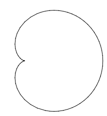
MAC 2313 Calculus 3 **Test 3** 30 Nov 2000

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. For the vector field $\mathbf{F} = \langle ye^{xy} + \cos(x+y), xe^{xy} + \cos(x+y)\rangle$ find a scalar field f so that $\mathbf{F} = \nabla f$ (that is, grad f) and use f to compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve that starts at the origin, follows the x-axis to $(3,0)$, then counter-clockwise along $x^2 + y^2 = 9$ to $(-3,0)$, then along the line $x = -3$ to its ending point $(-3,5)$.

2. For the helix $\mathbf{r}(t) = \langle \cos t, \sin t, t\rangle$ with $0 \le t \le 4\pi$.
   a. Compute the velocity and acceleration.
   b. Compute the arclength.
   c. Compute the line integral $\int_H \mathbf{F} \cdot d\mathbf{r}$ where H is the helix and $\mathbf{F} = \langle -y, x, 5\rangle$



3. Find the area of the cardiod (see graph to right) $r = 1 + \cos\theta$.

4. Compute the flux of the vector field $\mathbf{F} = \langle 2, 3, 5\rangle$ through each of the rectangular regions S in (a)–(d).
   a. S is a horizontal square of side 1 with one corner at $(0,0,2)$, above the first quadrant of the xy-plane oriented upward.
   b. S is a horizontal square of side 2 with one corner at $(0,0,3)$, above the third quadrant of the xy-plane oriented downward.
   c. S is a square of side $\sqrt{2}$ with one corner at the origin, one edge along the positive x-axis, one along the negative z-axis, oriented in the negative y-direction.
   d. S is a square of side $\sqrt{2}$ with one corner at the origin, one edge along the positive y-axis, one corner at $(1,0,1)$, oriented upward.

5. For the integral below, sketch the region of integration and then evaluate the integral by reversing the order of integration.

$$\int_0^1 \int_{e^y}^{e} \frac{x}{\ln x} \quad dx\,dy$$

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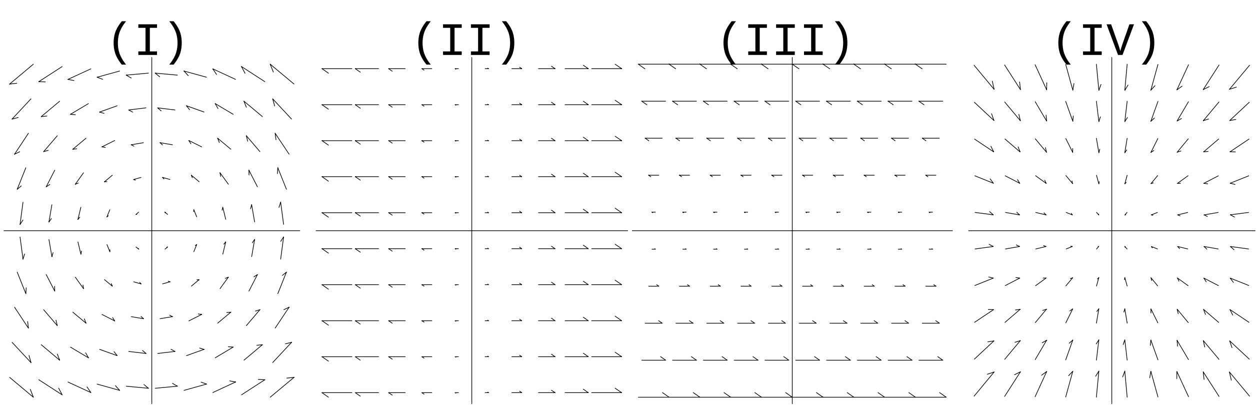
6. Find formula's for the vector fields below. (There are many possible answers)



There is more test on the otherside

7. By computing both sides in Green's Theorem, find the area of ellipse $x^2/a^2+y^2/b^2=1$. [Hint: $\mathbf{F}=\langle 0,x\rangle$, $x=a\cos t$ and $y=b\sin t$.]

8. Sketch the region and re-write the triple integral

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} x^2+y^2+z^2 \quad dzdydx$$

in both cylinderical and spherical co-ordinates but do **NOT** evaluate the integrals.

9. Give parameteric equations for C (including the range for t) when C is the curve described below.
a. The circle of radius 5, counter-clockwise, starting and ending at the x-axis.
b. The line starting at $(1,5,2)$ and ending at $(7,-3,12)$.
c. The unit circle starting at $\theta=\pi/2$ and going **clockwise** to $\theta=0$.
de. A counter-clockwise spiral starting at $(1,0)$ and going throught the point $(2,0)$ and ending at the point $(3,0)$. (The spiral makes two complete laps.)

10. Let $\mathbf{V}$ be the path-independent vector field pictured below. The vector field $\mathbf{V}$ associates with each point a unit vector pointing radially outward. The curves $A,B,\ldots G$ all have the direction shown. Consider the line integrals $\int_X \mathbf{V}\cdot d\mathbf{r}$ for $X=A,B,\ldots G$.
a. List all the line integrals which you expect to be zero.
b. List all the line integrals which you expect to be negative.
c. List all the line integrals which you expect to be positive in ascending order.