Program
UF/FSU Topology and Geometry Meeting

Florida State University
November 3 – November 4, 2023

Sponsored by the the De Witt Sumners Endowed Fund.

Program

Friday November 3rd, 2023:

2:30 *Refreshments - room 204*

3:05 Henry Adams - *Bridging metric geometry and topology.*

6:00 Banquet - Student Services Building (874 Traditions Way, Tallahassee, FL) room 203

Saturday November 4th, 2023:

8:30-9:50 **Talks Section 1**

8:30 Jonathan Bush - *Topological feature selection for time series*

9:00 Florian Stecker - *Anosov triangle reflection groups in SL(3,R)*

9:30 Deep Kundu - *Maps of Degree 1 and Critical Points*

9:40 Jared Miller - *Exploring Infinite Type Surfaces*

9:50-10:30 **Coffee Break**

10:30-12:00 **Talks Section 2**

10:30 Alex Dranishnikov - *On Gromov's PSC conjecture*

11:00 Jeremy Usatine - *Motivic integration for Artin stacks*

11:30 Hayden Hunter - *On Higher Dimensional Milnor Frames*

11:40 Aditya De Saha - *Fixed point-free involutions on boundary of RACGs*

11:50 Jonathan Cerqueira - *Discretizations of Sobolev Metrics on the Space of Closed Curves*

12:00-1:30 **Lunch**

1:30-3:00 **Talks Section 3**

1:30 Oishee Banerjee - *Cohomological stability and some arithmetic consequences*

2:00 Hubert Wagner - *Persistence computations for 3D images with billions of voxels*

2:30 Emmanuel Hartman - *Formulations of the Wasserstein-Fisher-Rao Distance and their Implications*

2:40 Mao Nishino - *Unbalanced Constrained Optimal Transport*

2:50 Audrey Nash - *Deciphering Taste Sensation: Neural Spike Train Analysis for Chemosensory Coding*

3:00-3:30 **Coffee Break**

3:30-4:30 **Talks Section 4**

3:30 Thang Nguyen - *Local rigidity of lattice actions on flag manifolds*

4:00 Nursultan Kuanyshov - *The LS-category of homomorphism of almost nilpotent groups*

4:10 Ethan Semrad - *Simplicial Complexes from Weighted Hypergraphs*

4:20 Sucharita Mallick - *Chromatic numbers of Borsuk graphs using anti-Vietoris-Rips thickenings*

Colloquium Talk

Henry Adams, University of Florida
Bridging metric geometry and topology

The goal of this talk is to show how tools from topology can be used to bound quantities arising in metric geometry. I'll begin by introducing the Gromov-Hausdorff distance, which is a way to measure the 'distance' between two metric spaces. Next, I will explain the nerve lemma, which says when a cover of a space faithfully encodes the shape of that space. Then, I'll use the nerve lemma to lower bound the Gromov-Hausdorff distance between a manifold and a finite subset thereof. I'll conclude by advertising a few other problems at the intersection of metric geometry and topology.

Talks and Abstracts

Jonathan Bush, University of Florida
Topological feature selection for time series

Given a time series observed from a potentially unknown dynamical system, Takens' theorem implies that a sliding window embedding of the time series will preserve the topology of an orbit of the underlying dynamical system. The topology of such an orbit is then summarized by persistent homology. In this talk, we will consider the problem of identifying components of time series most responsible for cyclic dynamics observed in sliding window embeddings. In this setting, I will show that derivatives of the persistent homology may be computed explicitly and describe a simple algorithm for gradient descent. As an example, we will consider neuronal data from the model organism C. elegans and identify subsets of neurons driving global cyclic brain dynamics in the spirit of dimensionality reduction.

Florian Stecker, Florida State University
Anosov triangle reflection groups in SL(3,R)

We identify subgroups of the Lie group SL(3,R) which are discrete in the subspace topology and isomorphic to the abstract group generated by reflections along the sides of a hyperbolic triangle. If we require a slightly stronger property (Anosov groups), we can even find all of these groups, by studying the dynamics of their action on the projective plane. This gives us a simple model to help understand deformations of surface group representations. All of it is joint work with Gye-Seon Lee and Jaejeong Lee.

