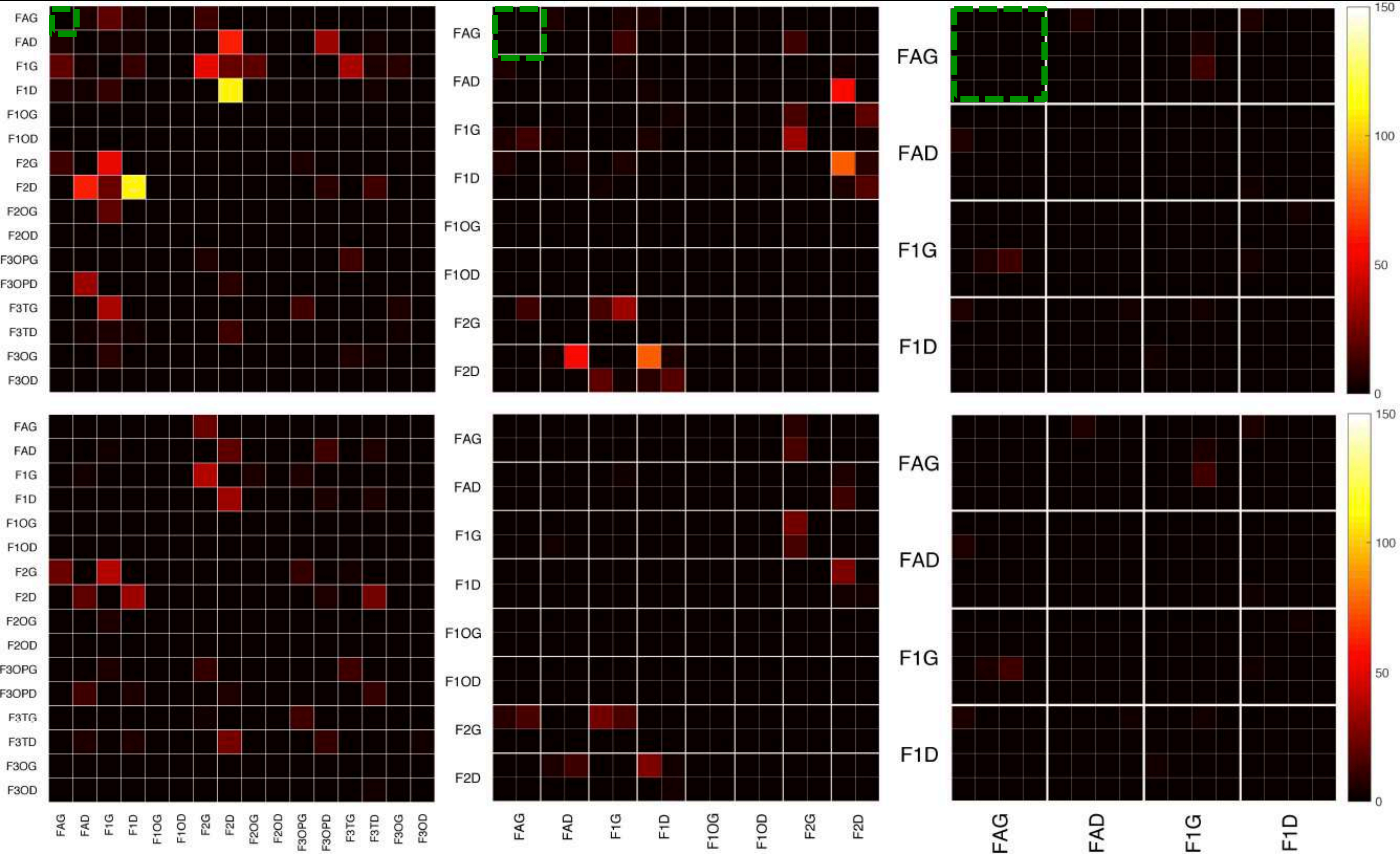
Standard brain parcellation with 116 regions