Deep Kundu, University of Florida
Maps of Degree 1 and Critical Points

Let ”Crit M” denote the minimal number of critical points (not necessarily non-degenerate) on a closed smooth manifold M. We are interested in the evaluation of ”Crit”. It is worth noting that we do not know yet whether ”Crit M” is a homotopy invariant of M. This makes the research of ”Crit” a challenging problem. In particular, we pose the following question: given a map f from M to N of degree 1 of closed manifolds, is it true that Crit M is greater than or equal to Crit N? We prove that this holds in dimension 3 or less. Some high dimension examples are considered. Note also that an affirmative answer to the question implies the homotopy invariance of ”Crit”; this simple observation is a good motivation for the research.

Jared Miller, Florida State University
Exploring Infinite Type Surfaces

A surface is said to be of finite type if its fundamental group is finitely generated; otherwise we say it is of infinite type. Infinite type surfaces are, in a sense, much more mysterious than finite type surfaces. There has recently been a surge of interest in mapping class groups for infinite type surfaces, but many questions remain open. In this talk we will discuss some recent results.

Alex Dranishnikov, University of Florida
On Gromov's PSC conjecture

Gromov's conjecture states that any positive scalar curvature closed n-manifold has the universal cover with macroscopic dimension at most n-2. We prove this conjecture for spin manifolds with certain fundamental groups. In particular, we prove it for right-angled Artin groups.

Jeremy Usatine, Florida State University
Motivic integration for Artin stacks

A standard method for studying a singular variety is to resolve it by a smooth variety and to then relate invariants of the singular variety to invariants of the smooth one. Motivic integration provides powerful tools for obtaining such a relationship. Motivated by the McKay correspondence, I will describe a context in which interesting varieties admit natural resolutions of singularities by Artin stacks. This suggests a need for versatile tools in studying these 'stacky' resolutions of singularities. I will discuss joint work with M. Satriano in which we use motivic integration to provide such tools.

Hayden Hunter, University of Florida
On Higher Dimensional Milnor Frames

A classic result of Milnor shows that any 3-dimensional unimodular metric Lie algebra admits an orthonormal frame with at most three non-zero structure constants. We refer to these frames as Milnor frames. We define extensions of Milnor frames into higher dimensions. We determine that the higher dimensional analogue of Milnor's result can only hold on the 3-dimensional Heisenberg Lie algebra directly summed with an abelian Lie algebra.

Aditya De Saha, University of Florida
Fixed point-free involutions on boundary of RACGs

A while back a question was asked by Bogdan Nica "Given a hyperbolic group G, does there exist a fixed-point-free involution on the visual boundary ∂G?" We don't have an answer to that, but we give a positive answer for the class of right-angled Coxeter groups.

Jonathan Cerqueira, Florida State University
Discretizations of Sobolev Metrics on the Space of Closed Curves

Sobolev metrics can be used to make the space of closed curves, and other spaces of shapes, into Riemannian manifolds. As these metrics are for infinite dimensional Riemannian manifolds, key theorems such as Hopf-Rinow fail, complicating their study. If related discrete metrics can be found, we regain these key tools by working with the discretizations in place of the smooth metrics. First, we will define the Sobolev metrics for closed curves in the plane and briefly explore their properties. Following, we will showcase some discretizations of these and their similar properties.

Oishee Banerjee, Florida State University
Cohomological stability and some arithmetic consequences

How do we describe the topology of the space of all nonconstant holomorphic (respectively, algebraic) maps F: X$\to$Y from one complex manifold (respectively, variety) to another? What is, for example, their cohomology? Such problems are old but difficult, and are nontrivial even when the domain and range are Riemann spheres. In this talk, I will give a broad overview of how these problems relate to other parts of mathematics such as spaces of polynomials, arithmetic (e.g., the geometric Batyerv-Manin type conjectures) and algebraic geometry (e.g., moduli spaces of elliptic fibrations, of smooth sections of a line bundle, etc). Time permitting, I will show how one can fruitfully attack such problems by incorporating techniques from homotopy theory to the holomorphic/algebraic world (e.g., by constructing a new spectral sequence).

Hubert Wagner, University of Florida
Persistence computations for 3D images with billions of voxels

Analyzing point-cloud data with persistent homology is perhaps the most popular TDA setup. However, there are interesting applications for image data, especially grayscale 3D images coming from medical scanners. Such datasets can be highly detailed and contain billions of voxels and more. I will report recent progress on efficient computations for such data, focusing on the new version of my 'Cubicle' software.

Emmanuel Hartman, Florida State University
Formulations of the Wasserstein-Fisher-Rao Distance and their Implications

In this talk we give a brief tour of different formulations of the Wasserstein-Fisher-Rao (WFR) unbalanced optimal transport distance. We will discuss the implications of each formulation, whether it be algorithms for computing the distance or other insights into the space of measures equipped with the WFR metric.