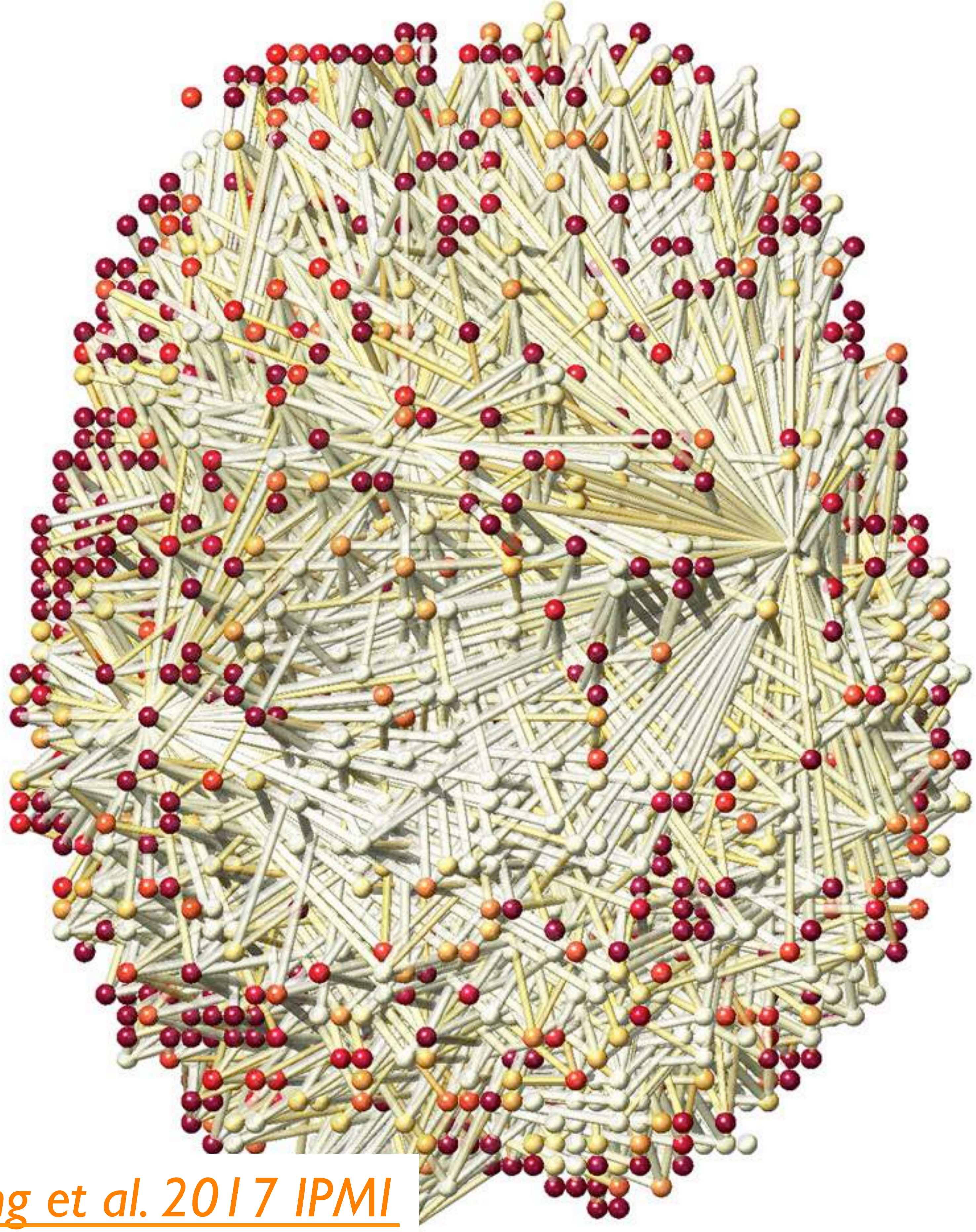
19-layer hierarchical brain parcellation