Mao Nishino, Florida State University
Unbalanced Constrained Optimal Transport

In this talk, we explore the intricacies and developments in unbalanced optimal transport (OT). Beginning with a fundamental introduction to unbalanced OT, the discussion progresses to delve into the Wasserstein Fisher-Rao distance, which offers a comprehensive perspective on the space of measures. A key highlight of the talk is the elucidation of the correspondence between convex closed curves and Radon measures called length measures. As we venture deeper, the conversation expands from the basic constraints of length measure to the complexities of more generalized constraints in the realm of OT. A significant portion of the talk is dedicated to the understanding of geodesics, focusing on the Fenchel-Rockafellar theorem and its implications. Concluding, we reflect upon the current state of knowledge regarding the existence of geodesics in this domain. This talk promises a profound understanding of unbalanced constrained OT, bridging foundational concepts with contemporary advancements.

Audrey Nash, Florida State University
Deciphering Taste Sensation: Neural Spike Train Analysis for Chemosensory Coding

Characterizing neural spike train data from individual neurons with a dual focus on spiking rate and temporal distribution allows us to discern the neuron's capacity to differentiate between stimuli. This presentation will provide a concise overview of a methodology employed in identifying chemosensory coding neurons. Subsequently, we will delve into the utilization of metric-based analysis techniques, coupled with optimal transport applications, to unveil patterns in tastant coding across neural populations and assess the respective influences of spiking rate and temporal phase on taste decoding.

Thang Nguyen, Florida State University
Local rigidity of lattice actions on flag manifolds

A lattice, as a subgroup of a linear group, naturally acts on projective space and more generally flag manifolds by homeomorphisms. Are these the only actions, up to (semi)conjugation, of lattices on flag manifolds? We show that the answer is yes locally around the natural actions. Namely, a sufficiently small perturbation of a natural action is necessarily conjugate to the natural one. From a joint work with C. Connell, M. Islam, and R. Spatzier.

Nursultan Kuanyshov, University of Florida
The LS-category of homomorphism of almost nilpotent groups

The Lusternik-Schnirelmann category (LS-category) is a numerical homotopy-invariant of a topological space. Nevertheless, the definition of the LS-category can be extended to discrete groups. In the 50s Eilenberg and Ganea proved that the LS-category of a discrete group G equals the cohomological dimension of G, $cat(G) = cd(G)$. Jamie Scott conjectured the same for discrete group homomorphism $\phi : G \to H$, $cat(\phi) = cd(\phi)$. In this talk, we discuss the recent progress on this conjecture. In particular, why the conjecture holds for homomorphisms between virtually nilpotent groups and for homomorphisms between almost nilpotent groups.

Ethan Semrad, Florida State University
Simplicial Complexes from Weighted Hypergraphs

Hypergraphs are a generalization of the standard graph that capture multi-way relationships in data. Data with potential hypergraph structure can show up in many biological systems. The Dowker simplicial complexes of hypergraphs have structural similarities to the clique-graphs and the line-graphs created by reduction techniques. In addition, the persistence homology of these Dowker complexes also allows us to get a lower bound between the Gromov-Hausdorff distance of the corresponding hypergraphs. We will explore these underlying similarities of the simplicial complexes and see how they can potentially assist in the interpretation of structured network data.

Sucharita Mallick, University of Florida
Chromatic numbers of Borsuk graphs using anti-Vietoris-Rips thickenings

In this talk, I will introduce Borsuk graphs, (circular) chromatic numbers, and anti-Vietoris-Rips thickenings. I will discuss the homotopy types of the anti-VR thickenings built on S^n in a range of scales. Using topological obstructions, I will show that for $k > n$, a graph homomorphisms from the Borsuk graph on S^k to the Borsuk graph on S^n can exist only if the scale for the latter Borsuk graph is sufficiently relaxed. Can similar techniques be used to provide new lower bounds on the chromatic number of Borsuk graphs? This is joint work with Henry Adams, Alex Elchesen and, Michael Moy.

Local info

- All talks will be held in room 101 in Love building.
- Refreshments, coffee and lunch breaks will take place in room 204 in Love building.
- The conference banquet will be held on Friday November 3rd in the Student Services Building (874 Traditions Way, Tallahassee, FL) room 203 at 6:00pm.
- On campus wifi is available through the eduroam network.