Hierarchical nested connectivity



Extremely dense brain network



+25000 nodes

+0.6 billion connections

HyperSPHARM representation in

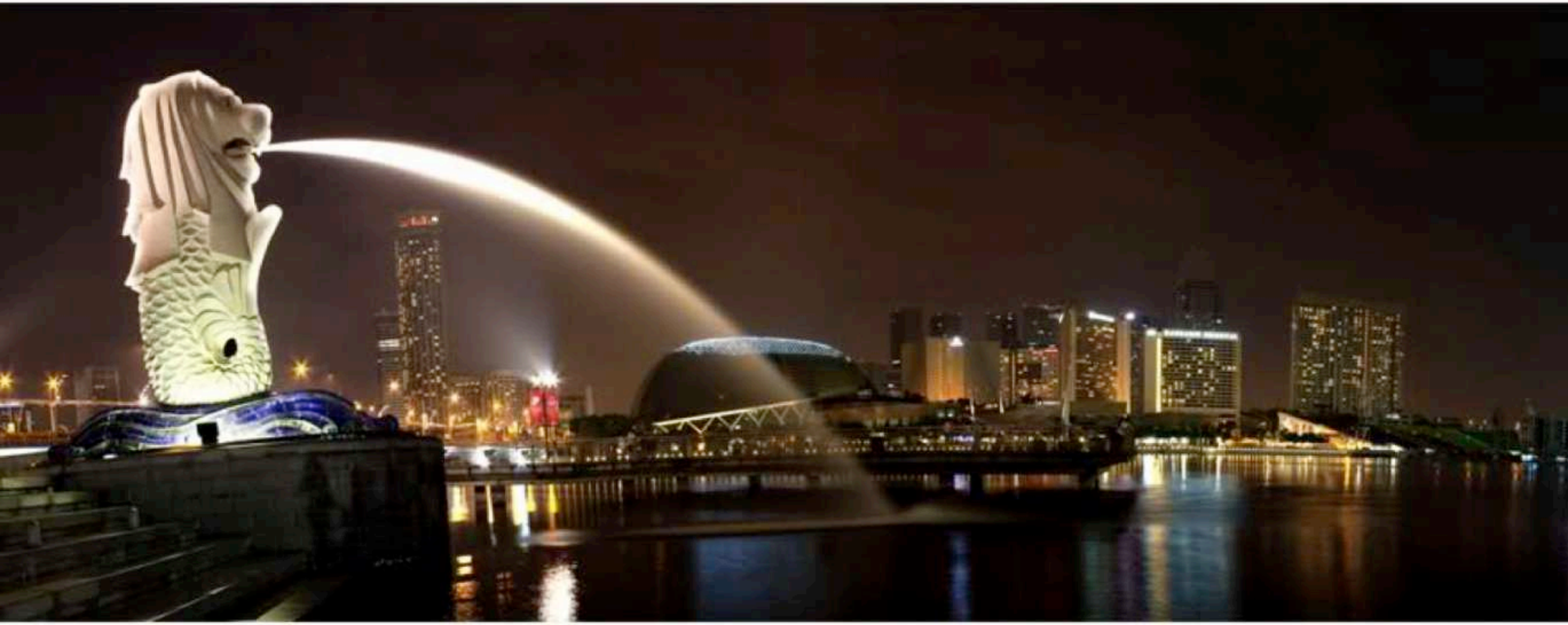
$$\mathbb{R}^3 \otimes \mathbb{R}^3$$

Chung et al. 2017 IPMI

http://nbiasite.wordpress.com

NONSTANDARD BRAIN IMAGE ANALYSIS

ORGANIZERS PROGRAM VENUE REGISTRATION



Satellite Meeting of 2018
OHBM Singapore

June 22-23, 2018